Straight Lines

EXERCISES

ELEMENTARY

Q.1 (1)
The vertices of triangle are the intersection points of these given lines. The vertices of Δ are $A(0,4)$, $B(1,2)$, $C(4,0)$

Now, $AB = \sqrt{(0-1)^2 + (4-1)^2} = \sqrt{10}$

$BC = \sqrt{(1-4)^2 + (0-1)^2} = \sqrt{10}$

$AC = \sqrt{(0-4)^2 + (0-4)} = 4\sqrt{2}$

$\because AB = BC$; $\therefore$ Δ is isosceles.

Q.2 (2) Mid point $\equiv \left(\frac{1+1}{2}, \frac{3-7}{2}\right) = (1,-2)$

Therefore required line is $2x - 3y = k \Rightarrow 2x - 3y = 8$.

Q.3 (1) Point of intersection $y = -\frac{21}{5}$ and $x = \frac{23}{5}$

$\therefore\ 3x + 4y = \frac{3(23) + 4(-21)}{5} = \frac{69-84}{5} = -3$.

Hence, required line is $3x + 4y + 3 = 0$.

Q.4 (1)

$(h-3)^2 + (k+2)^2 = \left|\frac{5h - 12k - 13}{\sqrt{25+144}}\right|$.

Replace (h, k) by (x, y), we get

$13x^2 + 13y^2 - 83x + 64y + 182 = 0$, which is the required equation of the locus of the point.

Q.5 (2)
Let point be (x_1, y_1), then according to the condition

$\frac{3x_1 + 4y_1 - 11}{5} = -\left(\frac{12x_1 + 5y_1 + 2}{13}\right)$

Since the given lines are on opposite sides with respect to origin, hence the required locus is $99x + 77y - 133 = 0$

Q.6 (1) Let the point be (x, y). Area of triangle with points $(x, y), (1,5)$ and $(3,-7)$ is 21 *sq. units*

$\therefore\ \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 5 & 1 \\ 3 & -7 & 1 \end{vmatrix} = 21$

Solving; locus of point (x,y) is $6x + y - 32 = 0$.

Q.7 (3) Here $c = -1$ and $m = \tan\theta = \tan 45^o = 1$
(Since the line is equally inclined to the axes, so $\theta = 45^o$)

Hence equation of straight line is $y = \pm(1.x) - 1$

$\Rightarrow x - y - 1 = 0$ and $x + y + 1 = 0$.

Q.8 (2)
A line perpendicular to the line $5x - y = 1$ is given by $x + 5y - \lambda = 0 = L$, (given)

In intercept form $\frac{x}{\lambda} + \frac{y}{\lambda/5} = 1$

So, area of triangle is $\frac{1}{2} \times$ (Multiplication of intercepts)

$\Rightarrow \frac{1}{2}(\lambda) \times \left(\frac{\lambda}{5}\right) = 5 \Rightarrow \lambda = \pm 5\sqrt{2}$

Hence the equation of required straight line is $x + 5y = \pm 5\sqrt{2}$.

Q.9 (2)
Let the required equation is $y = -x + c$ which is perpendicular to $y = x$ and passes through (3, 2). So $2 = -3 + c \Rightarrow c = 5$. Hence required equation is $x + y = 5$

Q.10 (1)The equation of any straight line passing through (3, –2) is $y + 2 = m(x - 3)$ .....(i)

The slope of the given line is $-\sqrt{3}$.

So, $\tan 60^o = \pm\frac{m - (-\sqrt{3})}{1 + m(-\sqrt{3})}$

On solving, we get $m = 0$ or $\sqrt{3}$

Putting the values of *m* in (i), the required equation of lines are $y + 2 = 0$ and $\sqrt{3}x - y = 2 + 3\sqrt{3}$.

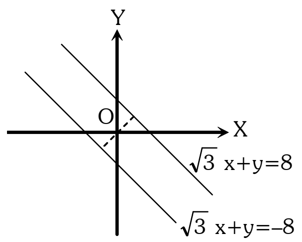
Q.11 (1)
Let the intercept be a and $2a$, then the equation of line is $\frac{x}{a} + \frac{y}{2a} = 1$, but it also passes through (1, 2), therefore $a = 2$.

Hence the required equation is $2x + y = 4$.

Q.12 (1)

Slope $=-\sqrt{3}$

$\therefore$ Line is $y=-\sqrt{3}x+c \Rightarrow \sqrt{3}x+y=c$



Now $\frac{c}{2}=|4| \Rightarrow c=\pm 8 \Rightarrow x\sqrt{3}+y=\pm 8$

Q.13 (1)

The point of intersection of $5x-6y-1=0$ and $3x+2y+5=0$ is $(-1,-1)$. Now the line perpendicular to $3x-5y+11=0$ is $5x+3y+k=0$, but it passes through $(-1,-1) \Rightarrow$ $-5-3+k=0 \Rightarrow k=8$

Hence required line is $5x+3y+8=0$.

Q.14 (4) The equation of a line passing through (2, 2) and perpendicular to $3x+y=3$ is $y-2=\frac{1}{3}(x-2)$ or $x-3y+4=0$.

Putting $x=0$ in this equation, we obtain $y=4/3$

So, *y*-intercept $=4/3$.

Q.15 (1)

Take two perpendicular lines as the coordinate axes. If *a*, *b* be the intercepts made by the moving line on the coordinate axes, then the equation of the line is

$\frac{x}{a}+\frac{y}{b}=1$ .....(i)

According to the question $\frac{1}{a}+\frac{1}{b}=\frac{1}{k}$, (say)

i.e., $\frac{k}{a}+\frac{k}{b}=1$ .....(ii)

The result (ii) shows that the straight line (i) passes through a fixed point (*k*, *k*).

Q.16 (4) Here equation of *AB* is $x+4y-4=0$ .....(i)

and equation of *BC* is $2x+y-22=0$ .....(ii)

Thus angle between (i) and (ii) is given by

$$\tan^{-1}\frac{-\frac{1}{4}+2}{1+\left(-\frac{1}{4}\right)(-2)}=\tan^{-1}\frac{7}{6}$$

Q.17 (3) $a_1a_2+b_1b_2=\frac{1}{ab'}+\frac{1}{a'b}=0$

Therefore, the lines are perpendicular

Q.18 (2)

$m_1=\frac{6+4}{-2-3}=\frac{10}{-5}=-2$ and $m_2=\frac{-18-6}{9-(-3)}=-2$

Hence the lines are parallel.

Q.19 (4)

Here,

Slope of Ist diagonal= $m_1=\frac{2-0}{2-0}=1 \Rightarrow \theta_1=45^o$

Slope of IInd diagonal= $m_2=\frac{2-0}{1-1}=\infty \Rightarrow \theta_2=90^o$

$\Rightarrow \theta_2-\theta_1=45^o=\frac{\pi}{4}$

Q.20 (1)

Let the point (*h*, *k*) then $h+k=4$ .....(i)

and $1=\pm\frac{4h+3k-10}{\sqrt{4^2+3^2}} \Rightarrow 4h+3k=15$ .....(ii)

and $4h+3k=5$ .....(iii)

On solving (i) and (ii); and (i) and (iii), we get the required points (3, 1) and (–7, 11).

Trick : Check with options. Obviously, points (3, 1) and (–7, 11) lie on x + y =4and perpendicular distance of these points from 4x + 3y = 10 is 1

Q.21 (1)

Required distance $=\frac{7}{\sqrt{(12)^2+5^2}}=\frac{7}{13}$

Q.22 (3)

Let *p* be the length of the perpendicular from the vertex (2, –1) to the base x + y = 2

Then $p=\left|\frac{2-1-2}{\sqrt{1^2+1^2}}\right|=\frac{1}{\sqrt{2}}$

If '*a*' be the length of the side of triangle, then

$p=a\sin 60^o \Rightarrow \frac{1}{\sqrt{2}}=\frac{a\sqrt{3}}{2} \Rightarrow a=\sqrt{\frac{2}{3}}$

Q.23 (1)

$L \equiv 2x+3y-4=0$, $L_{(-6,2)}=-12+6-4<0$

$L'=6x+9y+8=0$ $L'_{(-6,2)}=-36+18+8<0$

Hence the point is below both the lines..

Q.24 (1)

Equation of the line passing through (3, 8) and perpendicular to $x+3y-7=0$ is $3x-y-1=0$. The intersection point of both the lines is (1, 2).

Now let the image of $A(3,8)$ be $A'(x_1,y_1)$, then point (1, 2) will be the mid point of AA'.

$\Rightarrow \frac{x_1+3}{2}=1 \Rightarrow x_1=-1$ and $\frac{y_1+8}{2}=2 \Rightarrow y_1=-4$.

Hence the image is (–1, –4).

Q.25 (2) Here the lines are, $3x+4y-9=0$ ......(i)

and $6x+8y-15=0$ ......(ii)

Now distance from origin of both the lines are

$\frac{-9}{\sqrt{3^2+4^2}}=-\frac{9}{5}$ and $\frac{-15}{\sqrt{6^2+8^2}}=-\frac{15}{10}$

Hence distance between both the lines are

$\left|-\frac{9}{5}-\left(-\frac{15}{10}\right)\right|=\frac{3}{10}$

Ailter: Put $y=0$ in the first equation, we get $x=3$ therefore, the point (3, 0) lies on it. So the required distance between these two lines is the perpendicular length of the line $6x+8y=15$ from the point (3, 0). *i.e.*, $\frac{6\times 3-15}{\sqrt{6^2+8^2}}=\frac{3}{10}$.

Q.26 (3)

Here the given lines are

$ax+by+c=0$ .....(i)

$bx+cy+a=0$ .....(ii)

$cx+ay+b=0$ .....(iii)

The lines will be concurrent, if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0$

$\Rightarrow a^3+b^3+c^3-3abc=0$.

Q.27 (2)

The set of lines is $4ax+3by+c=0$, where $a+b+c=0$.

Eliminating *c*, we get $4ax+3by-(a+b)=0$

$\Rightarrow a(4x-1)+b(3y-1)=0$

This passes through the intersection of the lines $4x-1=0$ and $3y-1=0$ *i.e.* $x=\frac{1}{4}, y=\frac{1}{3}$ *i.e.*, $\left(\frac{1}{4},\frac{1}{3}\right)$.

Q.28 (3)

Required line should be,

$(3x - y + 2) + \lambda(5x - 2y + 7) = 0$ .....(i)

$\Rightarrow (3 + 5\lambda)x - (2\lambda + 1)y + (2 + 7\lambda) = 0$

$\Rightarrow y = \frac{3+5\lambda}{2\lambda+1}x+\frac{2+7\lambda}{2\lambda+1}$ .....(ii)

As the equation (ii), has infinite slope, $2\lambda + 1 = 0$

$\Rightarrow \lambda = -1/2$ putting $\lambda = -1/2$ in equation (i) we have $(3x - y + 2) + (-1/2)(5x - 2y + 7) = 0 \Rightarrow x = 3$.

Q.29 (1)

The equations of the bisectors of the angles between the lines are $\frac{x-2y+4}{\sqrt{1+4}}=\pm\frac{4x-3y+2}{\sqrt{16+9}}$

Taking positive sign, then

$(4-\sqrt{5})x-(3-2\sqrt{5})y-(4\sqrt{5}-2)=0$ .....(i)

and negative sign gives

$(4+\sqrt{5})x-(2\sqrt{5}+3)y+(4\sqrt{5}+2)=0$

Let θ be the angle between the line (i) and one of the given line, then $\tan\theta=\left|\frac{\frac{1}{2}-\frac{4-\sqrt{5}}{3-2\sqrt{5}}}{1+\frac{1}{2}.\frac{4-\sqrt{5}}{3-2\sqrt{5}}}\right|=\sqrt{5}+2>1$

Hence the line (i) bisects the obtuse angle between the given lines.

Q.30 (1)

Let the coordinates of *A* be (*a*, 0). Then the slope of the reflected ray is $\frac{3-0}{5-a}=\tan\theta$, (say).

The slope of the incident ray $=\frac{2-0}{1-a}=\tan(\pi-\theta)$

Since $\tan\theta+\tan(\pi-\theta)=0 \Rightarrow \frac{3}{5-a}+\frac{2}{1-a}=0$

$\Rightarrow 13-5a=0 \Rightarrow a=\frac{13}{5}$

Thus the coordinates of *A* are $\left(\frac{13}{5},0\right)$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

AB = $\sqrt{4+9}$ = $\sqrt{13}$

BC = $\sqrt{36+16}$ = $2\sqrt{13}$

CD = $\sqrt{4+9}$ = $\sqrt{13}$

AD = $\sqrt{36+16}$ = $2\sqrt{13}$

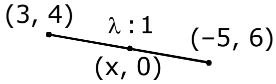
AC = $\sqrt{64+1}$ = $\sqrt{65}$

BD = $\sqrt{16+49}$ = $\sqrt{65}$

its rectangle

Q.2 (1)

$$\frac{-5\lambda+3}{\lambda+3} = x, \quad \frac{6\lambda-4}{\lambda+1} = 0$$



$\Rightarrow \lambda = \frac{2}{3}$

Q.3 **(4)**

since the points are collinear option D is correct

Q.4 (2)

$\Delta = 0$

$$\frac{1}{2}\begin{vmatrix} k & 2-2k & 1 \\ 1-k & 2k & 1 \\ -k-4 & 6-2k & 1 \end{vmatrix} = 0$$

k(2k – 6 + 2k) – (2 – 2k) (1 – k + k + 4) + 1 (1 – k) (6 – 2k) – 2k(–k – 4) = 0

$4k^2 - 6k - 10 + 10k + 6 - 8k + 2k^2 + 2k^2 + 8k = 0$

$8k^2 + 4k - 4 = 0 \Rightarrow 2k^2 + k - 1 = 0$

$2k^2 + 2k - k - 1 = 0$

2k (k + 1) – 1 (K + 1) = 0

$$\boxed{k = -1, \frac{1}{2}}$$

Q.5 (4)

(2a, 3a), (3b, 2b) & (c, c) are collinear

$$\Rightarrow \begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$$

$\Rightarrow$ (3bc – 2bc) – (2ca – 3ca) + (4ab – 9ab) = 0

$\Rightarrow$ bc + ca + 5ab = 0

$$\Rightarrow \frac{2}{2}\cdot\frac{5}{c} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{2}{\left(\frac{2c}{5}\right)} = \frac{1}{a} + \frac{1}{b}$$

$\Rightarrow$ a, $\frac{2c}{5}$, b are in H.P.

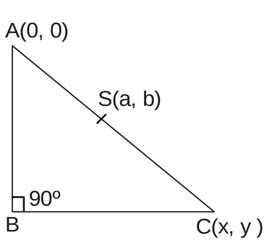
Q.6 (1)

By given information

Since in ΔABC, B is other centre. Hence ∠B = 90º

Cercum centre is S (a, b)

$$\frac{x+0}{2} = a \Rightarrow x = 2a$$



$$\frac{y+0}{2} = b \Rightarrow y = 2b$$

Hence, c(x, y) ≡ (2a, 2b)

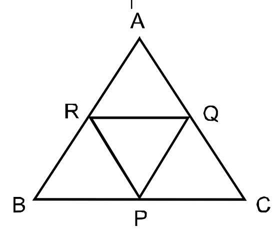
Q.7 (4)

If H is orthocentre of triangle ABC, then orthocentre of triangle BCH is point A

Q.8 (1)

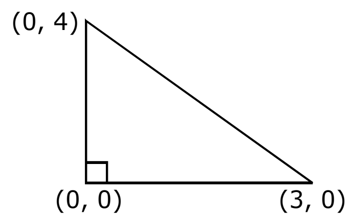
Area of the triangle formed by joining the mid points of the sides of the triangle = $\frac{1}{4}$(area of the triangle)

$$= \frac{1}{4}\times\frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ -2 & 3 & 1 \\ 4 & -3 & 1 \end{vmatrix} = \frac{1}{4}\times 6 = 1.5 \text{ sq.units}$$



Q.9 (**3**)

Δ right angled

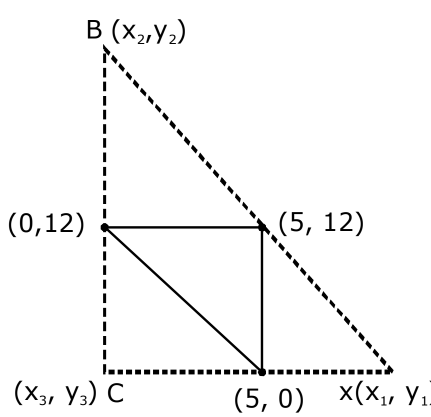


$\Rightarrow$ circum centre

= mid point of hypotaneous = $\left(\frac{3}{2}, 2\right)$

Q.10 (1)

$$\begin{cases} x_1 + x_3 = 10 \quad , \quad y_1 + y_3 = 0 \\ x_2 + x_3 = 0 \quad , \quad y_2 + y_3 = 24 \\ x_1 + x_2 = 10 \quad , \quad y_2 + y_2 = -24 \end{cases}$$



$x_1 = x_2 = 10,\ y_1 - y_2 = -24$
$x_1 = 10,\ y_1 = 0$
$x_2 = 0,\ y_2 = 24$
$x_3 = 0,\ y_3 = 0$

$$\left.\begin{array}{lll} x_1 = 10 & , & y_1 = 0 \\ x_2 = 0 & , & y_2 = 24 \\ x_3 = 0 & , & y_3 = 0 \end{array}\right\} \Rightarrow \begin{array}{lll} A(10,\ 0) & \text{on} & x-\text{axis} \\ B(a,\ 24) & \text{on} & y-\text{axis} \\ C(0,\ 0) & \text{is} & \text{origin} \end{array}$$

ΔABC is right angled $\Rightarrow$ orthocentre is (0, 0)

Q.11 (4)

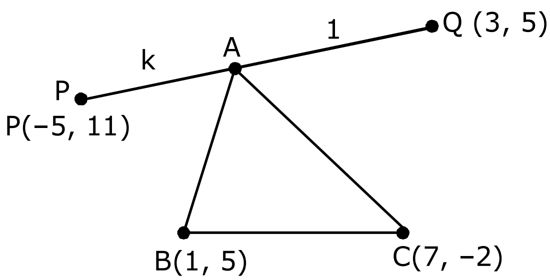
$$\Delta = \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{vmatrix}$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left\|\begin{matrix} 0 & 0 & 2 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}.\ 2\ (ab\sin^2\theta + ab\cos^2\theta) = ab$$

Q.12 (3)

$$\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$$



$$\frac{1}{2}\begin{vmatrix} \frac{3k-5}{k+1} & \frac{5k+1}{k+1} & 1 \\ 1 & 5 & 1 \\ 7 & -2 & 1 \end{vmatrix} = |2|$$

$$\Rightarrow\ 1.\ (-2-3) - 1.\left(\frac{-6k+10}{k+1} - \frac{35k+7}{k+1}\right)$$

$$+\left(\frac{15k-25}{k+1} - \frac{5k+1}{k+1}\right) = \pm 4$$

$\Rightarrow$ 6k – 10 + 35 k + 7 + 15k – 25 – 5k – 1
= ± 4 + 37 (k + 1)
$\Rightarrow$ 51 k – 29 = 41 k + 41 or 51 k – 29
= 33k + 33
$\Rightarrow$ 10 k = 70 or 18 k = 62

$$k = 7\ k = \frac{31}{9}$$

Q.13 (1)

$$AP = \sqrt{x^2 + (y-4)^2}$$

$$BP = \sqrt{x^2 + (y+4)^2}$$

$\because$ |AP – BP| = 6
AP – BP = ± 6

$$\sqrt{x^2 + (y-4)^2} - \sqrt{x^2 + (y+4)^2} = \pm 6$$

On squaring we get the locus of P
$9x^2 - 7y^2 + 63 = 0$

Q.14 (2)

Let coordinate of mid point is m(h, k)

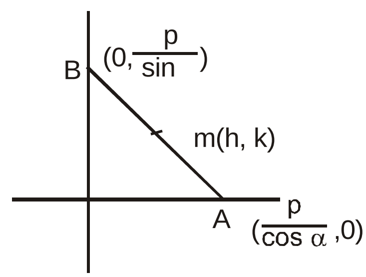
$$2h = \frac{p}{\cos d} \Rightarrow \cos\alpha = \frac{p}{2h}$$

$$2k = \frac{p}{\sin d} \Rightarrow \sin\alpha = \frac{p}{2k}$$

Squareing and add.

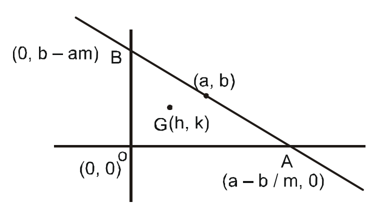
$$\frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$$

Locus of p(h, k) $\Rightarrow \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$



Q.15 (1)

equation of line AB
y – b = m (x – a)



$$\therefore\ G\left(\frac{a - \frac{b}{m}}{3}, \frac{b-am}{3}\right) \Rightarrow h = \frac{a - \frac{b}{m}}{3},$$

$$k = \frac{b-am}{3}$$

on eleminating 'm' we get required locus
bh + ak – 3hk = 0 $\Rightarrow$ bx + ay – 3xy = 0

Q.16 (3)

Let centroid is (h, k)

then $h = \frac{\cos\alpha + \sin\alpha + 1}{3}$ & $k = \frac{\sin\alpha - \cos\alpha + 2}{3}$

$\cos\alpha + \sin\alpha = 3h - 1$ & $\sin\alpha - \cos\alpha = 3k - 2$

squaring & adding

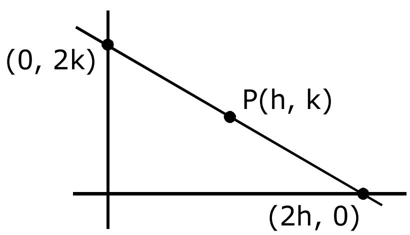
$2 = (3h - 1)^2 + (3k - 2)^2$ Locus of (h, k)

$\Rightarrow (3x - 1)^2 + (3k - 2)^2 = 2$

$\Rightarrow 3(x^2 + y^2) - 2x - 4y + 1 = 0$

Q.17 (2)

P is a mid point AB



AB = 10 units

$(2h)^2 + (2k)^2 = 10^2$

$h^2 + k^2 = 25$

Locus of (h, k)

$x^2 + y^2 = 25$

Q.18 (4)

P(1, 0), Q(–1, 0), R(2, 0), Locus of s (h, k) if $SQ^2 + SR^2 = 2SP^2$

$\Rightarrow (h + 1)^2 + k^2 + (h - 2)^2 + k^2$
$= 2(h - 1)^2 + 2k^2$

$\Rightarrow h^2 + 2h + 1 + h^2 - 4h - 4 = 2h^2 - 4h + 2$

$\Rightarrow 2h + 3 = 0$ Locus of s(h, k)

$\Rightarrow 2x + 3 = 0$

Parallel to y-axis.

Q.19 (2)

Slope $= \frac{k + 1 - 3}{k^2 - 5} = \frac{1}{2}$ $\Rightarrow k^2 - 5 - 2k + 4 = 0$

$\Rightarrow k = 1 \pm \sqrt{2}$ $\Rightarrow k^2 - 2k - 1 = 0$

$\Rightarrow k = \frac{2 \pm \sqrt{4 + 4}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2}$

Q.20 (2)

Let $B(x_1, y_1)$ and $C(x_2, y_2)$

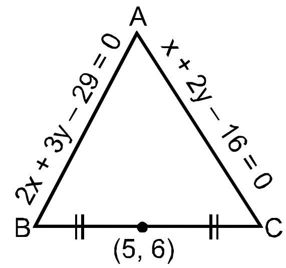
$\therefore\ 2x_1 + 3y_1 - 29 = 0$ ......(i)

and $x_2 + 2y_2 - 16 = 0$ ......(ii)

$\because$ mid-point of BC is (5, 6)

$\therefore\ x_1 + x_2 = 10$ ......(iii)

and $y_1 + y_2 = 12$ ......(iv)



Put the value of x_2 and y_2 in (ii), we get

$10 - x_1 + 2(12 - y_1) - 16 = 0$

$x_1 + 2y_1 = 18$ ......(v)

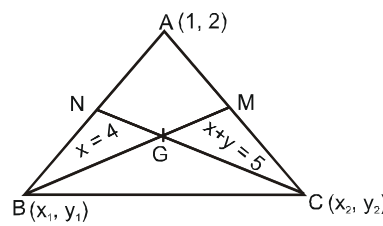
Now on solving (i) and (v), we get $x_1 = 4$ and $y_1 = 7$

$\therefore$ B(4, 7)

$\therefore$ equation of line BC is $y - 6 = \frac{7 - 6}{4 - 5}(x - 5)$

$\Rightarrow x + y = 11$

Q.21 (2)



$x_1 + y_1 = 5$ ... (i)

$x_2 = 4$ ... (ii)

co - ordinates of G are ≡ (4, 1)

$\Rightarrow \frac{1 + x_1 + x_2}{3} = 4$ ....(iii)

and $\frac{y_1 + y_2 + 2}{3} = 1$ ... (iv)

solving above equations, we get B & C.

Q.22 (4)

Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

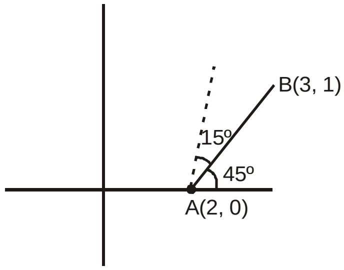
$\frac{a}{2} = 1 \Rightarrow a = 2$

$\frac{b}{2} = 2 \Rightarrow b = 4$

Hence $\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow$ $2x + y - 4 = 0$

Q.23 (3)

Slope of AB is $\tan\theta = \frac{1 - 0}{3 - 2} = 1$



$\theta = 45^{\circ}$

Hence equation of new line is

$y - 0 = \tan 60^{\circ}(x - 2)$

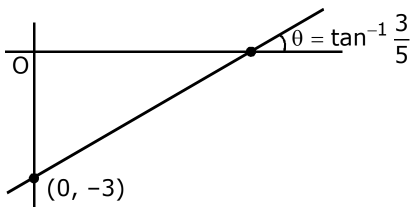
$y = \sqrt{3}\,x - 2\sqrt{3}$

$\Rightarrow \sqrt{3}\,x - y - 2\sqrt{3} = 0$

Q.24 (1)

$\theta = \tan^{-1}\dfrac{3}{5}$, $C = -3$

$\tan\theta = \dfrac{3}{5}$



$y = \dfrac{3}{5}x - 3$

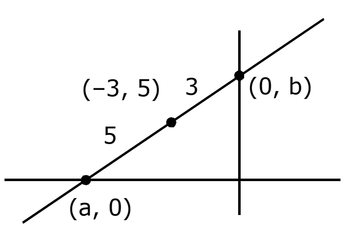
$3x - 5y - 15 = 0$

Q.25 (4)

$-3 = \dfrac{3a + 0}{5 + 3}$, $5 = \dfrac{0 + 5b}{5 + 3}$

$\Rightarrow a = -3, b = 8$

$\dfrac{x}{-8} + \dfrac{y}{8} = 1$



$-x + y = 8$

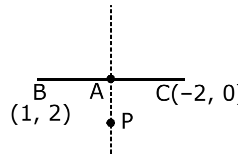
$x - y + 8 = 0$

Q.26 (3)

Perpendicular bisector of slopoe of line BC

$m_{BC} = \dfrac{2 - 0}{1 + 2} = \dfrac{2}{3}$

$m_{AP} = \dfrac{-3}{2}$



$A = \left(\dfrac{1-2}{2}, \dfrac{2+0}{2}\right) \Rightarrow \left(-\dfrac{1}{2}, 1\right)$

$y - 1 = \dfrac{-3}{2}\left(x + \dfrac{1}{2}\right) \Rightarrow 4y - 4 = -6x - 3$

$\Rightarrow 6x + 4y = 1$

locus of P

Q.27 (3)

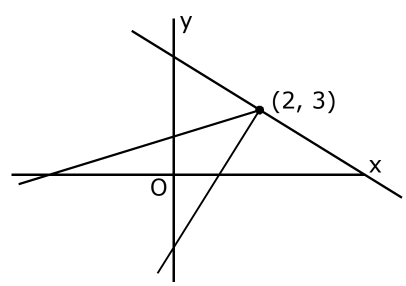
Equation $y - 3 = m(x - 2)$

cut the axis at

$\Rightarrow y = 0 \;\&\; x = \dfrac{2m - 3}{m}$

$\Rightarrow x = 0 \;\&\; y = -(2m - 3)$

$\text{Area } \Delta = 12 = \left|\dfrac{1}{2}\cdot\dfrac{(2m-3)}{m}\{-(2m-3)\}\right|$



$(2m - 3)^2 = \pm 24m$

$4m^2 - 12m + 9 = 24m$

or $4m^2 - 12m + 9 = -24m$

$4m^2 - 3y\,m + 9 = 0$

$D > 0$

or $4m^2 + 12m + 9 = 0$

$(2m + 3)^2 = 0$

two distinct root of m

no. of values of m is 3.

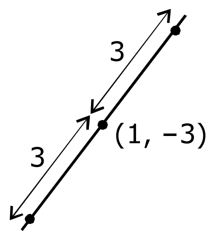
Q.28 (2)

$2x + 3y + 7 = 0$

$\tan\theta = \dfrac{-2}{3} \Rightarrow \sin\theta = \dfrac{2}{\sqrt{13}}$, $\cos\theta = \dfrac{-3}{\sqrt{13}}$

$$\frac{x-1}{\frac{-3}{\sqrt{13}}} = \frac{y+3}{\frac{2}{\sqrt{13}}} = \pm 3$$

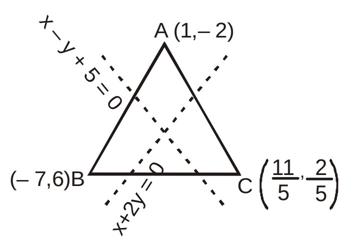
$$\left(1-\frac{9}{\sqrt{13}}, -3+\frac{6}{\sqrt{13}}\right)$$



or $\left(1+\frac{9}{\sqrt{13}}, -\frac{3-6}{\sqrt{13}}\right)$

Q.29 (1)

Image of A in x – y + 5 = 0 is



$$\frac{x-1}{1} = \frac{y+2}{-1} = -2\left(\frac{1+2+5}{2}\right) = -8$$

x = – 7, y = 6

Image of A(1, – 2) in x + 2y = 0

$$\frac{x-1}{1} = \frac{y+2}{2} = -2\left(\frac{1-4}{5}\right) = \frac{6}{5}$$

$x = \frac{11}{5}$, $y = \frac{2}{5}$

Hence equation of BC is $y - 6 = \frac{(6-2/5)}{(-7-11/5)}(x+7)$

$$y - 6 = \frac{28}{-28}(x+7)$$

$$y - 6 = \frac{-14}{23}(x+7)$$

$\Rightarrow$ 14x + 23y – 40 = 0

Q.30 (4)

$\perp$ to 3x + y = 3, passes (2, 2)

$m = +\frac{1}{3}$ & (2, 2)

$y - 2 = +\frac{1}{3}(x-2)$

$\Rightarrow -x + 3y = 4 \Rightarrow \frac{x}{-4} + \frac{y}{\frac{4}{3}} = 1 \Rightarrow b = \frac{4}{3}$

Q.31 (3)

required line should be

ax + by + λ = 0 satsify (c, d)

ac + bd + λ = 0 $\Rightarrow$ λ = – (ac + bc)

ax + by – (ac + bc) = 0

$\Rightarrow$ a (x – c) + b (y – d) = 0

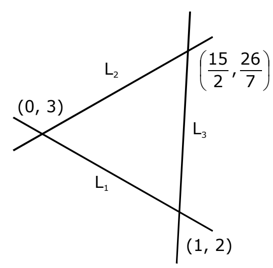
Q.32 (2)

L_1 : x + y – 3 = 0,

L_2 : x – 3y + 9 = 0

L_3 : 3x – 2y + 1 = 0

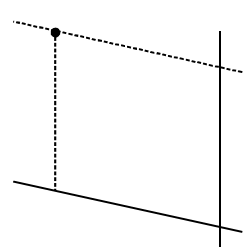
$$\Delta = \frac{1}{2}\begin{vmatrix} \frac{15}{7} & \frac{26}{7} & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$



$$= \frac{1}{2}\left[\frac{15}{7}(3-2) + 0 + 1\left(\frac{26}{7}-3\right)\right]$$

$$= \frac{1}{2}\left[\frac{15}{7}+\frac{5}{7}\right] = \frac{10}{7} \text{ sq.units}$$

Aliter : by parallelogram



$$\Delta = \frac{1}{2}\left|\frac{(c_1-c_2)(d_1-d_2)}{(m_1-m_2)}\right|$$

Q.33 (1)

$y - x + 5 = 0$, $\sqrt{3}\,x - y + 7 = 0$

$m_1 = 1$, $m_2 = \sqrt{3}$

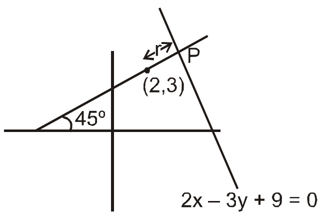
$\theta_1 = 45^o$, $\theta_2 = 60^o$

$\theta = 60^o - 45^o = 15^o$

Aliter $\tan\theta = \dfrac{\sqrt{3}-1}{1+\sqrt{3}} = \dfrac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$

$\Rightarrow\ \theta = 15^o$

Q.34 (2)



Let coordinates of point P by parametric

P(2 + r cos 45º, 3 + r sin 45º)

It satisfies the line $2x - 3y + 9 = 0$

$2\left(2+\dfrac{r}{\sqrt{2}}\right) - 3\left(3+\dfrac{r}{\sqrt{2}}\right) + 9 = 0 \Rightarrow r = 4\sqrt{2}$

Q.35 (2)

$a^2x + a\,by + 1 = 0$

origin and (1, 1) lies on same side.

$a^2 + ab + 1 > 0$ $\quad\forall\ a \in R$

$D < 0 \Rightarrow b^2 - 4 < 0$ $\quad\Rightarrow b \in (-2, 2)$

but $b > 0 \Rightarrow b \in (0, 2)$

Q.36 (1)

$L_1 : 2x + 3y - 4 = 0$

$L_2 : 6x + 96 + 8 = 0$, P(8, – 9)

$L_1(P) = 2.8 - 3.9 - 4 = 16 - 27 - 4 = -15 < 0$

$L_2(O) = 48 - 81 + 8 + 8 = -25 < 0$

point (8, –9) lies same side of both lines.

Q.37 (1)

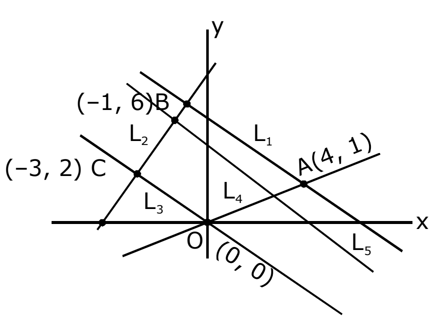
$L_1 : x + y = 5$, $L_2 : y - 2x = 8$

$L_3 : 3y + 2x = 0$, $L_4 : 4y - x = 0$

$L_5 : (3x + 2y) = 6$

vertices of quadrilateral

0(0, 0), A (4, 1), B (–1, 6), C(–3, 2)



$L_5(0) = -6 < 0$

$L_5(A) = 12 + 2 - 6 = 8 > 0$

$L_5(B) = -3 + 12 - 6 = 3 > 0$

$L_5(C) = -9 + 4 - 6 = -11 < 0$

O & C points are same side

& A & B points are other same side w.r.t to L_5

So L_5 divides the quadrilateral in two quadrialteral

Aliter :

If abscissa of A is less then abscissa of B

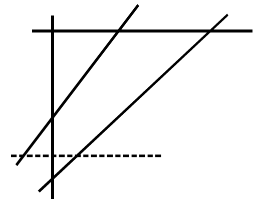
$\Rightarrow$ A lies left of B

otherwise A lies right of B

Q.38 (2)

P(a, 2) lies between

$L_1: x - y - 1 = 0$ &



$L_2: 2(x - y) - 5 = 0$

Method-I

$L_1(P)\ L_2(P) < 0$

$(a - 3)(2a - 9) < 0$

$\Rightarrow$ P(a, 2) lies on y = 2

intersection with given lines

$x = 3$ & $x = \dfrac{9}{2}$

$a > 3$ & $a < \dfrac{9}{2}$

(gemetrically)

$a \in \left(3, \dfrac{9}{2}\right)$

Q.39 (4)

$ax + by + c = 0$

$\dfrac{3a}{4} + \dfrac{b}{2} + c = 0$

compare both $(x, y) \equiv \left(\dfrac{3}{4}, \dfrac{1}{2}\right)$

Hence given family passes through $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$

Q.40 (2)

$$\begin{vmatrix} \sin^2 A & \sin A & 1 \\ \sin^2 B & \sin B & 1 \\ \sin^2 C & \sin C & 1 \end{vmatrix} = 0$$

$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin C) = 0$

$\Rightarrow$ A = B or B = C or C = A
any two angles are equal
$\Rightarrow$ Δ is isosceles

Q.41 (4)

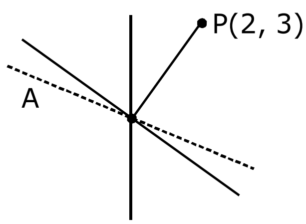
$(p + 2q)x + (p - 3q)y = p - q$
$px + py - p + 2qx - 3qy + q = 0$
$p(x + y - 1) + q(2x - 3y + 1) = 0$
passing through intersection of

$x + y - 1 = 0$ & $2x - 3y + 1 = 0$ is $\left(\frac{2}{5}, \frac{3}{5}\right)$

Q.42 (1)

PM is maximum if required
line $\perp$ intersection of
$3x + 4y + 6 = 0$
$\Rightarrow (-2, 0)$
$x + y + 2 = 0$

$m_{AP} = \frac{3-0}{2+2} = \frac{3}{4}$



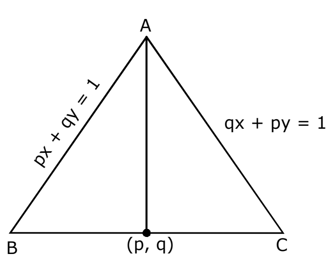
Slope $m = -\frac{4}{3}$

$y - 0 = -\frac{4}{3}(x + 2) \Rightarrow 4x + 3y + 8 = 0$

Q.43 (3)

$L_1 : Px + qy = 1$
$L_2 : qx + py = 1$

$L_1 + \lambda L_2 = 0$
$(px + qy - 1) + \lambda(qx + py - 1) = 0$

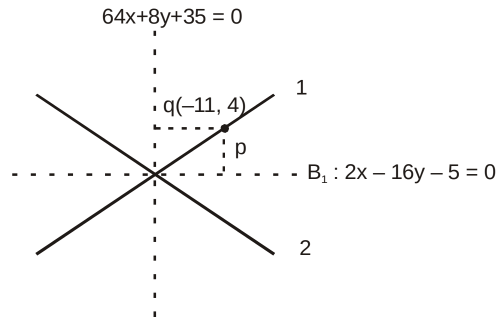


$\Rightarrow \lambda = \frac{(p^2 + q^2 - 1)}{(2pq - 1)} \Rightarrow (2pq-1)(px + qy - 1)$

$= (p^2 + q^2 - 1)(qx + py - 1)$

Q.44 (1)

$p = \left|\frac{-22 - 64 - 5}{2^2 + (-16)^2}\right| = \frac{91}{260}$



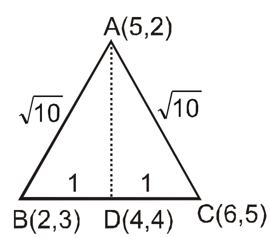
$q = \left|\frac{-64 \times 11 + 8 \times 4 + 35}{64^2 + 8^2}\right|$

$p < q$ Hence $2x - 16y - 5 = 0$ is a cute angle bisector

Q.45 (3)

Equation of AD : $y - 4 = \frac{2}{-1}(x - 4)$

$\Rightarrow$ $y - 4 = -2x + 8$



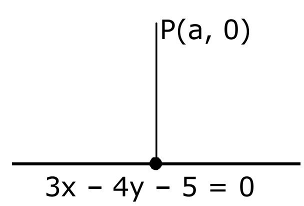
$\Rightarrow$ $2x + y = 12$

Q.46 (4)

$m = \frac{3}{4} \Rightarrow m_{PQ} = -\frac{4}{3}$

equation of PQ

$y - 5 = -\frac{4}{3}x$



$4x + 3y - 15 = 0$
$\Rightarrow$ $25x = 75$
& $3x - 4y - 5 = 0$ $\Rightarrow$ $x = 3$ & $y = 1$
Q(3, 1)

Q.47 (2)

Point of reflection of (0, 0)

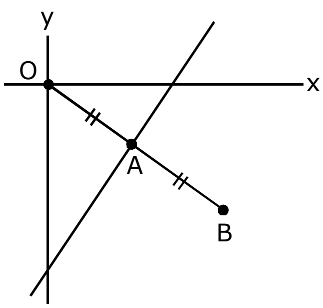
w.r.t. to $4x - 2y - 5 = 0$

$$OA = \left|\frac{-5}{\sqrt{4^2+2^2}}\right| = \frac{2}{2\sqrt{5}}$$

$$= \frac{\sqrt{5}}{2} = AB$$

equtaion of line OB

$$\frac{x-0}{-\frac{2}{\sqrt{5}}} = \frac{y-0}{\frac{1}{\sqrt{5}}} = \pm\sqrt{5}$$



$\Rightarrow OB = \sqrt{5}$

$x = \mp\sqrt{2}$, $y = \pm 1$ $\Rightarrow$ B (2, –1)

Aliter :

Image of origin w.r. to line

$$\frac{x-0}{4} = \frac{y-0}{-2} = \frac{-2(4.0-2.0-5)}{4^2+(-2)^2}$$

$$\Rightarrow \frac{x}{4} = \frac{y}{-2} = \frac{10}{20} \Rightarrow x = 2,\ y = -1,\ B(2, -1)$$

Q.48 (4)

$m_1 + m_2 = -10$

$m_1m_2 = \frac{a}{1}$

given $m_1 = 4m_2 \Rightarrow m_2 = -2$, $m_1 = -8$,

a = 16

Q.49 (1)

$\sqrt{3}\,x^2 - 4xy + \sqrt{3}\,y^2 = 0$

part of angle besection is $\frac{x^2-y^2}{\sqrt{3}-\sqrt{3}} = \frac{xy}{(-2)}$

$\Rightarrow x^2 - y^2 = 0$

$\Rightarrow y^2 - x^2 = 0$

Q.50 (1)

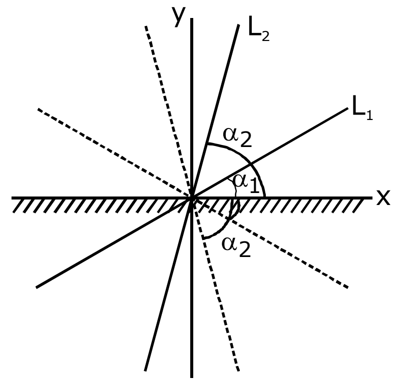
$ax^2 + 2hxy + by^2 = 0$

$m_1 + m_2 = \frac{-2h}{b}$, $m_1m_2 = \frac{a}{b}$

Relation of slopes of image lines

$(m_1' + m_2') = -(m_1 + m_2)$

$= -\left(\frac{-2h}{b}\right) = \frac{2h}{b}$ $\{m_1' = \tan(\alpha_1)$



$m_1'm_2' = (-m_1)(-m_2)$

$= m_1m_2 = \frac{a}{b}$

$$\left(\frac{y}{x}\right)^2 - (m_1' + m_2')\left(\frac{y}{x}\right) + m_1'm_2' = 0$$

$$\Rightarrow \left(\frac{y}{x}\right)^2 - \frac{2h}{b}\left(\frac{y}{x}\right) + \frac{a}{b} = 0$$

$\Rightarrow by^2 - 2hxy + ax^2 = 0$

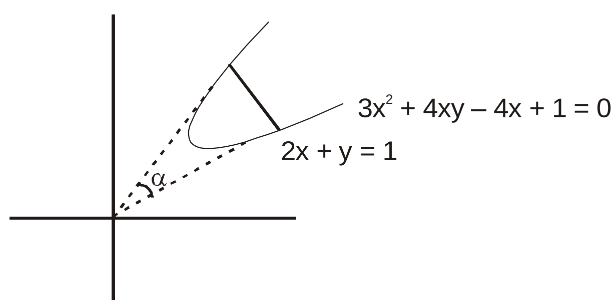
$\Rightarrow ax^2 - 2hxy + by^2 = 0$

Q.51 (1)

Homogenize given curve with given line

$3x^2 + 4xy - 4x(2x + y) + 1(2x + y)^2 = 0$

$3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy =$



$-x^2 + 4xy + y^2 =$

coeff. x^2 + coeff. $y^2 = 0$

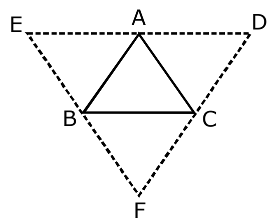
Hence angle is 90º

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 (C)

$A(x_1, y_1), B(x_2, m\, y_2), C(x_3, y_3)$



only three sides can be made parallel to corresponding sides of triangle passing through vertex of triangle respectively

$\Rightarrow$ So no. of IIgrams is 3.

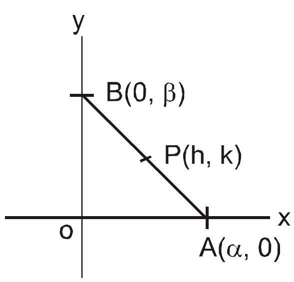
Q.2 (A)

By geometry

$a^2 + b^2 = (a + b)^2$ ....(i)

By section formula

$h = \frac{\alpha}{a+b} \Rightarrow \alpha = \frac{n(a+b)}{a}$



$k = \frac{\beta}{a+b} \Rightarrow \beta = \frac{k(a+b)}{b}$

Put value of α and β in (i)

$\frac{h^2(a+b)^2}{a^2} + \frac{k^2(a+b)^2}{b^2} = (a + b)^2$

$\Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

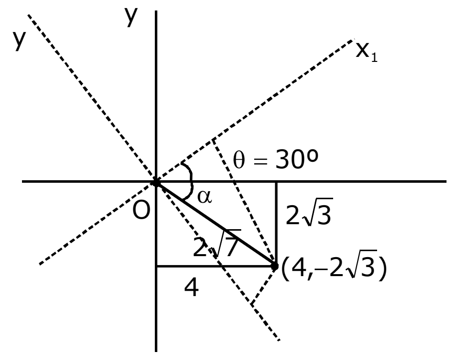
Locus of 'p' is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Q.3 (B)

First position

$(4, -2\sqrt{3}) = (4 \cos(-\alpha), r \sin(-\alpha))$

$r \cos \alpha = 4$



$r \sin \alpha = +2\sqrt{3}$

& $\sin \theta^o = \frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$

Last position w.r.t is x′

$(r \cos(-\theta - a, r \sin(-\theta - \alpha))$

$= (r \cos(\theta + \alpha), - r(\sin(\theta + \alpha))$

$= ((4 \cos \theta \cos \alpha - r \sin \alpha \sin \alpha)),$

$m\, (-r \cos\alpha \sin \theta - r \sin\alpha \cos\theta)$

$= \left(\left(4.\frac{\sqrt{3}}{2} - 2\sqrt{3}.\frac{1}{2}\right), \left(-4.\frac{1}{2} - 2\sqrt{3}.\frac{\sqrt{3}}{2}\right)\right)$

$= ((2\sqrt{3} - \sqrt{3}), (-2 - 3)) = (\sqrt{3}, -5)$

Q.4 (B)

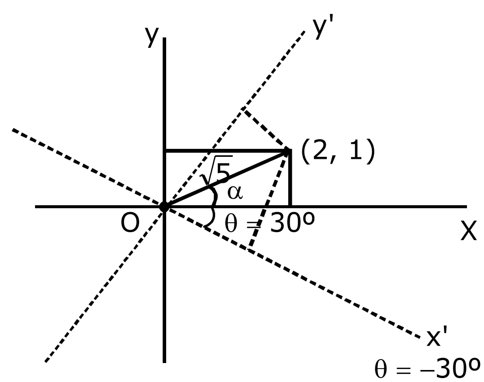
Before rotation

$(2, 1) = (4 \cos\alpha, r \sin\alpha)$

$r \cos \alpha = 2, r \sin \alpha = 1$

new position

$\Rightarrow x' = 4 \cos\alpha \cos \alpha - r \sin \alpha \sin \theta$



$= 2.\frac{\sqrt{3}}{2} + 2.\left(\frac{-1}{2}\right) = \frac{\sqrt{3} - 2}{2}$

$(x', y') = \left(\frac{2\sqrt{3}+1}{2}, \frac{\sqrt{3}-2}{2}\right)$

Q.5 (D)

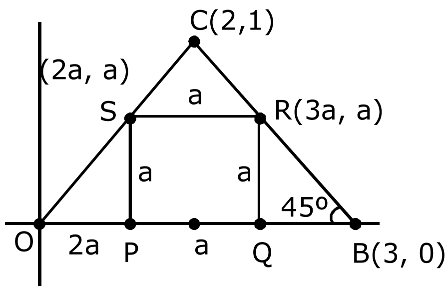
Let side of square is a units

equation of OC is 2y = x

S(2a, a) ⇒ R(3a, a)

Slope $m_{BC} = \frac{0-1}{3-2} = -1$

⇒ ∠B = 45º in ΔQBR



QB = a

OB = OP + PQ = QB

$3 = 2a + a + a \Rightarrow a = \frac{3}{4}$

$P\left(\frac{3}{2}, 0\right)$, $Q\left(\frac{9}{4}, 0\right)$, $R\left(\frac{9}{4}, \frac{3}{4}\right)$ & $S\left(\frac{3}{2}, \frac{3}{4}\right)$

Q.6 (D)

OA line y = x, $m_1 = \tan\theta_1 = 1$

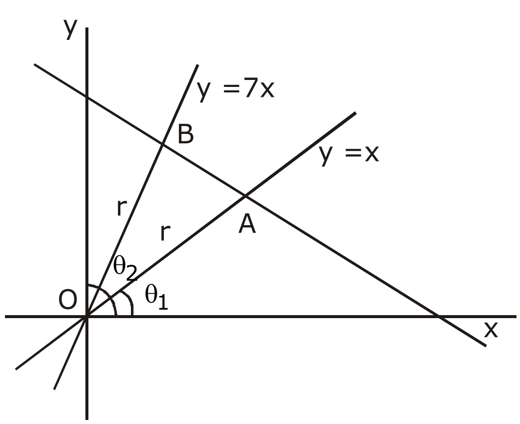
OB line y = 7m, $m_2 = \tan\theta_2 = 7$

A, B lies in I^{st} quadrant

OA = OB = r (let)

OA line $\frac{x}{\cos\theta_1} = \frac{y}{\sin\theta_1} = r \Rightarrow \frac{x}{\frac{1}{\sqrt{2}}} = \frac{y}{\frac{1}{\sqrt{2}}} = r$

$A\left(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right)$



OB line $\frac{x}{\frac{1}{5\sqrt{2}}} = \frac{y}{\frac{7}{5\sqrt{2}}} = r \Rightarrow B.\left(\frac{r}{4\sqrt{2}}, \frac{7r}{5\sqrt{2}}\right)$

Slope $m_{AB} = \frac{\frac{7r}{5\sqrt{2}} - \frac{r}{\sqrt{2}}}{\frac{1}{5\sqrt{2}} - \frac{r}{\sqrt{2}}} = \frac{7r-5r}{r-5r} = \frac{2}{-4} = -\frac{1}{2}$

Q.7 (D)

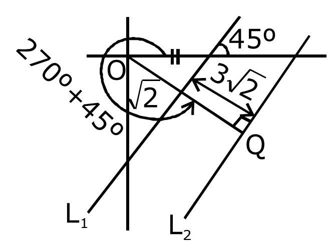
OP = $\sqrt{2}$, PQ = $3\sqrt{2}$ OQ = $4\sqrt{2}$

OQ makes angle with (+) x-axis in anti clockwise θ = 270º + 45º

equation L_2

$x\cos\theta + y\sin\theta = 4\sqrt{2}$

$x\cos(270º + 45º) + y\sin(270º+45º) = 4\sqrt{2}$



$x\sin 45º + y(-\cos 45º) = 4\sqrt{2}$

x – y = 8

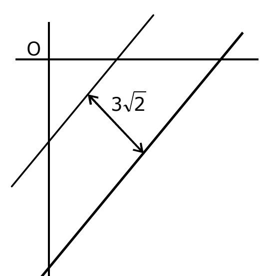
Aliter :

y – x + 2 = 0

⇒ x – y – 2 = 0

Parallel lines x – y + λ = 0

$3\sqrt{2} = \left|\frac{\lambda + 2}{\sqrt{2}}\right|$



⇒ λ + 2 = ± 6

⇒ λ = –8, 4

Line shift to (+) x-axis

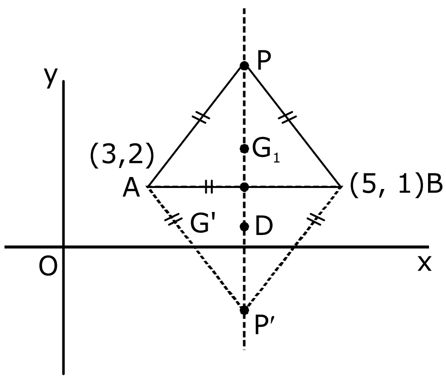
So line is x – y – 8 = 0

Q.8 (D)

$D\left(4, \frac{3}{2}\right)$, AB = $\sqrt{4+1} = \sqrt{5}$

$PD = \sqrt{5 - \frac{5}{4}} = \sqrt{\frac{15}{2}}$

$G.D. = \frac{1}{3} . \frac{\sqrt{15}}{2} = \frac{\sqrt{15}}{2}$



[Centroid ≡ orthocentre in equilateral]

$m_{PD} = \frac{-1}{m_{AB}} = \frac{-1}{-\frac{1}{2}} = 2$

$= \tan\theta \Rightarrow \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$

equation of pp′ is

$\frac{x - y}{\frac{1}{\sqrt{5}}} = \frac{y - \frac{3}{2}}{\frac{2}{\sqrt{5}}} = \pm \frac{\sqrt{5}}{2\sqrt{3}}$

$x = 4 \pm \frac{1}{2\sqrt{3}}, y = \frac{3}{2} \pm \frac{1}{\sqrt{3}}$

$G\left(4 + \frac{\sqrt{3}}{6}, \frac{3}{2} + \frac{\sqrt{3}}{3}\right), G'\left(4 - \frac{\sqrt{3}}{6}, \frac{3}{2} - \frac{\sqrt{3}}{3}\right)$

$OG > OG' \Rightarrow \left(4 + \frac{\sqrt{3}}{6}, \frac{3}{2} + \frac{\sqrt{3}}{3}\right)$

Q.9 (C)

P(2, 0), Q (4, 2)

line PQ is x – y = 2

$m_{PQ} = +1$

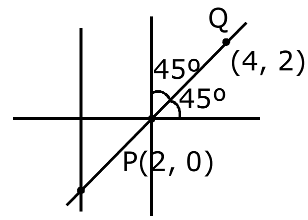
$\Rightarrow \theta = 45^o$

required line is

parallel to y-axis

(according questions)

$\Rightarrow x = 2$



Q.10 (B)

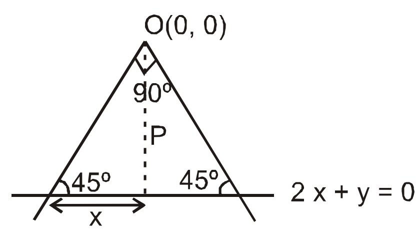
here $\tan\theta = \frac{1}{5}$

$\therefore \tan 2\theta = \frac{2\left(\frac{1}{5}\right)}{1 - \left(\frac{1}{5}\right)^2} = \frac{5}{12}$

$\therefore$ required line $y = \frac{5x}{12}$

Q.11 (C)

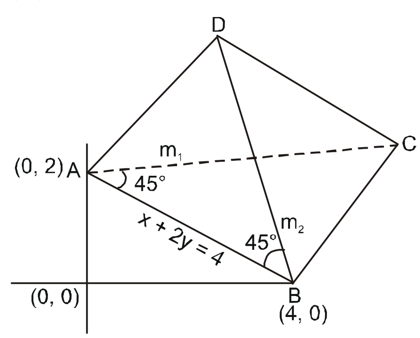
$p = \left|\frac{0 + 0 - a}{\sqrt{5}}\right| = \frac{a}{\sqrt{5}}$



$\tan 45^o = \frac{p}{x} \Rightarrow p = x$

Hence area $= \frac{1}{2}(2x)(p) = px = p^2 = a/5$

Q.12 (C)



$\tan 45° = \left|\frac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right| \Rightarrow \pm 1 = \frac{2m + 1}{2 - m}$

$\Rightarrow m = \frac{1}{3}, -3$

$\therefore$ Equation of AC

$y - 2 = \frac{1}{3}(x) \Rightarrow x - 3y + 6 = 0$ .... (i)

Equation of BD $y = -3(x - 4) \Rightarrow 3x + y - 12 = 0$ .... (ii)

From (i) & (ii)

$x = 3$ & $y = 3$

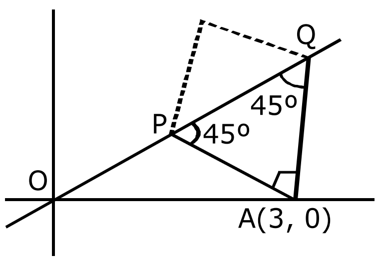
Q.13 (D)

$x = 2y$, A(3, 0)

$y = m(x - 3)$

$m_1 = \frac{1}{2}$ (given line)

$$\tan 45^\circ = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right|$$



$\Rightarrow \left|1 + \frac{m}{2}\right| = \left|m - \frac{1}{2}\right| \quad \Rightarrow \left(1 + \frac{m}{2}\right)$

$= \left(m - \frac{1}{2}\right)$ or $\frac{3m}{2} = -\frac{1}{2}$

$\Rightarrow m = 3$

$m = -\frac{1}{3}$

lines are $y = 3(x - 3)$

$\Rightarrow 3x - y - 9 = 0$ &

$y = -\frac{-1}{3}(x - 3)$

$\Rightarrow x + 3y - 3 = 0$

Q.14 (B)

$L_1 : x + \sqrt{3}y = 2$, $L_2 : ax + by = 1$, $q = 45^\circ$,

$L_3 = y\ \sqrt{3}x$

$$\begin{vmatrix} 1 & \sqrt{3} & -2 \\ a & b & -1 \\ \sqrt{3} & -1 & 0 \end{vmatrix} = 0$$

$\Rightarrow \sqrt{3}(-\sqrt{3} + 2b) + (-1 + 2a) = 0$

$\Rightarrow a + \sqrt{3}b = 2$ ....(i)

$m_1 = \frac{-1}{\sqrt{3}}$, $m_2 = -\frac{a}{b}$

$$\tan 45^\circ = \left|\frac{-\frac{1}{\sqrt{3}} + \frac{a}{b}}{1 + \frac{a}{\sqrt{3}b}}\right|$$

$\Rightarrow |a + \sqrt{3}b| = |\sqrt{3}a - b|$

$\Rightarrow (a + \sqrt{3}b)^2 + 2\sqrt{3}ab = 3a^2 + b^2 - 2\sqrt{3}ab$

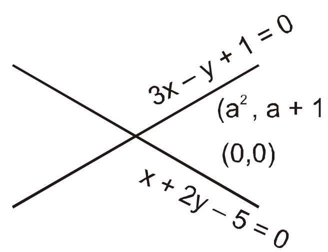
$\Rightarrow a^2 + b^2 - 2\sqrt{3}ab$ ....(ii)

squaring (i) & adding (ii)

$2a^2 + ab^2 = 4 \Rightarrow a^2 + b^2 = 2$

Q.15 (B)

Oragin R(a^2, a + 1) lies same side w.r.t. to given lines



$a^2 + 2a + 2 - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a + 3)(a - 1) < 0$

$\Rightarrow a \in (-3, 1)$

$3a^2 - (a + 1) + 1 > 0$

$\Rightarrow 3a^2 - a > 0$

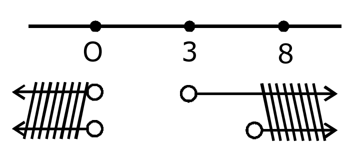
$\Rightarrow a(3a - 1) > 0$

$\Rightarrow a \in (\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

take intersection we get $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

Q.16 (A)

L_1 ; $2x + y - a = 0$ 0 (0, 0), P (3, 2)



$L_2 : x - 3y + a = 0$

$\Rightarrow L_1(0)\, L_1(P) > 0$ & $L_2(0)\, L_2(P) > 0$

$-a(8 - a) > 0$ & $a(-3 + a) > 0$

$a(a-8) > 0$ & $a(a-3) > 0$

$a \in (-\infty, 0) \cup (8, \infty)$ & $a \in (-\infty, 0) \cup (3, \infty)$

$\Rightarrow a \in (-\infty, 0) \cup (8, \infty)$

Q.17 (B)

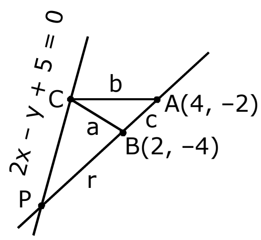
P lies on $2x - y + 5 = 0$

$|PA - PB|$ is maximum

we know

$b < a + c$

$b - a < c$



If $b - a = c$

then $(P - PB)$ is max.

$\Rightarrow$ PBA colinear

Slope $m_{AB} = 1 = \tan\theta$ If $PB = r$

$$\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y+4}{\frac{1}{\sqrt{2}}} = r \Rightarrow x = \frac{r}{\sqrt{2}} + 2,\ y = \frac{r}{\sqrt{2}} - 4$$

Satisfy given equation

$$2\left(\frac{r}{\sqrt{2}} + 2\right) - \left(\frac{r}{\sqrt{2}} - 4\right) + 5 = 0$$

$$2\frac{r}{\sqrt{2}} + 4 - \frac{r}{\sqrt{2}} + 4 + 5 = 0$$

$$\frac{r}{\sqrt{2}} = -13 \qquad \Rightarrow r = -13\sqrt{2}$$

$$P\left(\frac{-13\sqrt{2}}{\sqrt{2}} + 2, \frac{-13\sqrt{2}}{\sqrt{2}} - 4\right) \equiv (-11, -17)$$

Q.18 (D)

$L_1 : 2x - 3y - 6 = 0$

$L_2 : 3x - y + 3 = 0$

$L_3 : 3x + 4y - 12 = 0$ P(a, 0), Q(0, β)

By geometry origin lies in Δ

$L_1(0) < 0$ & $L_2(0) > 0$ $L_3(0) < 3$

$\Rightarrow L_1(P) \le 0$ & $L_2(P) \ge 0$ & $L_3(P) \le 0$

$\alpha - 3$ & $a + 1 \ge 0$ & $a \le 4$

$\Rightarrow a \in [-1, 3]$

$\Rightarrow L_1(Q) \le 0$ & $L_2(Q) \ge 0$ & $L_3(Q) \le 0$

$-3\beta - 6 \le 0$ & $-b + 3 \ge 0$ & $4\beta - 12 \le 0$

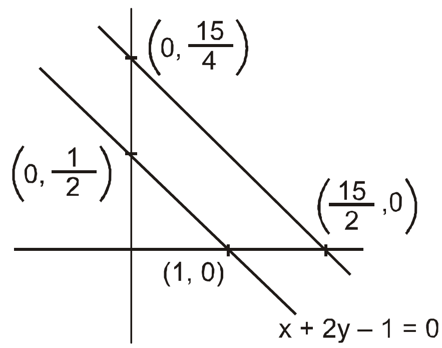
$\beta \ge -2$ & $\beta \le 3$ $\beta \le 3$ & $\beta \le 3 \Rightarrow \beta \in [-2, 3]$

Q.19 (A)

Point P $\left(1 + \frac{t}{\sqrt{2}}, 2 + \frac{t}{\sqrt{2}}\right)$ lies between given line

Hence $\left(1 + \frac{t}{\sqrt{2}}\right) + 2\left(2 + \frac{t}{\sqrt{2}}\right) - 1 = 0$

$$5 + \frac{3t}{\sqrt{2}} - 1 = 0 \Rightarrow t = -\frac{4\sqrt{2}}{3}$$



Now, $2\left(1 + \frac{t}{\sqrt{2}}\right) + 4\left(2 + \frac{t}{\sqrt{2}}\right) - 15 = 0$

$$\Rightarrow 10 + \frac{6t}{\sqrt{2}} - 15 = 0 \Rightarrow t = \frac{5\sqrt{2}}{6}$$

Hence $t \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$.

Q.20 (D)

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0\ a, b \in R, a \ne 1, b \pm 1, c \ne c$$

$C_2 \to C_2 \to C_1$ & $C_3 \to C_3 \to C_1$

$\Rightarrow a(b-1)(c-1) - (1-a)(c-1)$

$+1(0 - (1-a)(b-1)) = 0$

$$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

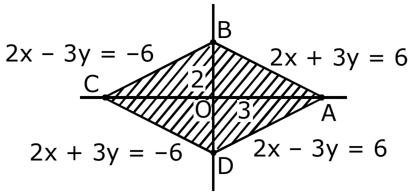
$$\Rightarrow \left(1 + \frac{a}{1-a}\right) + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Q.21 **(C)**

$2|x| + 3|y| \le 6$

area ABCD = 4 (ΔOAB)



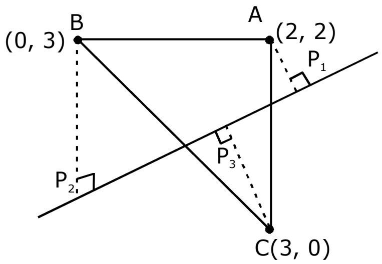
$= 4\left(\frac{1}{2}.2\times 3\right)$ = 12 sq. units

Q.22 (D)

Let a line ax + by + c = 0

$P_1 + P_2 + P_3 = 0$

$$\frac{3a+c}{\sqrt{a^2+b^2}} + \frac{3b+c}{\sqrt{a^2+b^2}} + \frac{2a+2b+c}{\sqrt{a^2+b^2}} = 0$$



5a + 5b + 3c = 0

$a\left(\frac{5}{3}\right) + b\left(\frac{5}{3}\right) + C = 0$

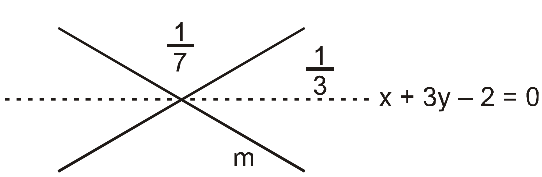
$\Rightarrow \left(\frac{5}{3},\frac{5}{3}\right)$ satisfy the given line

$\Rightarrow$ fix point is $\left(\frac{5}{3},\frac{5}{3}\right)$ which is centroid of ΔABC

Q.23 (C)

point of intersection of x + 3y – 2 = 0 and x – 7y + 5 = 0 is $\left(-\frac{1}{10},\frac{7}{10}\right)$

$$\left(\frac{-\frac{1}{3}-m}{1-\frac{m}{3}}\right) = -\left(\frac{-\frac{1}{3}-\frac{1}{7}}{1-\frac{1}{21}}\right)$$



$\Rightarrow \frac{-1-3m}{3-m} = \frac{10}{20} = \frac{1}{2}$

$\Rightarrow$ – 2 – 6m = 3 – m

$\Rightarrow$ m = – 1

Hence requred equation

$$y - \frac{7}{10} = -1\left(x+\frac{7}{10}\right)$$

$\Rightarrow$ 10y – 7 = – 10x – 1 $\Rightarrow$ 10x + 10y = 6 $\Rightarrow$ 5x + 5y = 3

Q.24 (B)

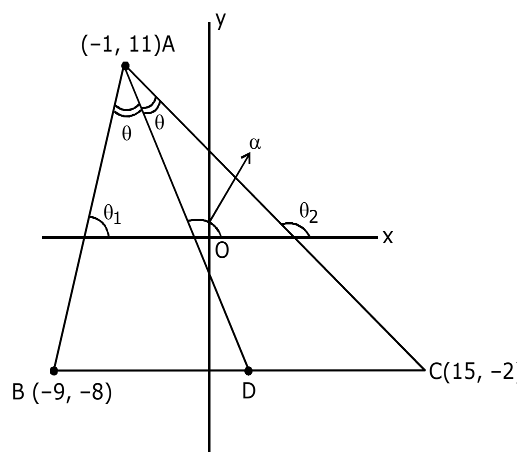
By geometry

Angle bisector of A is origin containing

line AB : 19x – 8y + 107 = 0

Line AC : –13x – 16y + 163 = 0

$$\frac{19x-8y+107}{\sqrt{19^2+8^2}} = \frac{-13x-16y+163}{\sqrt{13^2+16^2}}$$



$\{19^2 + 8^2 = 13^2 + 16^2 = 425$

$\Rightarrow$ 32x + 8y – 56 = 0 $\Rightarrow$ 4x + y = 7

Aliter :

$m_{AB} = \frac{19}{8} = \tan\theta_1$, $m_{AC} = \tan\theta_2 = \frac{-13}{16}$

$$\tan 2\theta = \left|\frac{\frac{19}{8}+\frac{13}{16}}{1-\frac{19}{8}.\frac{13}{6}}\right| = \left|\frac{-136}{13}\right|$$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{136}{13}$ {θ is acute tan θ > 0

$\Rightarrow 68\tan^2\theta + 13\tan\theta - 68 = 0 \Rightarrow \tan\theta = 0.9$

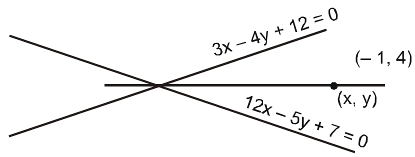
$\alpha = \theta + \theta_1$

$\tan\alpha = \dfrac{\tan\theta + \tan\theta_1}{1 - \tan\theta\tan\theta_1}$

equation is $(y - 11) = \tan\alpha\ (x + 1)$

Q.25 (A)

at (– 1, 4)



$3x - 4y + 12 < 0$ and $12x - 5y + 7 < 0$

$\Rightarrow \dfrac{3x - 4y + 12}{12x - 5y + 12} > 0$ at (– 1, 4)

So we have to take the bisector with + sign

$\dfrac{3x - 4y + 12}{5} = \dfrac{12x - 5y + 7}{13}$

$21x + 27y - 121 = 0$

Q.26 (C)

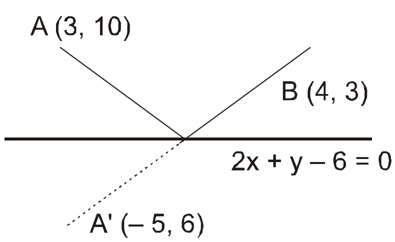
Image of A(1, 2) in line mirror y = x is (2, 1)

Image of b(2, 1) in y = 0 (x – axes) is 2, – 1)

Hence, $\alpha = 2, \beta = -1$

Q.27 (B)

Image of A(3, 10) in $2x + y - 6 = 0$



$\dfrac{x-3}{2} = \dfrac{y-10}{1} = -2\left(\dfrac{6+10-6}{2^2+1^2}\right)$

$\dfrac{x-3}{2} = \dfrac{y-10}{1} = -4$

A' = (– 5, 6)

Equation of A'B is $y - 3 = \left(\dfrac{6-3}{-5-4}\right)(x - 4)$

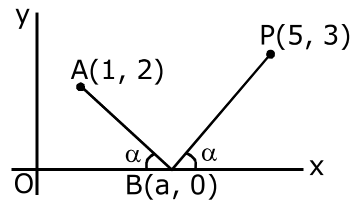
$y - 3 = -\dfrac{1}{3}(x - 4)$

$3y - 9 = -x + 4 \Rightarrow x + 3y - 13 = 0$

Q.28 (A)

$m_{AB} + m_{PB} = 0$

$\dfrac{2}{1-a} + \dfrac{3}{5-a} = 0$



$\Rightarrow a = \dfrac{13}{5}$

$m_{AB} = \dfrac{2}{1 - \frac{13}{5}} = \dfrac{10}{-8} = \dfrac{5}{-4}$

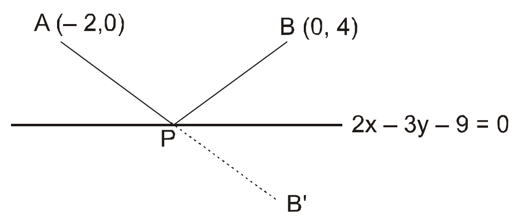
equation of AB $\Rightarrow y - 2 = -\dfrac{-5}{4}(x - 1)$ $5x + 4y = 13$

Q.29 (C)

Both A & B are same side of line $2x - 3y - 9 = 0$

Now, permeter of ΔA pm weel be least when pts A, P, B wees be collinear. Let B' is image of B

Then $\dfrac{x-0}{2} = \dfrac{y-4}{-3} = -2\left(\dfrac{0-12-9}{2^2+(-3)^2}\right)$



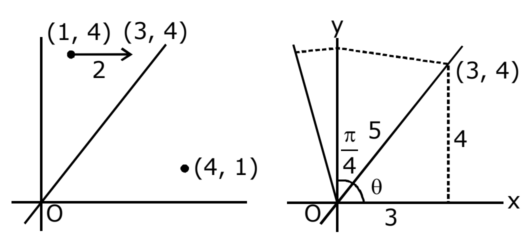
$\Rightarrow$ B' $\left(\dfrac{84}{13}, \dfrac{-74}{13}\right)$

Now equation of AB' is $y = \dfrac{-74}{110}(x + 2)$

point of intersection of given line & Q is P $\left(\dfrac{21}{17}, \dfrac{-37}{17}\right)$.

Q.30 (C)

(i) Reflection about y = x of (4, 1) is (1, 4)



(ii) Now 2 units along (+) x direction

(1 + 2, 4 + 0) ≡ (3, 4)

(iii) we wish to find

$$\left(5\cos\left(\theta+\frac{\pi}{4}\right),5\sin\left(\theta+\frac{\pi}{4}\right)\right)$$

$$x = 5\frac{\cos\theta}{\sqrt{2}} - \frac{5\sin\theta}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$y = 5\frac{\sin\theta}{\sqrt{2}} + \frac{5\cos\theta}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

$$(x, y) \Rightarrow \left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$$

Q.31 (C)

$2x^2 + 4xy / py^2 + 4x + 4x + qy + 1 = 0$

$a = 2, b = –p, c = 1, f = -\frac{q}{2}, y = 2, h = 2$

$abc + 2fgh – af^2 – bg^2 – ch^2 = 0$

$\Rightarrow -2p + 4q - \frac{q^2}{2} + 4P - 4 = 0$

$\Rightarrow 2P + 4q - \frac{q^2}{2} - 4 = 0$

$\perp \Rightarrow a + b = 0$

$\Rightarrow 4 + 4q - \frac{q^2}{2} - 4 = 0$

$2 – p = 0$

$\Rightarrow q\left(4-\frac{q}{2}\right) = 0$

$p = 2$

$\Rightarrow q = 0 , q = 8$

Q.32 (B)

Let equations of lines represented by the line pair $xy – 3y^2 + y – 2x + 10 = 0$ are

$y + c_1 = 0, x – 3y + c_2 = 0$

lines $\perp$ to these lines and passing through origin are

$x = 0, y = – 3x$

Joint equation

$x (3x + y) = 0$

$\Rightarrow xy + 3x^2 = 0$

Q.33 (C)

$x^2 – 2pxy – y^2 = 0$

pair of angle bisector of this pair $\frac{x^2-y^2}{1-(-1)} = \frac{xy}{-p}$

$\Rightarrow x^2 - y^2 + \frac{2}{p}xy = 0$

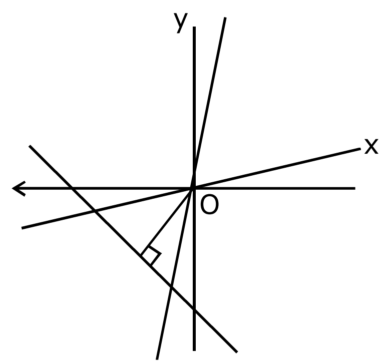
compare this bisector pair with $x^2 – 2qxy – y^2 = 0$

$\frac{2}{p} = -2q \Rightarrow pq = -1.$

Q.34 (D)

$x^2 – 4xy + y^2 = 0, x + y + 4\sqrt{6} = 0$

angle bisector of given pair of st. lines



$$\frac{x^2-y^2}{a-b} = \frac{xy}{h} \Rightarrow \frac{x^2-y^2}{1-1} = \frac{xy}{-2}$$

$\Rightarrow x^2 – y^2 = 0$

$\Rightarrow (x + y) (x – y) = 0$

$x + y = 0$ is $\parallel$ to third side

altitude $\equiv$ angle bisector $\Rightarrow$ isosceles Δ

Now $\tan\theta = \left|\frac{2\sqrt{h^2ab}}{a+b}\right| = \left|\frac{2\sqrt{4-1}}{2}\right| = \sqrt{3}$

$\Rightarrow \theta = 60^o$

$\Rightarrow$ angle between two equal sides is 60^o

$\Rightarrow$ equiliteral Δ

Q.35 (B)

$x^2 – 4xy + 4y^2 + x – 2y – 6 = 0$

$(x – 2y + C) (x – 2y + d) = 0$

$(x – 2y)^2 + (C + d) x – 2 (c + d) y + cd=0$

$c + d = 1, cd = – 6$

$c = 3, d = –2$

lines are $(x – 2y + 3) = 0, (x – 2y – 2) = 0$

distance $= \left|\frac{3-(-2)}{\sqrt{1^2+2^2}}\right| = \frac{5}{\sqrt{5}} = \sqrt{5}$

Q.36 (A)

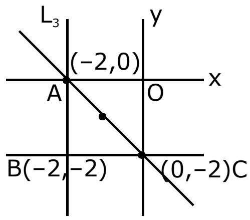
$xy + 2x + 2y + 4 = 0$ & $x + y + 2= 0$

$(x + c) (y + d) = 0$

$xy + dx + cy + cd=0$

$d = 2, c = 2$

$$\frac{x+z=0}{L_1} \& \frac{y+z=0}{L_2}$$



& $\dfrac{x+y+z=0}{L_3}$

$L_1 \perp L_2$ L_2
hypotaneous line L_3
mid point of hypotenous is circumcentre

$\left(\dfrac{0-2}{2}, \dfrac{-2-0}{2}\right)$ = (–1, –1)

Q.37 (B)

ax ± by ± C = 0

$m_1 = -\dfrac{a}{b}$, $m_2 = \dfrac{a}{b}$

$d_1 = -\dfrac{c}{b}$, $d_2 = \dfrac{c}{b}$

$d_1 = \dfrac{c}{b}$, $d_2 = -\dfrac{c}{b}$

Area of rhombus = $\left|\dfrac{(c_1-c_2)(d_1-d_2)}{(m_1-m_2)}\right|$

$= \left|\dfrac{2\dfrac{c}{b}\times\dfrac{2c}{b}}{2\dfrac{a}{b}}\right| = \dfrac{2c^2}{|ab|}$ sq. units

Q.38 (B)

Homogenize $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ by x + ky = 1

$5x^2 + 12xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$

it is equally indined with x-axes hence coeff. xy = 0

12 + 4H – 2 + 6H = 0

k = – 1

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, C)

Let requred point is P & Q

P divides in 1 : 2

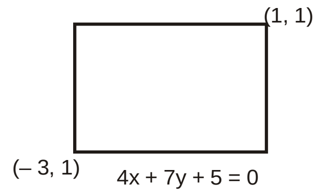
$P\left(\dfrac{9+2\times 0}{1+2}, \dfrac{1\times 12+2\times 0}{1+2}\right) \equiv (3, 4)$

Q divides in 2 : 1

Hence $Q\left(\dfrac{2\times 9+1\times 0}{2+1}, \dfrac{2\times 12+1+0}{2+1}\right) \equiv Q\ (6, 8)$

Q.2 (A, C, D)

Line ⊥ to 4x + 7y + 5 = 0 is



$7x - 4y + \lambda = 0$

It passes through (– 3, 1) and (1, 1)

$-11 - 4 + \lambda = 0 \Rightarrow \lambda = 25$

$7 - 4 + \lambda = 0 \Rightarrow \lambda = -3$

Hence lines are 7x – 4y + 25 = 0, 7x – 4y 3 = 0

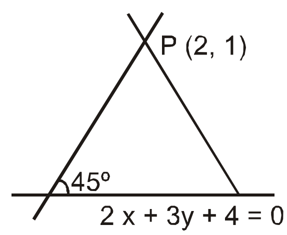
line 11 to 4x + 7y + 5 = 0 passing through (1, 1) is 4x + 7y + λ = 0

$\Rightarrow \lambda = -11$

$\Rightarrow 4x + 7y - 11 = 0$

Q.3 (A, C)

Let slope of requered line is m



Now,y – 1 = m(x – 2)

$\tan 15 = \left|\dfrac{m+\dfrac{2}{3}}{1-\dfrac{2m}{3}}\right| = \left|\dfrac{3m+2}{3-2m}\right|$

$\Rightarrow \dfrac{3m+2}{3-2m} = \pm 1 \Rightarrow 3m + 2 = \pm (3 - 2m)$

$\Rightarrow m = \dfrac{1}{5}, -5$

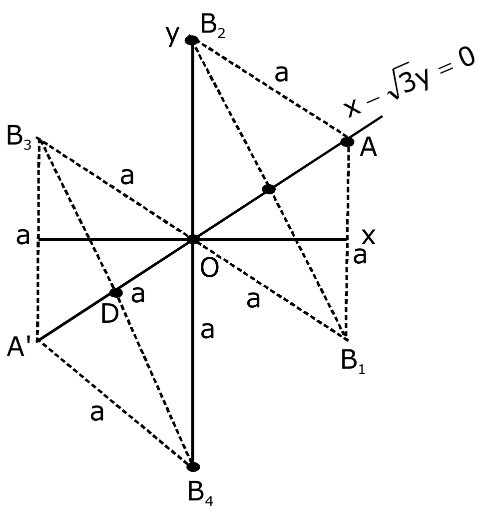
Hence, $y - 1 = \dfrac{1}{5}(x - 2) \Rightarrow x - 5y + 3 = 0$

$y - 1 = -5(x - 2) \Rightarrow 5x + y - 11 = 0$

Q.4 (A,B,C,D)

$y = \dfrac{1}{\sqrt{3}}\, x$

$\tan\theta = \dfrac{1}{\sqrt{3}}$,



$\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

$$\frac{x}{\frac{\sqrt{3}}{2}} = \frac{y}{\frac{1}{2}} = \pm a$$

$$\Rightarrow A\left(\frac{a\sqrt{3}}{2}, \frac{a}{2}\right), A'\left(\frac{-a\sqrt{3}}{2}, \frac{-a}{2}\right)$$

$$D\left(\frac{\sqrt{3}a}{4}, \frac{a}{4}\right), D'\left(-\frac{\sqrt{3}a}{4}, \frac{a}{4}\right)$$

equation of B_1B_2, $m_{B_1B_2} = -\sqrt{3}$

$$\frac{x \mp \frac{\sqrt{3}a}{4}}{-\frac{1}{2}} = \frac{y \mp \frac{a}{4}}{\frac{\sqrt{3}}{2}} = \pm \frac{\sqrt{3}a}{2}$$

$$B_1\left(\frac{\sqrt{3}a}{2}, \frac{-a}{2}\right), B_2(0, a), B_3\left(\frac{-\sqrt{3}a}{2}, \frac{a}{2}\right),$$

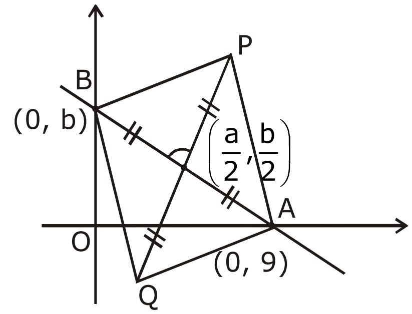
$B_4(0, -a)$

Q.5 (A,C)

$m_{AB} = \frac{-b}{a}$

$m_{PQ} = \frac{a}{b}$

parametric form of PQ



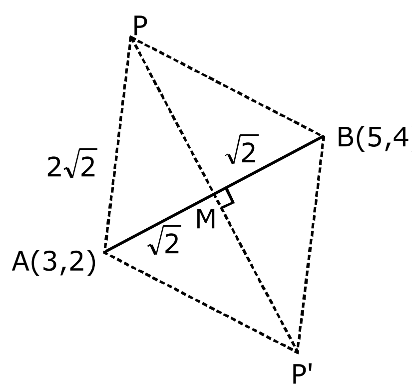
$$\frac{x - \frac{a}{2}}{\frac{b}{\sqrt{a^2+b^2}}} = \frac{y - \frac{b}{2}}{\frac{a}{\sqrt{a^2+b^2}}} = \pm\left(\frac{\sqrt{a^2+b^2}}{2}\right)$$

$$\frac{x - \frac{a}{2}}{b} = \frac{y - \frac{b}{2}}{a} = \pm\frac{1}{2}$$

$$\Rightarrow x = \frac{a}{2} \pm \frac{b}{2}, y = \frac{b}{2} \pm \frac{a}{2}$$

$$\left(\frac{a \pm b}{2}, \frac{b \pm a}{2}\right)$$

Q.6 (A,B)
Mid point M (4, 3)



$m = \frac{2}{2} = 1 m_{PQ} = -1$

$AB = \sqrt{2^2 + q^2} = 2\sqrt{2}$ $\frac{PM = \sqrt{6}}{\text{line } pp'}$

$$\frac{x - 4}{\frac{1}{\sqrt{2}}} = \frac{y - 3}{\frac{1}{\sqrt{2}}} = \pm\sqrt{6}$$

$x = 4 \pm \sqrt{3}$ $y = 3 \pm \sqrt{3}$

$x = 4 \pm \sqrt{3}$, $y = 3 \pm \sqrt{3}$

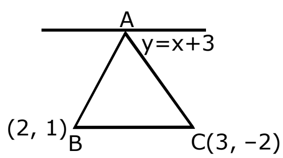
$(4 + \sqrt{3}, 3 - \sqrt{3})$ & $(4\sqrt{3}, 3 + \sqrt{3})$

Q.7 (C, D)

Let vertex A (a, a + 3)

$\Delta ABC = 5$ sq. units

$$\frac{1}{2}\begin{vmatrix} a & a+3 & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{vmatrix} = \pm 5$$



$\Rightarrow$ (3) a – (a + 3) (–1) + (–4 – 3) = ± 10

$\Rightarrow$ 4a = ± 10 + 4 $\qquad \Rightarrow$ $a = \frac{7}{2}, \frac{-3}{2}$

A $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

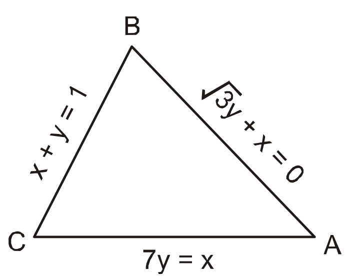
Q.8 (B, C)

Let slope of given lines

$m_1 = \frac{1}{7}$, $m_2 = \frac{-1}{\sqrt{3}}$, $m_3 = -1$

Hence interior angle of triangle

$$\tan A = \frac{m_1 - m_2}{1 + \frac{m}{m_2}} = \frac{\frac{1}{7} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{7\sqrt{3}}} = \frac{\sqrt{3} + 7}{7\sqrt{3} - 1} > 0$$



$$\tan B = \frac{m_2 - m_1}{1 + m_2 m_3} = \frac{-\frac{1}{\sqrt{3}} + 1}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} > 0$$

$$\tan C = \frac{m_3 - m_1}{1 + m_2 m_1} = \frac{-1 - \frac{1}{7}}{1 - \frac{1}{7}} = \frac{-8}{6} < 0$$

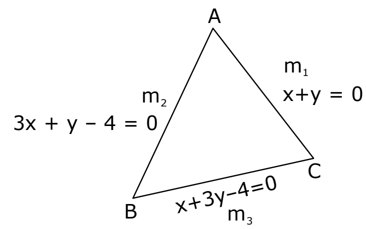
Hence angle C is obt. Therefore circumcentre and orthocentre less outside the triangle.

Q.9 (A,C)

$L_1 : x + y = 0$ $m_1 = -1$

$L_2 : 3x + y - 4 = 0$ $\qquad m_2 = -3$

$L_3 : x + 3y - 4 = 0$ $\qquad m_3 = -\frac{1}{3}$



Slope is decreasing order

$m_3 > m_1 > m_2$

$-\frac{1}{3} > -1 > -3$

$m_3 > m_1 > m_2$

$-\frac{1}{3} > -1 > -3$

$$\tan C = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{-\frac{1}{3} + 1}{1 + \frac{1}{3}} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-1 + 3}{1 + 3} = \frac{2}{4} = \frac{1}{2}$$

A = C & B is obtuse.

A = C & B is obtuse.

Obtuse isosceles triangle.

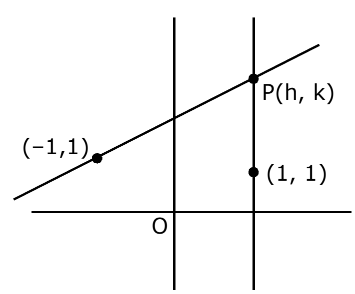
Q.10 (C, D)

$|m_1 - m_2| = 2$

$m_1 = \frac{k-1}{h-1}$, $m_2 = \frac{k-1}{h+1}$

$\Rightarrow \left(\frac{k-1}{h-1} - \frac{k-1}{h+1}\right)^2 = 4$

$\Rightarrow (k-1)^2 \left(\frac{2}{h^2 - 1}\right)^2 = 4$



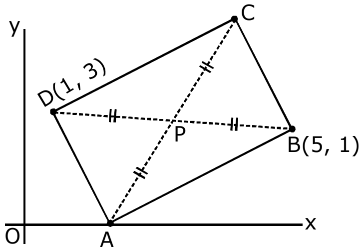
$\Rightarrow (k - 1)^2 = (h^2 - 1)^2$

$\Rightarrow$ $(y - 1) = \pm (x^2 - 1)$
$\Rightarrow$ $y = x^2$ or $y = 2 - x^2$

Q.11 (A, B, D)

Q.12 (A,D)

$y = 2x + c$



Diagonal bisect each other
mid point of BD is P (3, 2)
y = 2x + C passing through P
$\Rightarrow$ $2 = 6 + c$ $\Rightarrow$ $c = -4$

AP=BPO=CP=DP, BP = $\sqrt{2^2 + (-1)^2} = \sqrt{5}$

parametric form of AC
$\tan\theta = 2$, P(3, 2)

$$\frac{x-3}{\sqrt{5}} = \frac{y-2}{\frac{2}{\sqrt{5}}} = \pm\sqrt{5}$$

$x = 3 \pm 1$, $y = 2 \pm 2$ $\Rightarrow$ A(2, 0), C(4, 4)

Q.13 (A,C)

Lengths from origin

$$\left|\frac{cd}{\sqrt{c^2+d^2}}\right| = \left|\frac{ab}{\sqrt{a^2+b^2}}\right|$$

$$\Rightarrow \frac{c^2d^2}{c^2+d^2} = \frac{a^2b^2}{a^2+b^2} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{c^2}+\frac{1}{d^2}$$

all three lines will be concurrent

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{b} & -1 \\ \frac{1}{b} & \frac{1}{a} & -1 \\ \frac{1}{c} & \frac{1}{d} & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{Q}\left(\frac{-1}{a}+\frac{1}{d}\right)-\frac{1}{b}\left(\frac{-1}{b}+\frac{1}{c}\right)-1\left(\frac{1}{bd}-\frac{1}{ac}\right) = 0$$

$$\Rightarrow -\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{b^2}-\frac{1}{bc}-\frac{1}{bd}+\frac{1}{ac} = 0$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{1}{c}\left(\frac{1}{a}-\frac{1}{b}\right)-\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{b}\right) = 0$$

Q.14 (A,B)

B should be (0, 0)
given diagonal AC is
$11x + 7y = 9$ ....(i)
equation of AC $(4x + 5y + C)(7x + 2y + d)$
$- (4x + 5y)(7x + 2y) = 0$
$(7C + 4d)x + (2C + 5d)y + cd = 0$ . ..(ii)
compair (i) & (ii)

$$\underbrace{\frac{7c+4d}{11} = \frac{2c+5d}{7}} = \frac{cd}{-9}$$

$$49c + 28d = 22c + 55d \qquad \frac{7c+4d}{11} = \frac{cd}{-9}$$
$$\Rightarrow c = d \qquad \Rightarrow 9c + C^2 = 0$$
$$C(C+9) = 0$$

C = 0 not possible
$\Rightarrow$ $c = -9$ & $d = -9$
Diagonal BD is
$(4x + 5y)(7x + 2y - 9)$
$- (4x + 5y - 9)(7x + 2y) = 0$
$\Rightarrow$ $-9(4x + 5y) - (-9)(7x + 2y) = 0$
$\Rightarrow$ $3x - 3y = 0 \Rightarrow x - y = 0$

Q.15 (A, C)

The lines will pass through (4, 5) & parallel to the bisectors between them

$$\frac{3x-4y-7}{5} = \pm\frac{12x-5y+6}{13}$$

by taking + sign, we get $21x + 27y + 121 = 0$
Now by taking – sign, we get $99x - 77y - 61 = 0$
so slopes of bisectors are

$$-\frac{7}{9}, \frac{9}{7}$$

Equation of lines are

$$y - 5 = \frac{-7}{9}(x - 4)$$

and $y - 5 = \frac{9}{7}(x - 4)$

$\Rightarrow$ $7x + 9y = 73$and $9x - 7y = 1$

Q.16 (A,B)

$L_1 : 2x + y = 5$ $L_2 : x - 2y = 3$
Line BC passing throug (2, 3)
$(y - 3) = m(x - 2)$
m is equal to slope of

$$\frac{2x+y-5}{\sqrt{2^2+1}} = \pm\frac{x-2y-3}{\sqrt{1+2^2}}$$

$\Rightarrow$ $2x \mp x + y \pm 2y = 5 \mp 3$
A/B^2 are

$x + 3y = 2 \Rightarrow m = -\frac{1}{3}$

& $3x - y = 8 \Rightarrow m = 3$

BC line

$y - 3(x - 2) \Rightarrow 3x - y = 3$

& $y - 3 = -\frac{1}{3}(x - 2) \Rightarrow x + 3y = 11$

Comprehenssion # 1 (Q. No. 17 to 19)

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points (3, 4) and (1, 2) respectively and centroid of Δ ABC is G(3, 2), then answer the following questions :

Q.17 (A)

Q.18 (B)

Q.19 (C)

Sol. **(17, 18, 19)**

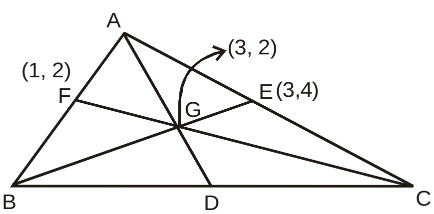
Let the co-ordinates of D(α, β)

then $\frac{\alpha + 1 + 3}{3} = 3 \Rightarrow \alpha = 5$

and $\frac{\beta + 2 + 4}{3} = 2 \Rightarrow \beta = 0$

$\therefore$ D(5, 0)

Taking $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$



then by $\frac{x_1 + x_2}{2} = 1$, $\frac{x_2 + x_3}{2} = 5$, $\frac{x_3 + x_1}{2} = 3$

and $\frac{y_1 + y_2}{2} = 2$, $\frac{y_2 + y_3}{2} = 0$, $\frac{y_1 + y_3}{2} = 4$

we get A(–1, 6) , B(3, –2) , C(7, 2)

equation of AB is 2x + y = 4

Height of altitude from A is = $\frac{2 \times \text{area}(\Delta ABC)}{BC}$

$= 6\sqrt{2}$

Comprehension # 2 (Q. No. 20 to 22)

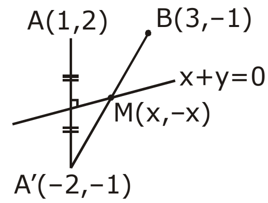
Q.20 (B)

Q.21 (D)

Q.22 (A)

Sol.20 AM+ BM $\geq$ AB

AM + BM=AB (minimum)



$\because$ AM = A'M

A'M + BM = AB

for A' $\Rightarrow \frac{x-1}{1} = \frac{y-2}{2} = \frac{-2(1+2)}{(1+1)}$

A'(– 2, – 1)

for AM + BM to be minimum, A', M, B are collinear

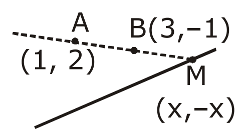
$\because \frac{1}{2}\begin{vmatrix} -2 & -1 & 1 \\ x & -x & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow x = 1$ $\qquad$ $y = -1$

M(1, –1)

Reflection of M in x = y is M'(– 1, 1)

Sol.21 $|AM - BM| \leq AB$



$|AM - BM|_{max} = AB$

Only possible when A, M, B are collinear

$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 1 \\ x & -x & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 2 & -3 & 0 \\ x-1 & -x-2 & 0 \end{vmatrix} = 0$

$\Rightarrow 2(-x - 2) + 3(x - 1) = 0 \Rightarrow x = 7$

M(7, – 7) & N(1, 1)

MN = $\sqrt{36 + 64} = \sqrt{100} = 10$

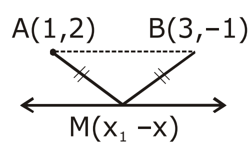
Sol.22 $|AM - BM| \geq 0 \Rightarrow AM - BM = 0$ (min)

$\Rightarrow$ AM = BM

$\Rightarrow (-x - 2)^2 + (x - 1)^2$

$= (-x + 1)^2 + (x - 3)^2$

$\Rightarrow 2x + 5 = -8x + 10$



$\Rightarrow 10x = 5 \Rightarrow x = \frac{1}{2}, y = -\frac{1}{2}$

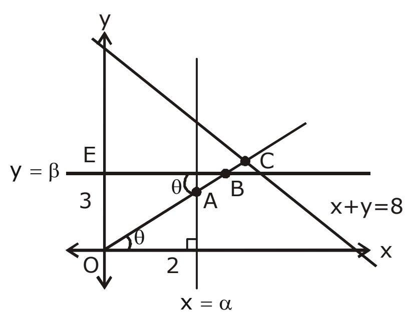
Area of ΔAMB = $\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 2 & -3 & 0 \\ -\frac{1}{2} & -\frac{5}{2} & 0 \end{vmatrix}$

$$\Rightarrow \frac{1}{2}\left[2\left(\frac{-5}{2}\right)+3\left(\frac{-1}{2}\right)\right] = \frac{1}{2}\left|\frac{-13}{2}\right| = \frac{13}{4}.$$

Comprehension # 3 (Q. No. 23 to 25)

Q.23 (B)
Q.24 (C)
Q.25 (A)

Sol.



Sol.23 $f(\alpha, \beta) = \left|\frac{\beta}{\alpha} - \frac{3}{2}\right| + (3\alpha - 2\beta)^6 + \sqrt{e\alpha + 2\beta - 2e - 6} \le 0$

$\therefore$ every term is zero.

$\frac{\beta}{\alpha} - \frac{3}{2} = 0 \Rightarrow 2\beta = 3\alpha$

& $e\alpha + 2\beta = 2e + 6$

$\alpha = 2 \therefore \beta = 3$

Sol.24 In ΔOAD,In ΔOBE,

$OA = \frac{2}{\cos\theta}$ $OB = \frac{3}{\sin\theta}$

for OC,

Let equation of OC be

$y = (\tan\theta)\, x$

.....(1)

& $x + y = 8$

....(2)

Solving (1) & (2)

$x(1 + \tan\theta) = 8$

$x = \frac{8}{1+\tan\theta}$, $y = \frac{8\tan\theta}{1+\tan\theta}$

are co-ordinates of C

$$OC = \sqrt{\frac{64}{(1+\tan\theta)^2} + \frac{64\tan^2\theta}{(1+\tan\theta)^2}}$$

$$OC = \frac{8\sec\theta}{1+\tan\theta} = \frac{8}{\cos\theta + \sin\theta}$$

Given OA. OB. OC = $48\sqrt{2}$

$\sin\theta.\cos\theta.(\sin\theta + \cos\theta) = \frac{1}{\sqrt{2}}$

$\frac{\sin 2\theta}{2}\sqrt{1+\sin 2\theta} = \frac{1}{\sqrt{2}}$

put $\sin 2\theta = t$

$\therefore t^3 + t^2 - 2 = 0$

$(t - 1)(t^2 + 2t + 2) = 0$

$t = 1 \Rightarrow \sin 2\theta = 1 \Rightarrow \theta = 45°$

$\therefore OA = 2\sqrt{2}$; $OB = 3\sqrt{2}$; $OC = 4\sqrt{2}$

Sol.25 $y = (\tan\theta)\, x$

$\Rightarrow y = x$

Comprehension # 4 (Q. No. 26 to 28)

Q.26 **(D)**

$c + f = 4$

Q.27 **(A)**

Equation of a straight line through (2, 3) and inclined at an angle of $(\pi/3)$ with y-axis ($(\pi/6)$ with x-axis) is

$\frac{x-2}{\cos(\pi/6)} = \frac{y-3}{\sin(\pi/6)} \Rightarrow x - \sqrt{3}y = 2 - 3\sqrt{3}$

Points at a distance c + f = 4 units from point P are

$(2 + 4\cos(\pi/6), 3 + 4\sin(\pi/6)) \equiv (2 + 2\sqrt{3}, 5)$

and $(2 - 4\cos(\pi/6), 3 - 4\sin(\pi/6)) \equiv (2 - 2\sqrt{3}, 1)$

only (A) is true out of given options

Q.28 (C)

Slopes of the lines

$3x + 4y = 5$ is $m_1 = -\frac{3}{4}$

and $4x - 3y = 15$ is $m_2 = \frac{4}{3}$

$\because$ $m_1 m_2 = -1$

$\therefore$ given lines are perpendicular and $\angle A = \frac{\pi}{2}$

Now required equation of BC is

$(y - 2) = \frac{m \pm \tan(\pi/4)}{1 \mp m\tan(\pi/4)}(x - 1)$......(1)

where m = slope of AB = $-\frac{3}{4}$

$\therefore$ equation of BC is (on solving (1))
$x - 7y + 13 = 0$ and $7x + y - 9 = 0$

$L_1 \equiv x - 7y + 13 = 0$

$L_2 \equiv 7x + y - 9 = 0$

Let required line be $x + y = a$

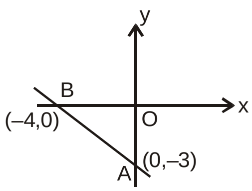
which is at $|b - 2a - 1| = |5 - 4 - 4\sqrt{3} - 1| = 4\sqrt{3}$ units from origin

$\therefore$ required line is $x + y - 4\sqrt{6} = 0$ (since intercepts are on positive axes only)

Q.29 (A) $\to$ (q, s),(B) $\to$ (r),(C) $\to$ (p), (D) $\to$ (q, s)

(A) Slope of such line is $\pm$ 1

(B) area of ΔOAB = $\frac{1}{2} \times 3 \times 4 = 6$ sq. units



(C) To represent pair of straight lines

$$\begin{vmatrix} 2 & -1 & -3 \\ -1 & -1 & 3 \\ -3 & 3 & c \end{vmatrix} = 0 \Rightarrow c = 3$$

(D) Lines represented by given equation are x + y + a = 0 and x + y – 9a = 0

$\therefore$ distance between these parallel lines is = $\frac{10a}{\sqrt{2}}$

$= 5\sqrt{2}a$

Q.30 **(A) $\to$ (R),(B) $\to$ (S),(C) $\to$ (Q)**

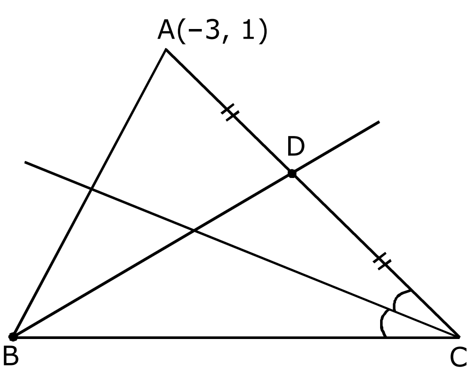
B median 2x + y – 3 = 0

angle bisector of C 7x – 4y –1 = 0

Let C on the line 7x – 4y – 100

$C\left(\lambda, \frac{7\lambda+1}{4}\right)$

D is mid point of AC lie median



$$D\left(\frac{-3+\lambda}{2}, \frac{1+\frac{7\lambda-1}{4}}{2}\right)$$

$$2\left(\frac{-3+\lambda}{2}\right) + \frac{3+7\lambda}{8} - 3 = 0$$

$-48 + 8\lambda + 3 + 7\lambda = 0 \Rightarrow \lambda = 3$

C (3, 5) & D(0, 3)

(C) line AC is y – 3 = 0 $\frac{2}{3}$(x – 0)

$\Rightarrow$ 2x –3y6 + 9 = 0 (Q)

(P) will not a side Q (It's given median)

(A) Line AB A(–3, 1) satisfy (R) 4x + 7y + 5 = 0

& (B) Line BC is only (S) 18x – y – 49 = 0

Q.31 **(A) $\to$ (Q),(B) $\to$ (P),(C) $\to$ (S), (D) $\to$ (R)**

$D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ and D = 0 is condition of concurrency

$D = -(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(A) if a + b + c = 0 but $\sum a^2 \neq \sum ab$ i.e. a,b,c are not all equal, then D = 0

hence lines are concurrent $\Rightarrow$ (Q)

(B) if a + b + c = 0 and $\sum a^2 = \sum ab \Rightarrow a = b = c$

$\therefore$ a = 0; b = 0; c = 0

$\Rightarrow$ lines becomes identical and of the form 0x + 0y + 0 = 0

any ordered pair (x, y) will satisfy $\Rightarrow$ complete xy plane $\Rightarrow$ (P)

(C) if a + b + c $\neq$ 0 and $\sum a^2 \neq \sum ab \Rightarrow$ a,b,c are not all equal $\Rightarrow$ D $\neq$ 0

In this case equations represents set of lines which are neither cncident nor concurrent $\Rightarrow$ (S)

(D) if a + b + c $\neq$ 0 and $\sum a^2 = \sum ab \Rightarrow a = b = c$

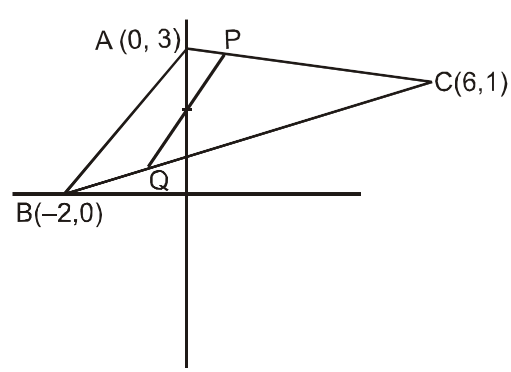
hence lines becomes identical or concident $\Rightarrow$ (R)

NUMERICAL VALUE BASED

Q.1 (**2**)

Since (λ, λ + 1) lies on y = x + 1

equation of A B : 3x – 2y + 6 = 0 ; BC : x – 8y + 2 = 0 ; AC : x + 3y – 9 = 0



Line y = x + 1 cuts AC at P $\left(\frac{3}{2},\frac{5}{2}\right)$ cut BC at

Q$\left(\frac{-6}{7},\frac{1}{7}\right)$. Hence $\lambda \in \left(\frac{-6}{7},\frac{3}{2}\right)$

Q.2 **(0)**

Let equation of line is $\ell x + my + n = 0$ ...(i)

given $\left(\frac{a^3}{a-1},\frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1},\frac{b^2-3}{b-1}\right)$

and $\left(\frac{c^3}{c-1},\frac{c^2-3}{c-1}\right)$ are collinear

$\left(\frac{t^3}{t-1},\frac{t^2-3}{t-1}\right)$ is general point which satisfies line (i)

$$\ell\left(\frac{t^3}{t-1}\right) + m\left(\frac{t^2-3}{t-1}\right) + n = 0$$

$\Rightarrow \quad \ell t^3 + mt^2 + nt - (3m + n) = 0$

$a + b + c = -\frac{m}{\ell} \Rightarrow \quad ab + bc + ac = \frac{n}{\ell}$

$\Rightarrow \quad abc = \frac{3m+n}{\ell}$

Now LHS = abc – (ab + bc + ac) + 3 (a + b + c) =

$$\frac{(3m+n)}{\ell} - \frac{n}{\ell} + 3\left(\frac{-m}{\ell}\right) = 0$$

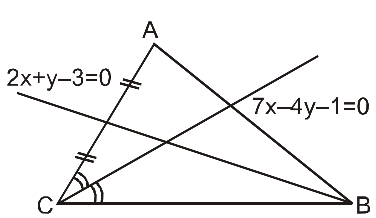
Q.3 18

Since C lies on 7x – 4y – 1 = 0, therefore let us choose its coordinates as $\left(h,\frac{7h-1}{4}\right)$.

The mid point of AC, i.e. $\left(\frac{h-3}{2},\frac{7h+3}{8}\right)$ lies on 2x + y – 3 = 0,

therefore we have $\left(\frac{h-3}{2}\right)+\left(\frac{7h+3}{8}\right) - 3 = 0$ gives h = 3

Hence, coordinates of C are (3, 5) and equation of AC is



$$y - 5 = \frac{5-1}{3+3}(x-3)$$

i.e., $\quad 2x - 3y + 9 = 0$ ....(1)

Let slope of BC = m. Since lines BC and AC $\left(\text{slope} = \frac{2}{3}\right)$ are equally inclined to the line 7x – 4y – 1 = 0 $\left(\text{slope} = \frac{7}{4}\right)$, therefore we have i.e., $\frac{m-\frac{7}{4}}{1+\frac{7m}{4}}$

$$= \frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{7}{6}} \text{ (see figure)}$$

i.e., $\frac{4m-7}{7m+4} = \frac{1}{2}$ gives m = 18.

Q.4 **(30)**

$9x^2(x + y - 5) = 4y^2(y + x - 5)$

$\Rightarrow \quad (x + y - 5)(3x - 2y)(3x + 2y) = 0$

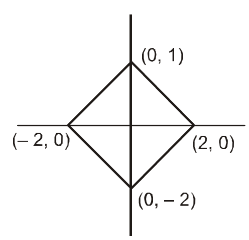
lines are $y = \frac{3x}{2}$; $y = \frac{-3x}{2}$; y = 5 – x

$\Rightarrow$ Area ≡ 30 sq. units.

Q.5 **(8)**

| x | + | y | = 2 represerts square of side = $2\sqrt{2}$

Hence area = 8

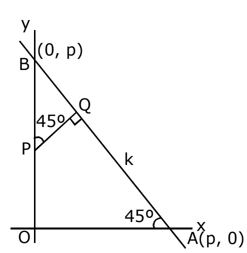


Q.6 **(3)**

x + y = p

Let Q divides AB in k : 1

$$\frac{\Delta Q}{QB} = \frac{k}{1}$$



$Q\left(\frac{p}{k+1}, \frac{pk}{k+1}\right)$, $m_{PQ} = 1$

line PQ . $y - \frac{kp}{k+1} = \left(x - \frac{p}{k+1}\right)$ (If cut y-axis)

then (x=0 put) $\Rightarrow y = \frac{(k-1)p}{(k+1)}$, $p\left(0, \frac{pk-p}{k+1}\right)$

$PQ = BQ = \sqrt{\left(\frac{p}{k+1}\right)^2 + \left(\frac{pk}{k+1} - \frac{pk}{k+1} + \frac{p}{k+1}\right)^2}$

$= \frac{\sqrt{2}pk}{k+1}$

Area $\Delta APQ = \frac{3}{8}\ \Delta OAB = \frac{3}{8} \cdot \frac{1}{2}p^2 = \frac{3}{16}p^2$

$\Rightarrow \frac{1}{2}\ \frac{\sqrt{2}pk}{(k+1)} \cdot \frac{\sqrt{2}p}{(k+1)} = \frac{3}{16}\ p^2$

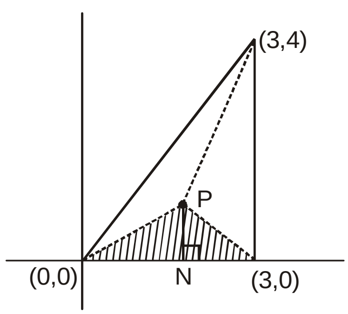
$\Rightarrow 16k = 3\,(k+1)^2 \Rightarrow 3k^2 + 6k + 3 = 16k$

$\Rightarrow k = 3\ k = \frac{1}{3}$ is reject

($\because$ P lies on OB only)

Q.7 (1)

Here BP and CP are angular bisectors. Maximum of d(P, BC) occurs, when P is incentre of ΔABC.



$\therefore$ Maximum of d(P, BC) = PN = ordinate of incentre = 1.

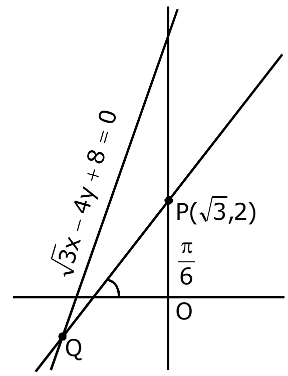
Q.8 (6)

Let PQ = r

equation of PQ

$\frac{x-\sqrt{3}}{\cos\frac{\pi}{6}} = \frac{y-2}{\sin\frac{\pi}{6}} = r$

$\Rightarrow Q\left(\sqrt{3} + \frac{\sqrt{3}r}{2}, 2 + \frac{r}{2}\right)$



satisfy given line

$\Rightarrow \sqrt{3}\left(\sqrt{3} + \frac{\sqrt{3}r}{2}, 2 + \frac{r}{2}\right) + 8 = 0$

$\Rightarrow 3 + \frac{3}{2}r - 8 - 2r + 8 = 0 \Rightarrow \frac{r}{2} = 3$

$\Rightarrow r = 6$

Q.9 (**19**)

Equation of family of curves passing through intersection of C_1 & C_2 is

$-\lambda x^2 + 4y^2 - 2xy - 9x + 3 + \mu(2x^2 + 3y^2 - 4xy + 3x - 1) = 0$ ....(i)

It will give the joint equation of pair of lines passing through origin,

if coefficient of x = 0 & Constant = 0

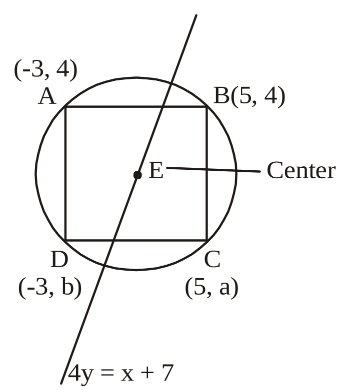
$\Rightarrow \mu = 3$

put $\mu = 3$ in equation (i), we get

$-\lambda x^2 + 4y^2 - 2xy + 6x^2 + 9y^2 - 12xy = 0$

It will subtend 90º at origin if coeff. of x^2 + coeff. of $y^2 = 0 \Rightarrow \lambda = -19$

Q.10 (**32**)



So C will be (5, a) ← D is (–3, b) Now Axa of two parts divided by diameter will be same.

get a and b and get Axa.

Q.11 **(52)**

Point be (x, y) but it lies on y = x + 2 So,
(x, x + 2)

$$F(x) = \left[\frac{3x-4(x+2)+8}{\sqrt{3^2+4^2}}\right]^2 + \left[\frac{3x-(x+2)-1}{\sqrt{3^2+1^2}}\right]^2$$

$$= \frac{2x^2+5[4x^2-12x+9]}{50}$$

$$= \frac{22\left[\left(x-\frac{30}{22}\right)^2-\frac{900}{484}\right]+45}{50}$$

F(x) is minimum at $x = \frac{15}{11}$. So point is $\left(\frac{15}{11}, \frac{37}{11}\right)$

= (a, b)

11 (a + b) = 52.

Q.12 **(2)**

$x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$

$(x + \sqrt{2}\,y + p)(x + \sqrt{2}\,y + q) = 0$

p + q = 4

pq = 1

Destance between 11 lines is $\left|\frac{p-q}{\sqrt{3}}\right|$

$$- \frac{\sqrt{(p+q)^2-4pq}}{\sqrt{3}} = \frac{\sqrt{16-4}}{\sqrt{3}} = 2$$

Q.13 **(2)**

Given lines are ax + y + 1 = 0 ....(i)

x + by = 0 ....(ii)

ax + by = 1 ....(iii)

Joint equation of (i) and (ii) is

(ax + y + 1) (x + by) = 0

$\Rightarrow ax^2 + by^2 + (ab + 1)\,xy + x + by = 0$

Making (iv) homogeneous with the help of equation (i) we have

$ax^2 + by^2 + (ab + 1)xy + x\,(ax + by) + by\,(ax + by) = 0$

since angle between these two lines is 90^0

$\therefore$ Coefficient of x^2 + Coefficient of y^2 = 0

$2a + b + b^2 = 0$ is the required condition.

Q.14 **(2)**

For collinearity of 3 points $\begin{vmatrix} -2 & 0 & 1 \\ -1 & \frac{1}{\sqrt{3}} & 1 \\ \cos 4\theta & \sin 4\theta & 1 \end{vmatrix} = 0$

$\Rightarrow \sqrt{3}\,\sin 4\theta - \cos 4\theta = 2 \Rightarrow \sin\left(4\theta - \frac{\pi}{6}\right) = 1$

$\Rightarrow 4\theta - \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi$

$\theta = \frac{\pi}{6} + \frac{k\pi}{2} \quad \Rightarrow \frac{\pi}{6}, \frac{2\pi}{3}$.

Q.15 **(2)**

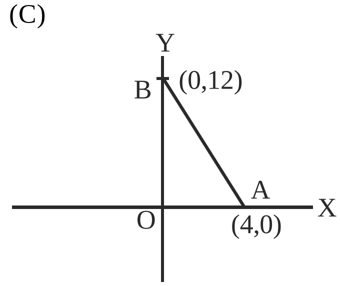
$x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$

$\Rightarrow \quad |m_1 - m_2| = \sqrt{(m_1+m_2)^2 - 4m_1m_2}$

$$\sqrt{\left(\frac{2\tan\theta}{\sin^2\theta}\right)^2 - 4\left(\frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}\right)} = 2$$

KVPY PREVIOUS YEAR'S

Q.1 (C)



$$\frac{1}{2}\begin{vmatrix} 1 & x & y \\ 1 & 0 & 12 \\ 1 & 4 & 0 \end{vmatrix} = \pm 18$$

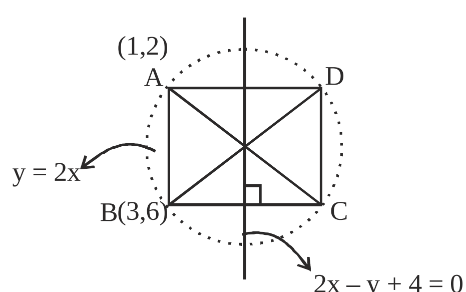
1(– 48) – x (–12) + y (4) = ± 36

12x + 4 y – 48 = ± 36

3x + y – 12 = ± 9

$(3x + y - 12)^2 = 81$

Q.2 (A)



Slope of AB = $\frac{4}{2} = 2$

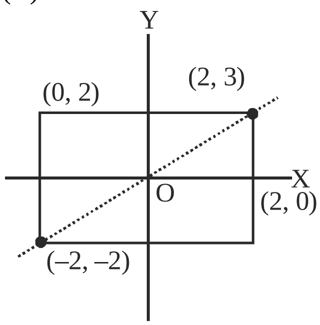
slope of BC = $-\frac{1}{2}$

$\ell(AB) = \sqrt{4+16} = 2\sqrt{5}$

distance between 2x – y + 4 = 0 & 2x – y = 0 $\Rightarrow \frac{4}{\sqrt{5}}$

Area = $2\sqrt{5}.\frac{8}{\sqrt{5}} = 16$

Q.3 (B)



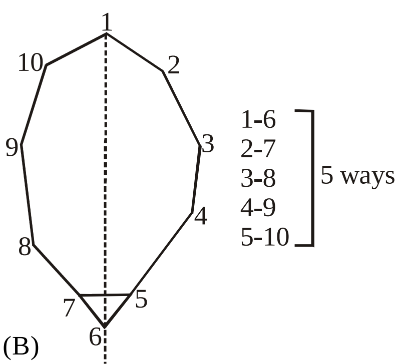
| x+ y |+ | x – y| = 4 represent a square

$x^2 + y^2 - 4x - 6y = (x-2)^2 + (y-3)^2 - 13$

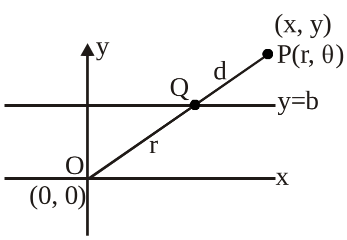
= (ditance point on square from (2, 3))^2 –13

Maximum = $(-2-2)^2 + (-2-3)^2 - 13 = 28$

Q.4 (B)



Q.5 (B)



equation of OP

y = x tan θ

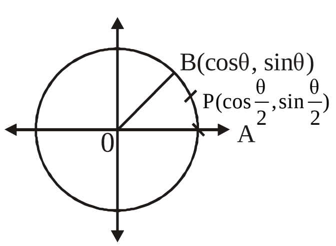
point Q is (b cot θ, b)

∴ point P is y = b ± sin θ

r sin θ = b ± d sin θ

(r ∓ d) sin θ = b

Q.6 (A)



$$\frac{\Delta(AOB)}{\Delta(APB)} = 2+\sqrt{5}$$

$$\frac{\frac{1}{2}\cdot 1\cdot \sin\theta}{\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 1 \\ \cos\theta & \sin\theta & 1 \end{vmatrix}} = 2+\sqrt{5} \text{ on solving}$$

$$\frac{\cos\frac{\theta}{2}}{1-\cos\frac{\theta}{2}} = 2+\sqrt{5} \Rightarrow \cos\frac{\theta}{2} = \frac{1+\sqrt{5}}{4}$$

So $\cos\theta = \frac{\sqrt{5}-1}{4}$

If $\theta \to 2\theta$

$$\frac{\Delta\ AOB}{\Delta\ APB} = \frac{\cos\theta}{1-\cos\theta} = \frac{1}{\sqrt{5}}$$

Q.7 (C)

AB = $\sqrt{(a_1-b_1)^2 + (a_2-b_2)^2}$

Square + Square = $\sqrt{65}$ possible when

= 64 + 1

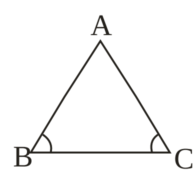
$\sqrt{74}$ = 49 + 25

$\sqrt{97}$ = 81 + 16

But $\sqrt{83}$ not possible

Q.8 **(D)**

Case (i) :

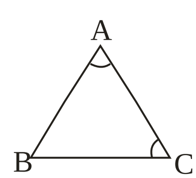


If ∠B = ∠C

locus of A is ⊥ bisector of BC

So it is straight line

Case (ii) :



If ∠A = ∠C

BC fixed B(a, 0), C(0, a)

BC = AB

So, $(x-a)^2 + y^2 + a^2$

Circle

Case (iii) :

∠A = ∠B

AC = BC

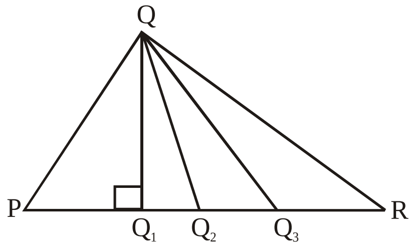
$\sqrt{h^2+(k-a)^2}=\sqrt{2a^2}$

$x^2 + (y - a)^2 = 2a^2$

also a circle

So union of two circle and a line.

Q.9 (A)



$PQ_3 = Q_3R$ ($\therefore$ QQ_3 is median)

$PQ_3 = \frac{1}{2}$ PR

$PQ_2 : Q_2R = r : p$ (By property of angle bisector)

$PQ_2 = \left(\frac{r}{r+P}\right)PR$

But r < P (Given)

$PQ_2 < \frac{1}{2}PR$

Comparison between Altitude and angle bisector

$\Rightarrow \angle QPQ_2 + \angle PQ_2Q + \angle PQQ_2 = \angle RQQ_2 + \angle QQ_2R + \angle QRQ_2$

$\therefore \angle PQQ_2 = \angle RQQ_2$ {Since angle bisector}

$\angle QPQ_2 + \angle PQ_2Q = \angle QQ_2R + \angle QRQ_2$

$\therefore PQ < QR$ then $\angle QPQ_2 > \angle QRQ_2$

Hence $\angle QQ_2P \angle QQ_2R$

But $\angle QQ_2P + \angle QQ_2R = 180^o$

Hence $\angle QQ_2P < 90^o$ & $\angle QQ_2R > 90^o$

$\Rightarrow$ Foot from Q to side PR lies inside ΔPQQ_2

$\Rightarrow PQ_1 < PQ_2 < PQ_3$

Q.10 (A)

$(a - 8)^2 - (b - 7)^2 = 5$

$(a - b - 1)(a + b - 15) = 5$

I_1	I_2

Four cases

I_1	I_2
5	1
1	5
–5	– 1
–1	– 5

Case - 1

a – b – 1 = 5 & a + b – 15 = 1

$\Rightarrow$ a = 11, b = 5

Case-2

a – b – 1 = – 5 & a + b – 15 = – 1

$\Rightarrow$ a = 11, b = 9

Case-3

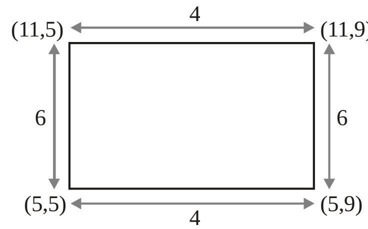
a – b – 1 = 1 & a + b – 15 = 5

$\Rightarrow$ a = 11, b = 9

Case-4

a – b –1 = – 1 & a + b – 15 = – 5

$\Rightarrow$ a = 5, b = 5



Perimeter = 4 + 4 + 6 + 6 = 20

Q.11 (A)

Equation of line passing throug (x_1,y_1) and (x_2,y_2) is

$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow (x_2 - x_1)\, y + (y_1 - y_2)\, x + y_1 (x_1 - x_2)+x_1(y_2 - y_1) =0$

$\Rightarrow ax + by + c = 0$ where a, b, c $\in$ I

$a = x_2 - x_1$, $b = y_1 - y_2$, $c = y_1(x_1 - x_2) + x_1(y_2 - y_1)$

square of distance of (0, 0) from

$\left(\frac{c}{\sqrt{x^2+b^2}}\right)^2=\frac{c^2}{a^2+b^2}$ = rational

Case-1: If n is not perfect square

And square of radius = $n^2\left(1+\left(1-\frac{1}{\sqrt{n}}\right)^2\right)$ = irrational

$\Rightarrow r^2 \neq \frac{c^2}{a^2+b^2}s$

$\Rightarrow$ ax + by + x = 0 never be tangent to given circle

$\Rightarrow \lim_{n\to\infty} P_n = 0$

Case-2 : If n is perfect square

In this case number of tangents passing through two points from given set are few, but total number of lines are in much quantity when n approaches to infinite.

$\Rightarrow \lim_{n\to\infty} P_n = 0$

Q.12 (C)

Area = $\frac{1}{2}$ $d_1d_2 \sin\theta$ is maximum when $\theta = 90°$

$\Rightarrow$ Parallelogram is a rhombus

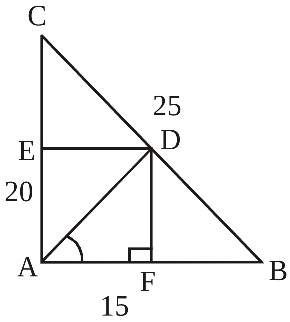
$\Rightarrow$ perimeter = $4\sqrt{\left(\frac{d_1}{2}\right)^2+\left(\frac{d_2}{2}\right)^2}=4\sqrt{29}$ $\in$ (21, 22]

Q.13 (A)

Required ways = total words – words formed with vowels only – words formed with consonants only

$= 26^4 - 5^4 - 21^4 = 456976 - 194481 - 625 = 261870$

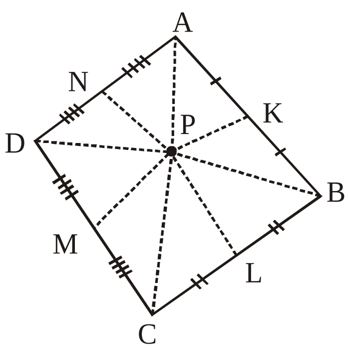
Q.14 (A)



$$AD = \frac{2(\text{Area of } ABC)}{BC} = \frac{20 \times 15}{25} = 12$$

Note that AFDE is a rectangle.
Hence AD = EF.

Q.15 (C)



Note : Area of ΔAPN = Area of ΔPDN
Area of ΔAPK = Area of ΔPBK
Area ΔPCL = Area of ΔPBL
Area of ΔPCM = Area of ΔPDM
Hence . Area (PKAN) + Area (PLCM)
= Area (PMDN) + Area (PLBK)
Hence Area (PLCM) = 36 + 41 – 25 = 52

Q.16 (C)

In ΔABC

$$\frac{15}{\sin 2\theta} = \frac{9}{\sin\theta} = \frac{BC}{\sin 3\theta}$$

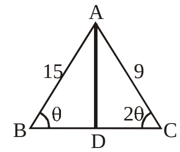
$$\frac{15}{\sin 2\theta} = \frac{9}{\sin\theta} \Rightarrow \cos\theta \frac{5}{6}$$

$$\frac{9}{\sin\theta} = \frac{BC}{\sin 3\theta}$$

$\Rightarrow$ BC = 9 [3 – 4 $\sin^2\theta$]
= 9 [4$\cos^2\theta$ – 1]

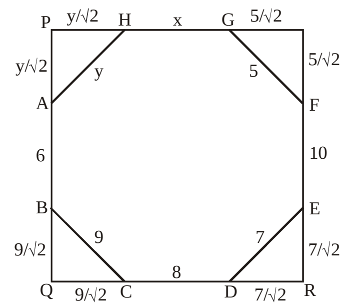
$$= 9\left[4 \times \frac{25}{36} - 1\right] = 16$$

$$\therefore \ BD = \frac{5}{8} BC = 10$$



5 : 3

Q.17 (B)



Let ABCDEFGH be the equiangular octagon as shown
PQ = SR

$$\Rightarrow \frac{y}{\sqrt{2}} + 6 + \frac{9}{\sqrt{2}} = \frac{5}{\sqrt{2}} + 10 + \frac{7}{\sqrt{2}}$$

$\Rightarrow$ y = 3 + 4 $\sqrt{2}$

Also : PS = QR

$$\Rightarrow \frac{y}{\sqrt{2}} + x + \frac{5}{\sqrt{2}} = \frac{9}{\sqrt{2}} + 8 + \frac{7}{\sqrt{2}}$$

$$\Rightarrow x = 4 + 4\sqrt{2}$$

$$\therefore x + y = 7 + 8\sqrt{2} = 18.313$$

$\therefore$ Nearest integer = 18.

JEE MAIN
PREVIOUS YEAR'S

Q.1 (1)

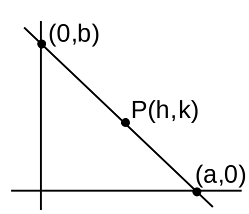
Image of P(3,5) on the line x – y + 1 = 0 is

$$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{2(3-5+1)}{2} = 1$$

x = 4, y = 4
$\therefore$ Image is (4,4)
Which lies on
$(x - 4)^2 + (y - 2)^2 = 4$

Q.2 (1)

Q.3 (1)



$$\frac{x}{a} + \frac{y}{b} = 1$$

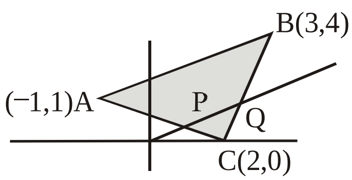
$$\frac{h}{a} + \frac{k}{b} = 1 \qquad .....(i)$$

$$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$$

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2} \qquad .....(ii)$$

$\therefore$ Line passes through fixed point (2, 2) (from (1) and (2))

Q.4 **(2)**



$P \equiv (x_1, mx_1)$
$Q \equiv (x_2, mx_2)$

$$A_1 = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{13}{2}$$

$$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$A_2 = \frac{1}{2}\left|2(mx_1 - mx_2)\right| = m\left|x_1 - x_2\right|$$

$$A_1 = 3A_2 \Rightarrow \frac{13}{2} = 3m\left|x_1 - x_2\right|$$

$$\Rightarrow \left|x_1 - x_2\right| = \frac{16}{6m}$$

AC : x + 3y = 2
BC : y = 4x − 8

$$P : x + 3y = 2 \;\&\; y = mx \Rightarrow x1 = \frac{2}{1+3m}$$

$$Q : y = 4x - 8 \;\&\; y = mx \Rightarrow x2 = \frac{8}{4-m}$$

$$\left|x_1 - x_2\right| = \left|\frac{2}{1+3m} - \frac{8}{4-m}\right|$$

$$= \left|\frac{-26m}{(1+3m)(4-m)}\right| = \frac{26m}{(3m+1)\,|\,m-4\,|}$$

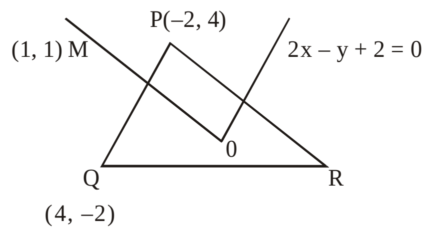
$$= \frac{26m}{(3m+1)(4-m)}$$

$$\left|x_1 - x_2\right| = \frac{13}{6m}$$

$$\frac{26m}{(3m+1)(4-m)} = \frac{13}{6m}$$

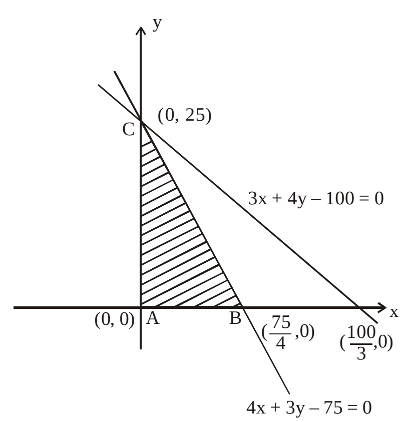
$\Rightarrow$ 12m2 = − (3m + 1)(m − 4)
$\Rightarrow$ 12m2 = − (3m2 − 11m − 4)
$\Rightarrow$ 15m2 − 11m − 4 = 0
$\Rightarrow$ 15m2 − 15m + 4m − 4 = 0
$\Rightarrow$ (15m + 4) (m − 1) = 0
$\Rightarrow$ m = 1

Q.5 **(2)**



Equation of perpendicular bisector of PR is y = x
Solving with 2x – y + 2 = 0 will give
(–2, 2)

Q.6 **(904)**



$z = 6xy + y^2 = y(6x + y)$
$3x + 4y \le 100$ .....(i)
$4x + 3y \le 75$ ......(ii)
$x \ge 0$
$y \ge 0$

$$x \le \frac{75 - 3y}{4}$$

Z = y (6x + y)

$$Z \le y\left(6.\left(\frac{75-3y}{4}\right) + y\right)$$

$$Z \le \frac{1}{2}(225y - 7y^2) \le \frac{(225)^2}{2 \times 4 \times 7}$$

$$= \frac{50625}{56}$$
≈ 904.0178
≈ 904.02

It will be attained at y $= \dfrac{225}{14}$

Q.7 **(144)**

Since orthocentre and circumcentre both lies on y-axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow \Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\ \cos\gamma}$$

$$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\ \cos\gamma}$$

$= 12$

Q.8 **(2)**

$3x + 4y = 9$

$y = mx + 1$

$\Rightarrow 3x + 4mx + 4 = 9$

$\Rightarrow (3 + 4m)x = 5$

$\Rightarrow$ x will be an integer when

$3 + 4m = 5, -5, 1, -1 \Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$

so, number of integral values of m is 2

Q.9 **(1)**

$y = mx + c$

$3 = m + c$

$$\sqrt{2} = \left|\frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m}\right| \quad = 6m + \sqrt{2} = m - 3\sqrt{2}$$

$$= \sin = -4\sqrt{2} \ \rightarrow\ m = \frac{-4\sqrt{2}}{5}$$

$$= \ = 6m - \sqrt{2} = m - 3\sqrt{2}$$

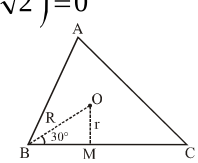
$$= 7m - 2\sqrt{2} \ \rightarrow\ m = \frac{2\sqrt{2}}{7}$$

According to options take $m = \dfrac{-4\sqrt{2}}{5}$

$$\text{So } \ y = \frac{-4\sqrt{2}x}{5} + \frac{3 + 4\sqrt{2}}{5}$$

$$4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$$

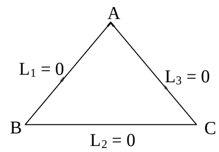
Q.10 **(1)**



$r = OM = \dfrac{3}{\sqrt{2}}$

$\&\ \sin 30° = \dfrac{1}{2} = \dfrac{r}{R} \Rightarrow R = \dfrac{6}{\sqrt{2}}$

$\therefore\ r + R = \dfrac{9}{\sqrt{2}}$



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : 2x + y = 6$

A (2, 2)

B (1, 1)

C (3, 0)

$\Rightarrow AB = \sqrt{2}$, $BC = \sqrt{5}$, $AC = \sqrt{5}$

$\therefore$ Triangle is isosceles

Q.11 (3)

Q.12 (2)

Q.13 (2)

Q.14 **(9)**

Q.15 (6)

Q.16 ((1250)

Q.17 (1)

Q.18 (3)

Q.19 (4)

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 **(B)**

Let slope of line L = m

$$\therefore \left|\frac{m - (-\sqrt{3})}{1 + m(-\sqrt{3})}\right| = \tan 60° = \sqrt{3} \Rightarrow \left|\frac{m + \sqrt{3}}{1 - \sqrt{3}m}\right| = \sqrt{3}$$

taking positive sign, $m + \sqrt{3} = \sqrt{3} - 3m$

$\Rightarrow m = 0$

taking negative sign $m + \sqrt{3} + \sqrt{3} - 3m = 0$

$\Rightarrow m = \sqrt{3}$

As L cuts x-axis

$\Rightarrow m = \sqrt{3}$

so L is $y + 2 = \sqrt{3}(x - 3)$

Q.2 **(A) or (C) or Bonus**

As $a > b > c > 0$	$\Rightarrow a - c > 0$ and $b > 0$
	$\Rightarrow a - c > 0$ and $b > 0$
	$\Rightarrow a + b - c > 0$

$\Rightarrow$ option (A) is correct

Further $a > b$ and $c > 0$

$\Rightarrow\ a - b > 0$	and $c > 0$
$\Rightarrow\ a - b > 0$	and $c > 0$
$\Rightarrow\ a - b + c > 0$	$\Rightarrow$ option (c) is

correct

Aliter

$(a-b)x + (b-a)y = 0 \quad \Rightarrow x = y$

$\Rightarrow$ Point of intersection $\left(\dfrac{-c}{a+b}, \dfrac{-c}{a+b}\right)$

Now $\sqrt{\left(1+\dfrac{c}{a+b}\right)^2 + \left(1+\dfrac{c}{a+b}\right)^2} < 2\sqrt{2}$

$\Rightarrow \sqrt{2}\left(\dfrac{a+b+c}{a+b}\right) < 2\sqrt{2} \Rightarrow a+b-c>0$

Q.3 **(6)**

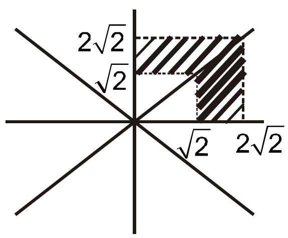
let p(h, k)

$2 \le \left|\dfrac{h-k}{\sqrt{2}}\right| + \left|\dfrac{h+k}{\sqrt{2}}\right| \le 4$

$\Rightarrow \quad 2\sqrt{2} \le |h-k| + |h+k| \le 4\sqrt{2}$

if $\quad h \ge k$

$\Rightarrow \quad 2\sqrt{2} \le x-y+x+y \le 4\sqrt{2}$ or $\sqrt{2} \le x \le 2\sqrt{2}$



similarly when k > h

we have $\sqrt{2} \le y \le 2\sqrt{2}$

The required area = $(2\sqrt{2})^2 - (\sqrt{2})^2 = 6$.

Q.4 **(B,C,D)**

(A) lines are parallel but not coincide (depends on λ and μ)

(B) lines are not parallel.

(C) lines coincide

(D) lines are parallel

Question Stem for Question Nos. 5 and 6

Question Stem

Consider the line L_1 and L_2 defined by

$L_1 :\ x\sqrt{2} + y - 1 = 0$ and $L_2 :\ x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance P form L_2 is λ^2. The line y = 2x + 1 meets C at two points R and S, where teh distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct point R' and S'. Let D be the **square** of the distance between R' and S'.

Q.5 (9.00)

P(x, y) $\left|\dfrac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\dfrac{\sqrt{2}x-y+1}{\sqrt{3}}\right| = \lambda^2$

$\left|\dfrac{2x^2-(y-1)^2}{\sqrt{3}}\right| = \lambda^2,\ C : \left|2x^2-(y-1)^2\right| = 3\lambda^2$

line y = 2x + 1, RS = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$, R(x_1, y_1) and S(x_2, y_2)

$y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1 \Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

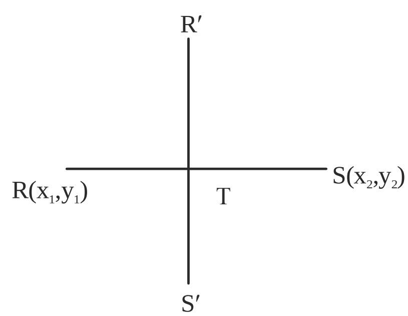
$RS = \sqrt{5(x_1-x_2)^2} = \sqrt{5}\,|x_1 - x_2|$

Solve curve C and line y = 2x + 1 we get

$\left|2x^2-(2x)^2\right| = 3\lambda^2 \Rightarrow x^2 = \dfrac{3\lambda^2}{2}$

$RS = \sqrt{5}\left|\dfrac{2\sqrt{3}\lambda}{\sqrt{2}}\right| = \sqrt{30}\lambda = \sqrt{270} \Rightarrow 30\lambda^2 = 270 \Rightarrow \lambda^2 = 9$

Q.6 (77.14)



⊥ bisectior pf RS

$T \equiv \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Here $x_1 + x_2 = 0$

T = (0, 1)

Equation of

$R'S' : (y-1) = -\dfrac{1}{2}(x-0) \Rightarrow x + 2y = 2$

$R'(a_1, b_1)\ S'(a_2, b_2)$

$D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

solve x + 2y = 2 and $\left|2x^2-(y-1)^2\right| = 3\lambda^2$

$\left|8(y-1)^2-(y-1)^2\right| = 3\lambda^2 \Rightarrow (y-1)^2 = \left(\dfrac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$

$y - 1 = \pm\dfrac{\sqrt{3}\lambda}{\sqrt{7}} \Rightarrow b_1 = 1 + \dfrac{\sqrt{3}\lambda}{\sqrt{7}}, b_2 = 1 - \dfrac{\sqrt{3}\lambda}{\sqrt{17}}$

$D = 5\left(\dfrac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2 = \dfrac{5\times4\times3\lambda^2}{7} = \dfrac{5\times4\times27}{7} = 77.14$

Circle

EXERCISES

ELEMENTRY

Q.1 (1)

Required equation is $(x-a)^2+(y-a)^2=a^2$

$\Rightarrow x^2+y^2-2ax-2ay+a^2=0$.

Q.2 (1)

The circle is $x^2+y^2-\frac{1}{2}x=0$.

Centre $(-g,-f)=\left(\frac{1}{4},0\right)$

and $R=\sqrt{\frac{1}{16}+0-0}=\frac{1}{4}$

Q.3 (2)

Let the centre of the required circle be (x_1, y_1) and the centre of given circle is (1, 2). Since radii of both circles are same, therefore, point of contact (5, 5) is the mid point of the line joining the centres of both circles. Hence $x_1 = 9$ and $y_1 = 8$. Hence the required equation is $(x-9)^2+(y-8)^2=25$

$\Rightarrow x^2+y^2-18x-16y+120=0$.

Trick : The point (5, 5) must satisfy the required circle. Hence the required equation is given by (2).

Q.4 (4)

Let the centre be (h, k), then radius = h

Also $CC_1=R_1+R_2$

or $\sqrt{(h-3)^2+(k-3)^2}=h+\sqrt{9+9-14}$

$\Rightarrow (h-3)^2+(k-3)^2=h^2+4+4h$

$\Rightarrow k^2-10h-6k+14=0$ or $y^2-10x-6y+14=0$

Q.5 (3)

The other end is $(t, 3-t)$

So the equation of the variable circle is

$(x-1)(x-t)+(y-1)(y-3+t)=0$

or $x^2+y^2-(1+t)x-(4-t)y+3=0$

$\therefore$ The centre (α, β) is given by

$\alpha=\frac{1+t}{2}, \beta=\frac{4-t}{2}$

$\Rightarrow\ 2\alpha+2\beta=5$

Hence, the locus is $2x+2y=5$.

Q.6 (4)

Here the centre of circle (3, –1) must lie on the line $x+2by+7=0$.

Therefore, $3-2b+7=0 \Rightarrow b=5$.

Q.7 (4)

Any line through (0, 0) be $y-mx=0$ and it is a tangent to circle $(x-7)^2+(y+1)^2=25$, if

$\frac{-1-7m}{\sqrt{1+m^2}}=5 \Rightarrow m=\frac{3}{4},-\frac{4}{3}$

Therefore, the product of both the slopes is –1.

i.e., $\frac{3}{4}\times-\frac{4}{3}=-1$.

Hence the angle between the two tangents is $\frac{\pi}{2}$.

Q.8 (3)

Equation of pair of tangents is given by $SS_1 = T^2$. Here

$S=x^2+y^2+20(x+y)+20,\ S_1=20$

$T=10(x+y)+20$

$\therefore SS_1=T^2$

$\Rightarrow 20\{x^2+y^2+20(x+y)+20\}=10^2(x+y+2)^2$

$\Rightarrow 4x^2+4y^2+10xy=0 \Rightarrow 2x^2+2y^2+5xy=0$.

Q.9 (2)

Accordingly, $\frac{3(2)-4(4)-\lambda}{\sqrt{3^2+4^2}}=\pm\sqrt{2^2+4^2+5}$

$\Rightarrow -10-\lambda=\pm 25 \Rightarrow \lambda=-35, 15$.

Q.10 (1)

Let $S_1 \equiv x^2+y^2-2x+6y+6=0$

and $S_2 \equiv x^2+y^2-5x+6y+15=0$,

then common tangent is $S_1-S_2=0$

$\Rightarrow 3x=9 \Rightarrow x=3$.

Q.11 (2)

Since normal passes through the centre of the circle.

$\therefore$ The required circle is the circle with ends of diameter as (3, 4) and (– 1, – 2).

It's equation is $(x-3)(x+1)+(y-4)(y+2)=0$

$\Rightarrow\ x^2+y^2-2x-2y-11=0.$

Q.12 (3) Length of each tangent

$L^2=(4)^2+(5)^2-(4\times4)-(2\times5)-11$

L = 2

$r=\sqrt{2^2+1^2-(-11)}$

r = 4

Area = L + r = 8 *sq. units*.

Q.13 (2)

Length of tangents is same *i.e.*, $\sqrt{S_1}=\sqrt{S_2}=\sqrt{S_3}$.

We get the point from where tangent is drawn, by solving the 3 equations for *x* and *y*.

i.e., $x^2 + y^2 = 1$,

$x^2 + y^2 + 8x + 15 = 0$ and $x^2 + y^2 + 10y + 24 = 0$

or $8x + 16 = 0$ and $10y + 25 = 0$

$\Rightarrow x = -2$ and $y = -\frac{5}{2}$

Hence the point is $\left(-2, -\frac{5}{2}\right)$.

Q.14 (2)

Suppose (x_1, y_1) be any point on first circle from which tangent is to be drawn, then

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c_1 = 0$ ....(i)

and also length of tangent

$= \sqrt{S_2} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$ ....(ii)

From (i), we get (ii) as $\sqrt{c - c_1}$.

Q.15 (1)

$S_1 = x^2 + y^2 + 4x + 1 = 0$

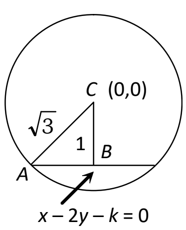
$S_2 = x^2 + y^2 + 6x + 2y + 3 = 0$

Common chord $\equiv S_1 - S_2 = 0 \Rightarrow 2x + 2y + 2 = 0$

$\Rightarrow x + y + 1 = 0$

Q.16 (3)

Obviously $BC = \sqrt{2}$



Hence, $\pm \frac{0 - 2.0 - k}{\sqrt{1^2 + (-2)^2}} = \sqrt{2} \Rightarrow k = \pm\sqrt{10}$

Q.17 (1)

We know that the equation of common chord is $S_1 - S_2 = 0$, where S_1 and S_2 are the equations of given circles, therefore

$(x-a)^2 + (y-b)^2 + c^2 - (x-b)^2 - (y-a)^2 - c^2 = 0$

$\Rightarrow 2bx - 2ax + 2ay - 2by = 0$

$\Rightarrow 2(b-a)x - 2(b-a)y = 0 \Rightarrow x - y = 0$

Q.18 (3)

Equation of common chord is $ax - by = 0$

Now length of common chord

$= 2\sqrt{r_1^2 - p_1^2} = 2\sqrt{r_2^2 - p_2^2}$

where r_1 and r_2 are radii of given circles and p_1, p_2 are the perpendicular distances from centres of circles to common chords.

Hence required length

$= 2\sqrt{a^2 - \frac{a^4}{a^2 + b^2}} = \frac{2ab}{\sqrt{a^2 + b^2}}$

Q.19 (4)

Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow 2x - 2y = 0$ *i.e.*, $x - y = 0$

$\because$ Length of perpencicular drawn from C_1

to $x - y = 0$ is $\frac{1}{\sqrt{2}}$

$\therefore$ Length of common chord $= 2\sqrt{\frac{19}{2} - \frac{1}{2}} = 6$

Q.20 (3)

Here the intersection point of chord and circle can be found by solving the equation of circle with the equation of given line, therefore, the points of intersection are (–4, –3) and $\left(\frac{24}{5}, \frac{7}{5}\right)$. Hence the midpoint is $\left(\frac{-4 + \frac{24}{5}}{2}, \frac{-3 + \frac{7}{5}}{2}\right) = \left(\frac{2}{5}, -\frac{4}{5}\right)$.

Q.21 (4)

Let the mid point of chord be (*h*, *k*), then its equation is $T = S_1$

i.e., $hx + ky - (x + h) - 3(y + k) - 10$

$= h^2 + k^2 - 2h - 6k - 10$

Since it passes through the origin, therefore

$h^2 + k^2 - h - 3k = 0$

or locus is $x^2 + y^2 - x - 3y = 0$.

Q.22 (1)

$SS_1 = T^2$

$\Rightarrow (x^2 + y^2 - 2x + 4y + 3)(36 + 25 - 12x - 20y + 3)$

$= (6x - 5y - x - 6 + 2(y - 5) + 3)^2$

$\Rightarrow 7x^2 + 23y^2 + 30xy + 66x + 50y - 73 = 0$.

Q.23 (1)

$C_1(1, 2), C_2(0, 4), R_1 = \sqrt{5}, R_2 = 2\sqrt{5}$

$C_1C_2 = \sqrt{5}$ and $C_1C_2 = |R_2 - R_1|$

Hence circles touch internally.

Q.24 (3)

Equation of radical axis, $S_1 - S_2 = 0$

i.e.,

$(2x^2 + 2y^2 - 7x) - (2x^2 + 2y^2 - 8y - 14) = 0$

$\Rightarrow -7x + 8y + 14 = 0$, $\therefore 7x - 8y - 14 = 0$

Q.25 (4)

$S_1 \equiv x^2 + y^2 - 16x + 60 = 0$ .....(i)

$S_2 \equiv x^2 + y^2 - 12x + 27 = 0$ .....(ii)

$S_3 \equiv x^2 + y^2 - 12y + 8 = 0$ .....(iii)

The radical axis of circle (i) and circle (ii) is

$S_1 - S_2 = 0 \Rightarrow -4x + 33 = 0$ ....(iv)

the radical axis of circle (ii) and circle (iii) is

$S_2 - S_3 = 0 \Rightarrow -12 + 12y + 19 = 0$ .....(v)

Solving (iv) and (v), we get the radical centre $\left(\frac{33}{4}, \frac{20}{3}\right)$.

Q.26 (2)

Required equation is

$(x^2 + y^2 + 13x - 3y) + \lambda(2x^2 + 2y^2 + 4x - 7y - 25) = 0$

which passes through (1, 1), so $\lambda = \frac{1}{2}$

Hence required equation is

$4x^2 + 4y^2 + 30x - 13y - 25 = 0$.

Q.27 (1)

Let equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ with $x^2 + y^2 = p^2$

cutting orthogonally,

we get $0 + 0 = +c - p^2$ or $c = p^2$

and passes through (*a*, *b*), we get

$a^2 + b^2 + 2ga + 2fb + p^2 = 0$ or

$2ax + 2by - (a^2 + b^2 + p^2) = 0$

Required locus as centre (–g, –f) is changed to (*x*, *y*).

Q.28 (2)

Given circle is $\left(2, \frac{3}{2}\right)$, $\frac{5}{2} = r_1$ (say)

Required normals of circlres are

$x + 3 = 0, x + 2y = 0$

which intersect at the centre $\left(-3, \frac{3}{2}\right)$, $r_2 =$ radius (say).

2nd circle just contains the 1st

i.e., $C_2C_1 = r_2 - r_1 \Rightarrow r_2 = \frac{15}{2}$.

Q.29 (2)

The polar of the point $\left(5, -\frac{1}{2}\right)$ is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$\Rightarrow 5x - \frac{1}{2}y - 2(x + 5) + 0 + 0 = 0$

$\Rightarrow 3x - \frac{y}{2} - 10 = 0 \Rightarrow 6x - y - 20 = 0$.

Q.30 (1)

Given two circles

$x^2 + y^2 - 2x + 22y + 5 = 0$

$x^2 + y^2 + 14x + 6y + k = 0$

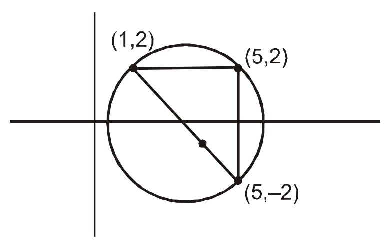
The two circles cut orthogonally, if

$2(g_1g_2 + f_1f_2) = c_1 + c_2$ *i.e.,* $2(-1.7 + 11.3) = 5 + k$

$2(-7 + 33) = 5 + k \Rightarrow 52 - 5 = k \Rightarrow k = 47$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)



diameter = $4\sqrt{2}$

r = $2\sqrt{2}$

Q.2 (1)

(3,4) & (2,5) are ends of diameter of circle

So, Equation (x – 3)(x –2) + (y –4)(y –5) = 0

$x^2 + y^2 - 5x - 9y + 26 = 0$

Q.3 (2)

Equation of circle (x – 0) (x – a) + (y – 1)(y – b) = 0

it cuts x-axis put y = 0 $\Rightarrow x^2 - ax + b = 0$

Q.4 (3)

Length of intercept on x-axis = $2\sqrt{g^2 - c}$

$= 2\sqrt{\frac{25}{4} + 14} = 2\sqrt{\frac{81}{4}} = 9$

on y-axis = $2\sqrt{f^2 - c} = 2\sqrt{\left(\frac{13}{2}\right)^2 + 14}$

$= 2\sqrt{\frac{169+56}{4}} = 2\sqrt{\frac{225}{4}} = 15$

Q.5 (4)
given circle $x^2 + y^2 - 4x - 6y = 0$
it cuts x-axis put y = 0, x = 0, 4
it cuts y-axis put x = 0, y = 0, 6
Hence mid points on x-axis (2, 0)
on y-axis (0, 3)

Equations of line $\frac{x}{2}+\frac{y}{3} = 1 \Rightarrow 3x + 2y - 6 = 0$

Q.6 (3)
Intersection of given lines is centre
$2x - 3y - 5 = 0$
$3x - 4y - 7 = 0$

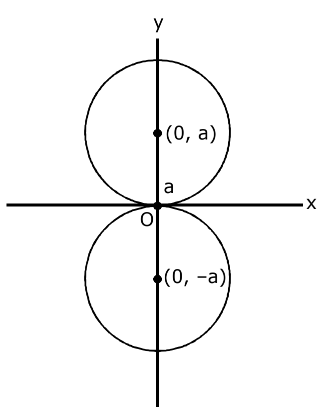
$\frac{x}{21-20} = \frac{y}{-15+14} = \frac{1}{-8+9}$

$\Rightarrow x = 1, y = -1$

$(1, -1)$, $\pi r^2 = 154 \Rightarrow$ $r^2 = \frac{154}{22} \times 7$

$\Rightarrow r = 7$
$g = -1, f = 1, c = g^2 + f^2 - r^2$
$= 1 + 1 - 49 = -47$
$x^2 + y^2 - 2x + 2y - 47 = 0$

Q.7 (2)
$x^2 + (y \pm a)^2 = a^2$
$x^2 + y^2 \pm 2ay = 0$

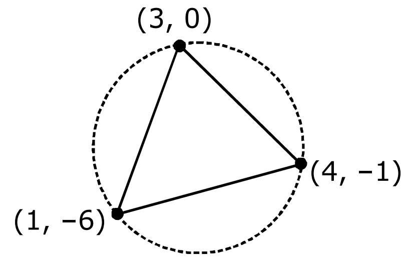


Q.8 (1)

Centre (2, –1), radius = $\sqrt{(3-2)^2+(6+1)^2}$

$= \sqrt{1+49} = \sqrt{50}$
$(x - 2)^2 + (y + 1)^2 = 50$
$x^2 + y^2 - 4x + 2y - 45 = 0$

Q.9 (4)
Let the centre (a, b)
$(a - 3)^2 + (b)^2 = (a - 1)^2 + (b + 6)^2$
$= (a - 4)^2 + (b + 1)$



(i) & (ii)
$- 6a + 9 = - 2a + 1 + 12b + 36$
$\Rightarrow 4a + 12b + 28 = 0$ $\Rightarrow a + 3b + 7 = 0$
(i) & (iii)
$- 6a + 9 = - 8a + 16 + 2b + 1$
$\Rightarrow 2a - 2b = 8$ $\Rightarrow a - b = 4$

$a = \frac{5}{4}, b = -\frac{11}{4}$ $r = \sqrt{\frac{49}{16}+\frac{121}{16}} = \frac{\sqrt{170}}{4}$

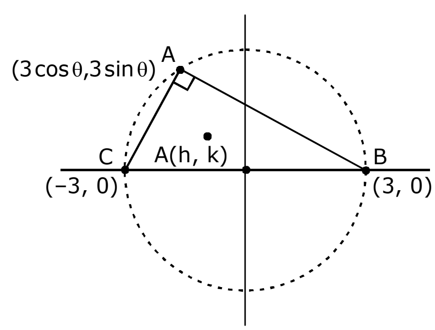
$g = -\frac{5}{4}, f = \frac{11}{4}, c = \frac{25}{16} + \frac{121}{16} - \frac{170}{16}$

$= \frac{-24}{16} = \frac{-3}{2}$

$x^2 + y^2 - 2 . \frac{5}{4} x + 2 . \frac{11}{4} y - \frac{3}{2} = 0$

$2x^2 + 2y^2 - 5x + 11y - 3 = 0$

Q.10 (1)
Circle is
$x^2 + y^2 = 9$
$\therefore$ co-ordinate of point
A (3 cosθ, 3 sinθ)

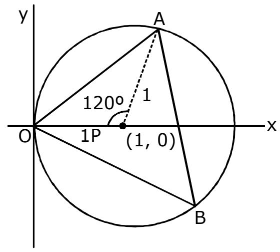


centroid of ΔABC is P(h, k) whose coordinate is

$\left(\frac{3+3\cos\theta-3}{3}, \frac{0+0+3\sin\theta}{3}\right) \equiv (\cos\theta, \sin\theta)$

$h = \cos\theta, k = \sin\theta$
$h^2 + k^2 = 1 \Rightarrow x^2 + y^2 = 1$

Q.11 (2)
$x^2 + y^2 - 2x = 0$
$(x - 1)^2 + y^2 = 1$
area ΔOAB = 3 or Δ(OAP)



$= 3 \times \frac{1}{2}\ 1.1 \sin 120^o$

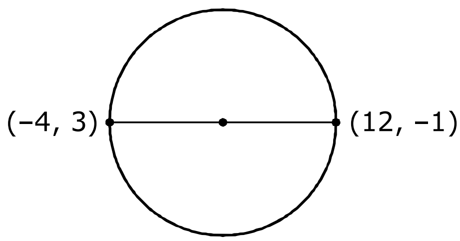
$= \frac{3}{2}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$ sq. units

Q.12 (3)

$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$

$x^2 + y^2 - 8x - 2y - 51 = 0$

$f = (-1), c = -51$

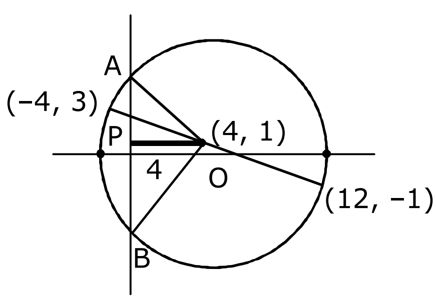


y intercept $= 2\sqrt{f^2 - c} = 2\sqrt{1+51}$

$= 2\sqrt{52} = 4\sqrt{13}$

Aliter

centre (4, 1), radius $= \sqrt{68}$



$AP = \sqrt{68 - 16} = \sqrt{52}$

$AB = 2(AP) = 2\sqrt{52} = 4\sqrt{13}$

Q.13 (1)

$y^2 - 2y + 2xy = 0$ represent normals.

$\{(y(y - 2) - 2x(y - 2) = 0)$

$(y - 2)(y - 2x) = 0\}$

Intersection point is centre

$y = 2$ & $y = 2x \Rightarrow x = 1, y = 2$

centre (1, 2), passing thorugh (2, 1)

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

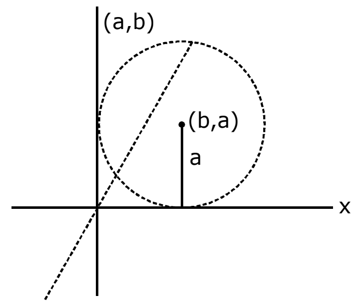
$(x - 1)^2 + (y - 2)^2 = 2$

$x^2 + y^2 - 2x - 4y + 3 = 0$

Q.14 (2)

Reflection of (a, b) in $y - x = 0$ is (b, a)

centre (b, a) touching x-axis.

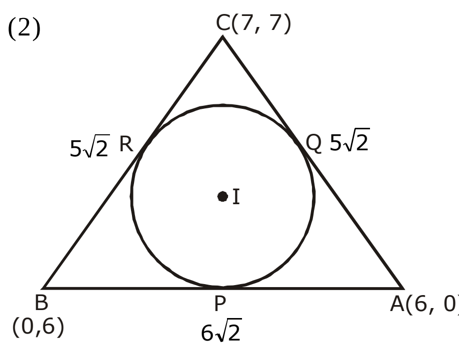


r = Q

$(x - b)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2bx - 2ay + b^2 = 0$

Q.15 (2)



$\because$ P(3, 3)

$\therefore$ I

$$\left(\frac{6.(5\sqrt{2}) + 0 + 7.6\sqrt{2}}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}, \frac{0 + 6.(5\sqrt{2}) + 7(6\sqrt{2})}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}\right)$$

$$I\left(\frac{9}{2}, \frac{9}{2}\right), r = IP = \sqrt{\left(\frac{9}{2} - 3\right)^2 + \left(\frac{9}{2} - 3\right)^2} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \left(x - \frac{9}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{9}{2}$$

$$\Rightarrow x^2 + y^2 - 9x - 9y + \frac{81}{2} - \frac{9}{2} = 0$$

$$\Rightarrow x^2 + y^2 - 9x - 9y + 36 = 0$$

Q.16 (1)

Point on the line $x + y + 13 = 0$ nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$ is foot of $\perp$ from centre

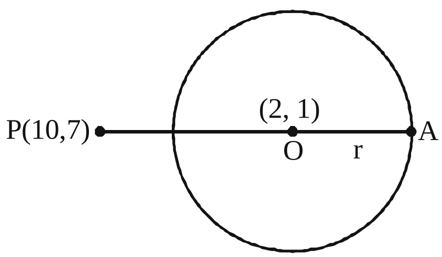
$$\frac{x+2}{1}=\frac{y+3}{1}=-\left(\frac{-2-3+13}{1^2+1^2}\right)=-4$$

x = –6, y = –7

Q.17 (2)

$x^2 + y^2 - 4x - 2y - 20 = 0$, P (10, 7)

$S_1 = 100 + 49 - 40 - 14 - 20 > 0$



P lies outside

O (2, 1), r = $\sqrt{4+1+20}$ ⇒ r = 5

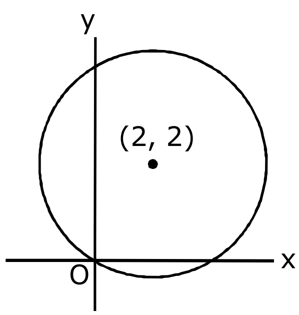
greatest distance = PA = PO + OA

$= \sqrt{8^2+6^2} + 5 = 10 + 5 = 15$

Q.18 (3)

$x^2 + y^2 - 4x - 4y = 0$

C(2, 2), r = $\sqrt{4+4-0} = 2\sqrt{2}$



Parametric Coordinate

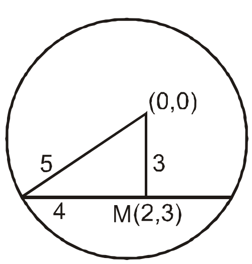
$(2 + 2\sqrt{2}\cos\alpha,\ 2 + 2\sqrt{2}\sin\alpha)$

Q.19 (2)

Let slope of required line is m

y – 3 = m(x – 2)

⇒ mx – y +(3 – 2m) = 0



length of ⊥ from origin

= 3

⇒ $9 + 4m^2 - 12m = 9 + 9m^2$

⇒ $5m^2 + 12m = 0$ ⇒ $m = 0, -\frac{12}{5}$

Hence lines are y – 3 = 0 ⇒ y = 3

$y - 3 = -\frac{12}{5}(x-2)$ ⇒ 5y – 15 = –12x + 24

⇒ 12x + 5y = 39.

Q.20 (2)

From centre (2, –3), length of perpendicular on line 3x + 5y + 9 = 0 is

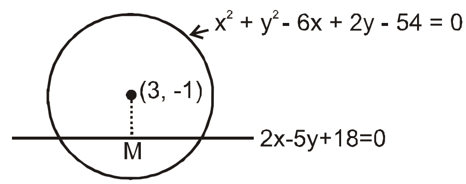
$p = \frac{6-15+9}{\sqrt{25+9}} = 0$; line is diameter.

Q.21 (1)

Required point is foot of ⊥

$$\frac{x-3}{2}=\frac{y+1}{-5}=-\left(\frac{6+5+8}{4+25}\right)=-1$$

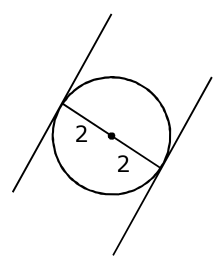
⇒ x = – 2 + 3 = 1



x = 1, y = 4

Q.22 (1)

$4 = \left|\frac{c_1-c_2}{\sqrt{1+3}}\right|$ ⇒ $|c_1 - c_2| = 8$



Q.23 (2)

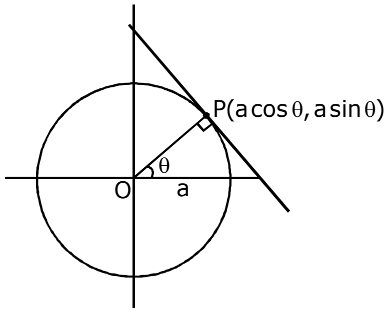
Point (8,6) lies on circle ; $S_1 = 0$ ⇒ one tangent.

Q.24 **(4)**

$x^2 + y^2 = a^2$

$m_N = \tan\theta$

$m_T = -\frac{1}{m_N} = \frac{1}{\tan\theta} = -\cot\theta$



Q.25 (3)

$\ell x + my + n = 0,\ x^2 + y^2 = r^2$

$r = \left|\frac{n}{\sqrt{\ell^2 + m^2}}\right| \Rightarrow r^2(\ell^2 + m^2) = n^2$

Q.26 (2)

Line parallel to given line $4x + 3y + 5 = 0$ is $4x + 3y + k = 0$

This is tangent to $x^2 + y^2 - 6x + 4y - 12 = 0$

$\left|\frac{12 - 6 + k}{5}\right| = 5$

$6 + k = \pm 25 \Rightarrow k = 19, -31$

Hence required line $4x + 3y - 31 = 0,\ 4x + 3y + 19 = 0$

Q.27 (1)

$p = \left|\frac{(-g + g)\cos\theta + (-f + f)\sin\theta - k}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$= \sqrt{g^2 + f^2 - c} \Rightarrow g^2 + f^2 = c + k^2$

Q.28 (4)

Equation of tangent $x - 2y = 5$

Let required point be (α, β)

$\alpha x + \beta y - 4(x + \alpha) + 3(y + \beta) + 20 = 0$

$x(\alpha - 4) + y(\beta + 3) - 4\alpha + 3\beta + 20 = 0$

Comparing

$\frac{\alpha - 4}{1} = \frac{\beta + 3}{-2} = \frac{4\alpha - 3\beta - 20}{5}$

Similarly $(\alpha, \beta) \equiv (3, -1)$

Q.29 (3)

Let tangent be $y = mx$

$\left|\frac{7m + 1}{\sqrt{1 + m^2}}\right| = 5$

$\Rightarrow 49m^2 + 1 + 14m = 25(1 + m^2)$

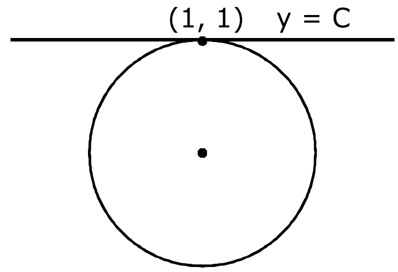
$24m^2 + 14m - 24 = 0$

$m_1 m_2 = -1$ angle $= 90°$

Q.30 (1)

$x^2 + y^2 - 2x + 2y - 2 = 0$

Tangent at (1, 1)



$x + y - (x + 1) + (y + 1) - 2 = 0$

$y - 1 + y + 1 - 2 = 0$

$2y - 2 = 0$

$y = 1 \Rightarrow c = 1$

Q.31 (2)

Tangent at (x_1, y_1) is

$xx_1 + yy_1 = 25$

$3x + 4y = 25 \Rightarrow x_1 = 3, y_1 = 4 \Rightarrow (x_1, y_1) = (3, 4)$

Q.32 (1)

Let tangent from (0, 1) on $x^2 + y^2 - 2x + 4y = 0$

$y - 1 = mx$ C(1, –2), $r = \sqrt{5}$

$\Rightarrow mx - y + 1 = 0$

$r = \sqrt{5} = \frac{|m + 2 + 1|}{\sqrt{m^2 + 1}} \Rightarrow 5(m^2 + 1) = (m + 3)^2$

$\Rightarrow 4m^2 - 6m - 4 = 0 \Rightarrow 2m^2 - 3m - 2 = 0$

$\Rightarrow (m - 2)(2m + 1) = 0 \Rightarrow m = 2, -\frac{1}{2},$

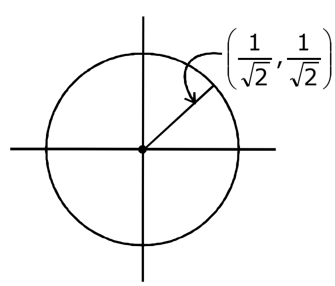
Tangents are $2x - y + 1 = 0$

$x + 2y - 2 = 0$

Q.33 (3)

Normal is diameter passing through centre (0, 0)

$\&\ m = \frac{\frac{1}{\sqrt{2}} - 0}{\frac{1}{\sqrt{2}} - 0} = 1$

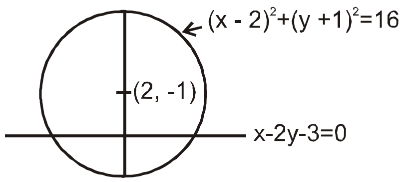


$y = x \Rightarrow x - y = 0$

Q.34 (2)

Required diameter is $\perp$ to given line.

Hence $y + 1 = -2(x - 2)$



$\Rightarrow 2x + y - 3 = 0$

Q.35 (1)

Normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ also pusses through centre.

Hence its equation is line joining (2, –2) and (1, 1)

$$(y - 1) = \frac{1+2}{1-2}(x - 1)$$

$y - 1 = -3x + 3$

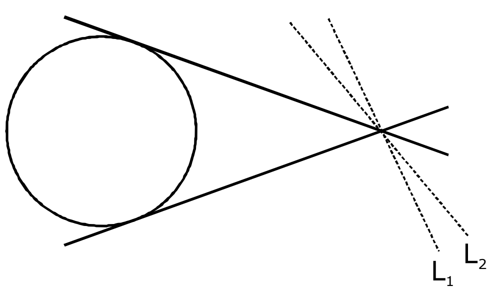
$\Rightarrow 3x + y - 4 = 0$

Q.36 (2)

Line passing thorough the intesection points of L_1 & L_2 is tangent of circle

$(2x - 3y + 1) + \lambda\,(3x - 2y - 1) = 0$

$(2 + 3\lambda)\, x - y\,(3 + 2\lambda) + (1 - \lambda) = 0$ is tangent of given circle



centre (–1, 2), r = $\sqrt{1+2^2-0} = \sqrt{5}$

$$\sqrt{5} = \left|\frac{-(2+3\lambda) - 2(3+2\lambda) + (1-\lambda)}{\sqrt{(2+3\lambda)^2 + (3+2\lambda)^2}}\right|$$

$$= \frac{|-8\lambda - 7|}{\sqrt{(2+3\lambda)^2 + (3+2\lambda)^2}}$$

$\Rightarrow\ 5\,[(2 + 3\lambda)^2 + (3 + 2\lambda)^2] = (8\lambda + 7)^2$

$\Rightarrow\ 65\,\lambda^2 + 120\lambda + 65 = 64\lambda^2 + 112\lambda + 49$

$\Rightarrow\ \lambda^2 + 8\lambda + 15 = 0 \qquad \Rightarrow\ (\lambda + 4)^2 = 0$

$\Rightarrow\ \lambda = -4\ \Rightarrow$ tangent $-10x + 5y + 5 = 0$

$\Rightarrow\ 2x - y - 1 = 0$

Aliter :

Point of intersection is (1, 1)

$2x - 3y + 1 = 0$

$3x - 2y - 1 = 0$

(1, 1) lies on circle

$\therefore$ tangent of circle is

$x \cdot 1 + y \cdot 1 + (x + 1) - 2y\,(y + 1) = 0$

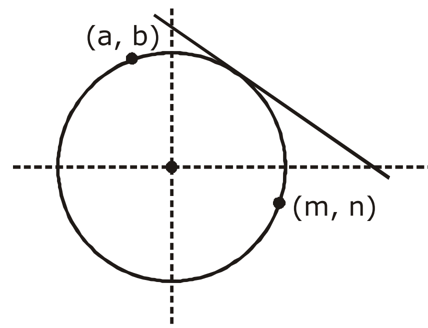
$2x - y - 1 = 0$

Q.37 (1)

Given $a^2 + b^2 = 1$, $m^2 + n^2 = 1$

i.e. points (a, b) & (m, n) on the circle

$x^2 + y^2 = 1$ tangent at (a, b)



$ax + by - 1 = 0$ point (0, 0) & (m, n)

so lie some side of the tangent

(0, 0) $\Rightarrow -1 < 0$

$\therefore$ (m, n) $\Rightarrow$ am + bn – 1 < 0 $\Rightarrow$ am + bn < 1

(m, n) & (a, b) can be equal

$\therefore$ am + bn ≤ 1

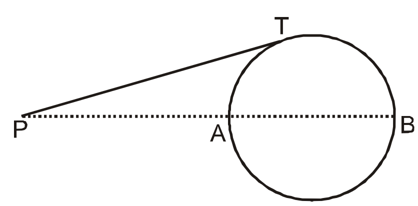
(m, n) & (a, b) can be negative

$\therefore$ $|am + bn| \le 1$

Q.38 (3)

As we know

$PA.PB = PT^2 = (\text{Length of tangent})^2$



Length of tangent = $\sqrt{16 \times 9} = 12$

Q.39 (1)

Let any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ (α, β)

This point satisfies $\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + p = 0$

Length of tangent from this point to circle $x^2 + y^2 + 2gx + 2fy + q = 0$

length = $\sqrt{S_1} = \sqrt{\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + q}$

$= \sqrt{q - p}$

Q.40 (3)

$2(x^2 + y^2) - 7x + 9y - 11 = 0$, P (2, 3)

Point lie outside

$\therefore\ x^2 + y^2 - \frac{7}{2}x + \frac{9}{2}y - \frac{11}{2} = 0$

Length of tangent

$T_1 = \sqrt{S_1} = \sqrt{4 + 9 - 7 + \frac{27}{2} - \frac{11}{2}}$

$= \sqrt{6 + 8} = \sqrt{14}$

Q.41 (2)

Let point on line be

(h, 4 – 2h) (chord of contact)

$hx + y(4 - 2h) = 1$

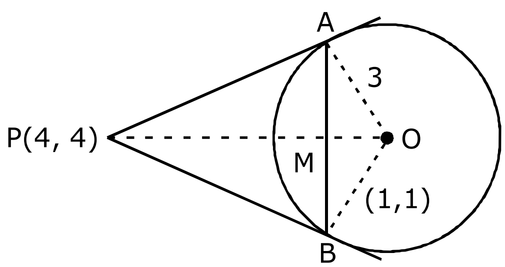
$h(x - 2y) + 4y - 1 = 0$ Point $\left(\frac{1}{2}, \frac{1}{4}\right)$

Q.42 (2)

$x^2 + y^2 - 2x - 2y - 7 = 0$

$0(1, 1),\ r = \sqrt{1 + 1 + 7} = 3$

Equation of AB



$4x + 4y - (x + 4) - (y + 4) - 7 = 0$

$3x + 3y = 15 \Rightarrow x + y = 5$

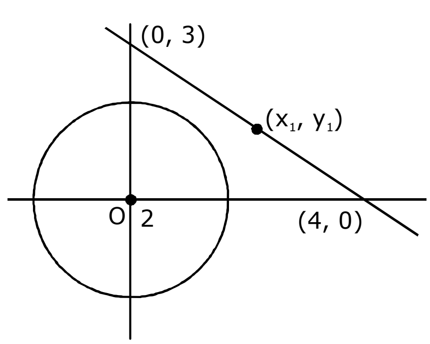
$OM = \frac{|1 + 1 - 5|}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}}$

$AM = \sqrt{3^2 - \frac{3^2}{2}} = \frac{3}{\sqrt{2}} \Rightarrow AB = 2.\frac{3}{\sqrt{2}} = 3\sqrt{2}$

Q.43 (4)

equation of pair of tangents and find angle betwen time.

$x^2 + y^2 = 4$ & line $3x + 4y = 12$



Let P (x_1, y_1) oin given line & C.O.C of P.

$xx_1 + yy_1 = 4$ ....(i)

P satisfy given line

$3x_1 + 4y_1 = 12$ ...(ii)

3(i) – (ii)

$\Rightarrow\ 3xx_1 + 3yy_1 = 12$

$3x_1 \pm 4y_1 = 12$

$-\quad -\quad\quad -$

$3x_1(x - 1) + y_1(3y - 4) = 0$

$(x - 1) + \lambda(3y - 4) = 0$

$\Rightarrow\ L_1 + \lambda L_2 = 0$

Find point $x = 1$ & $y = \frac{4}{3} \Rightarrow \left(1, \frac{4}{3}\right)$

Q.44 (**3**)

Chord of contant from (0, 0) & (g, f) are

$gx + fy + c = 0$

& $gx + fy + g(x + g) + f(y + f) + c = 0$

$\Rightarrow 2gx + 2fy + g^2 + f^2 + c = 0$

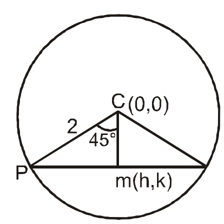
distance between C.O.C.'s

$= \frac{\left|\frac{g^2 + f^2 + c - c}{2}\right|}{\sqrt{g^2 + f^2}} = \frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$

$\{\because g^2 + f^2 - c \geq 0\}$

Q.45 (3)

$\cos 45° = \frac{cm}{cp} = \frac{\sqrt{h^2 + k^2}}{2}$



Hence locus $x^2 + y^2 = 2$

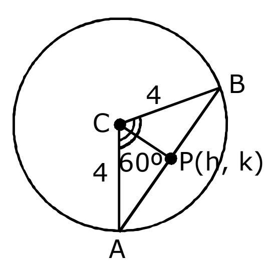
Q.46 (3)

Let mid point of cord P(h, k)

$x^2 + y^2 - 2x - 4y - 11 = 0$

C(1, 2), r = 4

$CP = 4\cos 30° = 4\,\frac{\sqrt{3}}{2} = 2\sqrt{3}$



We know that locus is circle
whose radius is CP & centre (1, 2)

$(x-1)^2 + (y-2)^2 = (2\sqrt{3})^2$

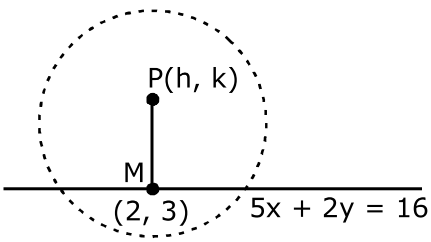
$\Rightarrow x^2 + y^2 - 2x - 4y - 7 = 0$

M-II equation of chord $T = S_1$ have a distance from centre is $2\sqrt{3}$ and get the locus.

Q.47 (1)

Let the centre P(h, k)

$m_{PH} = \frac{-1}{m_2} = \frac{-1}{-\frac{5}{2}} = \frac{2}{5}$



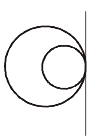
$\frac{k-3}{h-2} = \frac{2}{5}$

$2h - 5k + 11 = 0$

$2x - 5y + 11 = 0 \rightarrow$ Line PM.

Q.48 (2)

$C_1C_2 = 5$, $r_1 = 7_1$ $r_2 = 2$



$C_1C_2 = |r_1 - r_2| \Rightarrow$ one common tangent

Q.49 (2)

Equation of common tangent at point of contact is $S_1 - S_2 = 0$

$\Rightarrow 10x + 24y + 38 = 0$

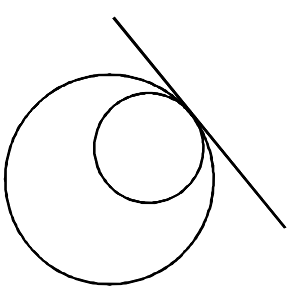
$\Rightarrow 5x + 12y + 19 = 0$

Q.50 **(A)**

$S_1 \Rightarrow C_1(1, 0)$, $r_1 = \sqrt{2}$

$S_2 \Rightarrow C_2(0, 1)$, $r_2 = 2\sqrt{2}$

$C_1C_2 = \sqrt{1^2 + 1^2} = \sqrt{2}$



$C_1C_2 = |r_2 - r_1|$

$\sqrt{2} = \sqrt{2}$

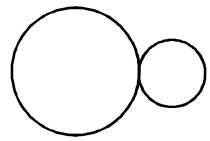
Internally touch $\therefore$ common tangent is one.

Q.51 (1)

$x^2 + y^2 = 9$

$\Rightarrow C_1(0, 0)$, $r_1 = 3$

$x^2 + y^2 + 6y + c = 0$

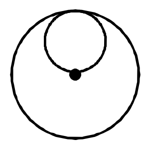


$C_2(0, -3)$, $r_2 = \sqrt{9-c}$

If circle are externally touching

$c_1c_2 = r_1 + r_2$

$B = 3 + \sqrt{9-c}$



$\Rightarrow c = 9$

If cirlce are internally touching

$C_1C_2 = |r_1 - r_2|$

$3 = +3 - \sqrt{9-c}$ or $3 = -3 + \sqrt{9-c}$

$\Rightarrow c = 9 \Rightarrow 6 = \sqrt{9-c}$

$\Rightarrow c = -27$

$c = 9, -27$

Aliter :

Common tangent of S_1 & S_2

$6y + c + 9 = 0$

$3 = \left|\frac{c+9}{\sqrt{6^2}}\right| \Rightarrow 18 = |c+9|$

$\Rightarrow c = 9, -27$

Q.52 (1)

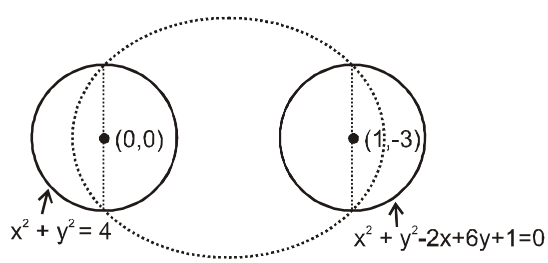
Let required circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Hence common chord with $x^2 + y^2 - 4 = 0$

is $2gx + 2fy + c + y = 0$

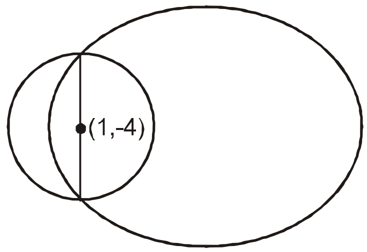
This is diameter of circle $x^2 + y^2 = 4$ hence $c = -4$.

Now again common chord with other circle



$2x(g + 1) + 2y(f – 3) + (c – 1) = 0$
This is diameter of $x^2 + y^2 – 2x + 6y + 1 = 0$
$2(g + 1) – 6(f – 3) + 5 = 0$
$2g – 6f + 15 = 0$
locus $2x – 3y – 15 = 0$ which is st. line.

Q.53 (3)
Common chord of given circle
$6x + 4y + (p + q) = 0$
This is diameter of $x^2 + y^2 – 2x + 8y – q = 0$



centre (1, –4)
$6 – 16 + (p + q) = 0 \Rightarrow p + q = 10$

Q.54 (3)
$S_1 – S_3 = 0 \Rightarrow 16y + 120 = 0$

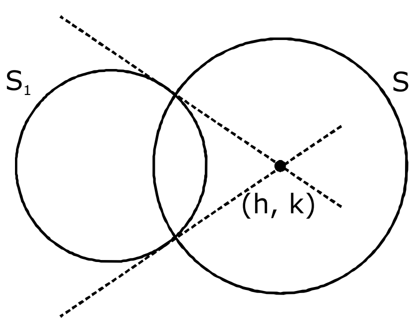
$\Rightarrow y = \frac{-120}{16}$

$\Rightarrow y = -\frac{15}{2} \Rightarrow x = 8$

Intersection point of radical axis is

$\left(8, \frac{-15}{2}\right)$

Q.55 (1)
Let point of intersection of tangents is (h, k) family of circle.



$x^2 + y^2 – (\lambda + 6) x + (8 – 2\lambda) y – 3 = 0$
Common chord is $S – S_1 = 0$
$\Rightarrow – (\lambda + 6) x + (8 – 2\lambda) y – 2 = 0$
$\Rightarrow (\lambda + 6) x + (2\lambda – 8) y + 2 = 0$
....(i)
C.O.C. from (h, k) to $S_1: x^2 + y^2 = 1$ is
$hx + ky = 1$ ....(ii)
(i) & (ii) are same equation

$\frac{\lambda+6}{h} + \frac{2(\lambda-4)}{k} = \frac{2}{-1}$

$\Rightarrow \lambda = – 2h – 6, \quad \lambda = –k + 4$
$\therefore –2h –6 = – k + 4$
$\Rightarrow 2h – k + 10 \Rightarrow$ Locus : $2x – y + 10 = 0$

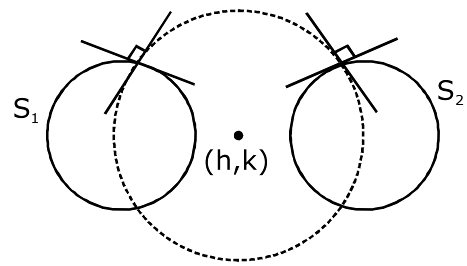
Q.56 (1)
$S_1 – S_2 = 0 \quad \Rightarrow \quad 7x – 8y + 16 = 0$

$S_2 – S_3 = 0 \quad \Rightarrow \quad 2x –4y + 20 = 0$
$S_3 – S_1 = 0 \quad \Rightarrow \quad 9x –12y + 36 = 0$
On solving centre (8, 9)
Length of tangent

$= \sqrt{S_1} = \sqrt{64+81-16+27-7} = \sqrt{149}$
$= (x – 8)^2 + (y – 9)^2 = 149$
$= x^2 + y^2 – 16x – 18y – 4 = 0$

Q.57 (3)
Let centre (h, k) & circle
$x^2 + y^2 + 2gx + 2fy + c = 0$
$h = –g, \ k = –f$
For $S_1 : g_1 = 2, f_1 = –3, c_1 = 9,$

For $S_2 : g_2 = -\frac{5}{2}, f_2 = 2, c_2 = –2$



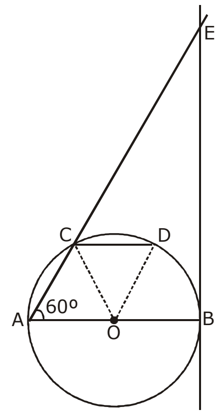
$\therefore 2.g.2 + 2.f (–3) = c + 9$
$\Rightarrow 4g – 6f = c + 9$ ...(1)

& $2g\left(\frac{-5}{2}\right) + 2.f (2) = c – 2$

$\Rightarrow –5g + 4f = c – 2$
...(2)
Subtract (2) from (1)
$– 9g + 10f = 11 \Rightarrow 9x – 10y + 11 = 0$

Q.58 (4)
2CD = AB
CD = OC = OD = AC

$\frac{AB}{AE} = \cos 60^\circ$



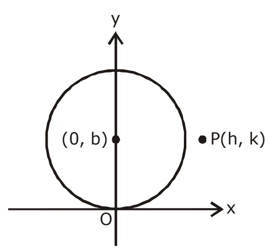
$AE = \frac{AB}{1/2} = 2AB$

Q.59 (1)

Circle $x^2 + (y - b)^2 = b^2$

$\Rightarrow x^2 + y^2 - 2by = 0$

Polar w.r.t. circle P(h, k)



$\therefore hx + ky - b(y + k) = 0$

$\Rightarrow hx + y(k - b) - bk = 0$

Compair with

$\ell x + my + n = 0$

$\Rightarrow \frac{\ell}{h} = \frac{m}{k-b} = \frac{n}{-bk}$

$\Rightarrow \ell = \frac{hn}{-bk}$ & $m = \frac{n(k-b)}{-bk}$

$\Rightarrow b = \frac{-hn}{\ell k}$ & $mbk + n(k - b) = 0$

$\therefore -mk\frac{hn}{\ell k} + n\left(k + \frac{hn}{\ell k}\right) = 0$

$\Rightarrow -\frac{mnh}{\ell} + \frac{n(k^2\ell + hn)}{k\ell} = 0$

$\Rightarrow -mnhk + nk^2\ell + hn^2 = 0$

$\Rightarrow -mhk + k^2\ell + hn = 0$

$\Rightarrow h(mk - n) - \ell k^2 = 0$

$\Rightarrow x(my - n) - \ell y^2 = 0$

JEE-ADVANCED

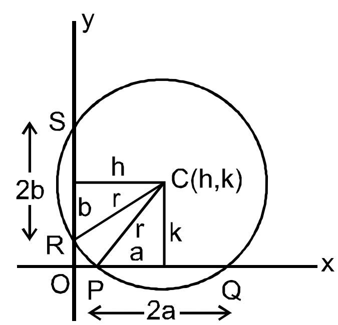
OBJECTIVE QUESTIONS

Q.1 (B)

$h^2 + b^2 = r^2$

$k^2 + a^2 = r^2$

$\Rightarrow h^2 - k^2 = a^2 - b^2$



$\therefore$ locus is $x^2 - y^2 = a^2 - b^2$

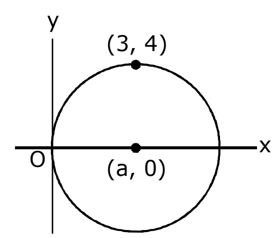
Q.2 (B)

Let centre (a, 0), radius = a

$(a - 3)^2 + 4^2 = a^2$

$-6a + 9 + 16 = 0$

$6a = 25 \Rightarrow a = \frac{25}{6}$



$g = -\frac{25}{6}$, f=0, c=0

$x^2 + y^2 - \frac{25}{3}x = 0$

Aliter :

c = 0, f = 0 Let circle

$x^2 + y^2 + 2gx = 0$ passes (3, 4)

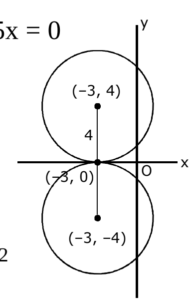
$9 + 16 + 6g = 0$

$g = \frac{-25}{3} \Rightarrow 3(x^2 + y^2) - 25x = 0$

Q.3 (C)

$(x + 3)^2 + (y \pm 4)^2 = 16$

$x^2 + y^2 + 6x \pm 8y + 9 = 0$

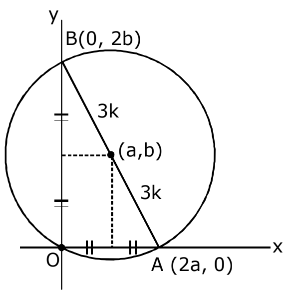


Q.4 (A)

Let centre (a, b)

$AB^2 = (6k)^2 = (2a)^2 + (-2b)^2$

$\Rightarrow a^2 + b^2 = 9k^2$



Let centroid of ΔOAB is (x_1, y_1)

$x_1 = \frac{2a}{3}, y_1 = \frac{2b}{3} \Rightarrow a = \frac{3}{2}x_1, b = \frac{3}{2}y_1$

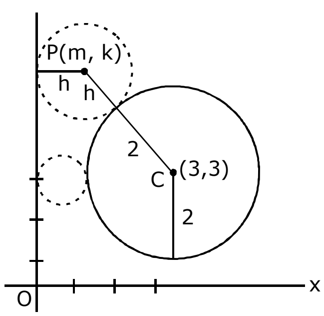
$\Rightarrow \left(\frac{3x_1}{2}\right)^2 + \left(\frac{3y_1}{2}\right)^2 = 9k^2$

$\Rightarrow x_1^2 + y_1^2 = (2k)^2 \Rightarrow x^2 + y^2 = (2k)^2$

Q.5 (D)

$x^2 + y^2 - 6x - 6y + 14 = 0$

centre (3, 3), radius = 2



$\Rightarrow$ radius is h ($\because$ touches y-axis)

PC = h + 2

$\sqrt{(h-3)^2 + (k-3)^2} = (h + 2)$

$\Rightarrow h^2 + k^2 - 6h + 18 = 4 + 4h + h^2$

$\Rightarrow k^2 - 10h - 6k + 14 = 0$

$\Rightarrow y^2 - 10x - 6y + 14 = 0$

Q.6 (D)

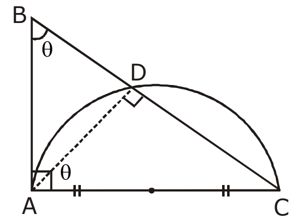
$AD \perp BC$

In $\Delta ACD \Rightarrow \frac{AD}{AC} = \sin\theta$ ....(i)

In $\Delta ABD \Rightarrow \frac{AD}{AB} = \cos\theta$ ....(ii)

$(i)^2 + (ii)^2$

$\Rightarrow \frac{AD^2}{AC^2} + \frac{AD^2}{AB^2} = 1$



$\Rightarrow \frac{1}{AC^2} + \frac{1}{AB^2} = \frac{1}{AD^2}$

$\Rightarrow AC^2 = \frac{AB^2 AD^2}{AB^2 - AD^2} \Rightarrow AC = \frac{AB.AD}{\sqrt{AB^2 - AD^2}}$

Q.7 (D)

Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

passes through (1, t), (t, 1) & (t, t)

$\Rightarrow 1 + t^2 + 2g + 2ft + c = 0$ ....(i)

$\Rightarrow t^2 + 1 + 2gt + 2f + c = 0$ ....(ii)

$\Rightarrow t^2 + t^2 + 2gt + 2ft + c = 0$ ....(iii)

by (i), (ii) & (iii) we get

$g = -\frac{(t+1)}{2}, f = -\frac{(t+1)}{2}, c = 2t$

$\therefore x^2 + y^2 - x(t + 1) - y(t + 1) + 2t = 0$

$(x^2 + y^2 - x - y) + t(-x - y + 2) = 0$

$\Rightarrow S + tL = 0$

Fixed point of intesection of S & L

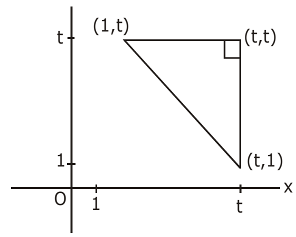
$\therefore x^2 + y^2 = 2$

& $x + y = 2$

$\Rightarrow x^2 + (2 - x)^2 = 2$

$\Rightarrow 2x^2 - 4x + 2 = 0$

$\Rightarrow (x - 1)^2 = 0$



$\Rightarrow x = 1$ & $y = 1$

Point (1, 1)

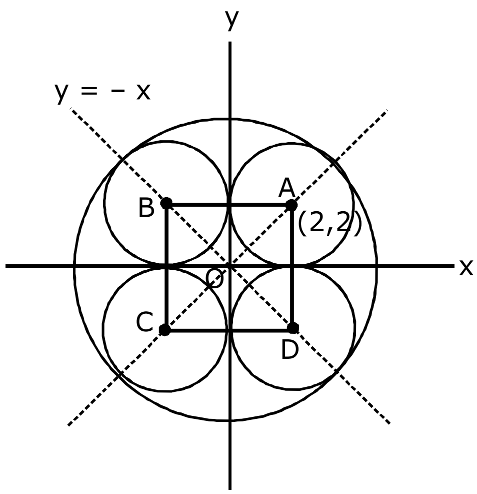
Q.8 (A,C,D)

Centres (2,2),(–2,2),(–2,–2),(2,–2) & radius =2

(A) Centres lies on $y^2 - x^2 = 0$

(B) not only y = x

(C) Area of quadrilateral ABCD



= 4 × 4 = 16 sq. units.

(D) Radius of such circle = OA + 2

$= \sqrt{2^2 + 2^2} + 2 = 2\sqrt{2} + 2$

$= 2(\sqrt{2} + 1)$

Area = $\pi 2^2 (\sqrt{2} + 1)^2 = \pi 4(3 + 2\sqrt{2})$

Q.9 (C)

Point $\left(t, \frac{1}{t}\right)$ lies on $x^2 + y^2 = 16$

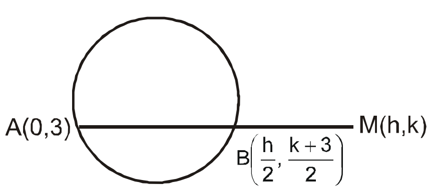
$t^2 + \frac{1}{t^2} = 16$

$\Rightarrow t^4 - 16t^2 + 1 = 0$ ....(i)

If roots are t_1, t_2, t_3, t_4 then

$t_1t_2t_3t_4 = 1$ ....(ii)

Q.10 (B)



B lies on circle

$\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0$

$\Rightarrow \frac{h^2}{4} + 2h + \frac{(k-3)^2}{4} = 0$

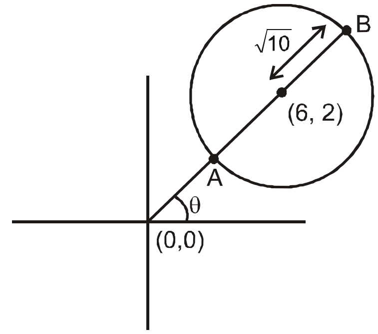
Hence locus of (h, k) $x^2 + 8x + (y - 3)^2 = 0$

Q.11 (A)

By parameteric

$B(6 + \sqrt{10}\cos\theta, 2 + \sqrt{10}\sin\theta)$

$\tan\theta = \frac{1}{3}$



$B\left(6 + \sqrt{10} \times \frac{3}{\sqrt{10}}, 2 + \sqrt{10} \times \frac{1}{\sqrt{10}}\right) \equiv B(9, 3)$

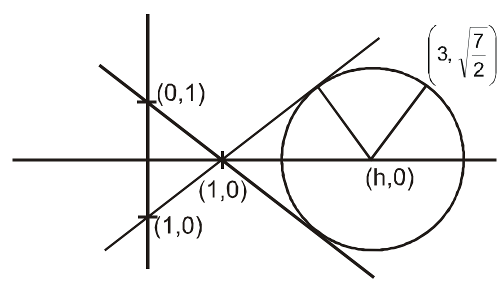
Q.12 (A)

$(x^2 - 2x + 1) - y^2 = 0 \Rightarrow (x + y - 1) = 0$

$x - y - 1 = 0$

$\left|\frac{h - 0 - 1}{\sqrt{2}}\right| = \sqrt{(h-3)^2 + \frac{7}{2}}$

$h^2 + 1 - 2h = 2\left(h^2 + 9 - 6h + \frac{7}{2}\right)$



$\Rightarrow h^2 - 10h + 24 = 0 \Rightarrow h = 6, 4$

But centre lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$

Hence required point (4, 0)

Q.13 (B)

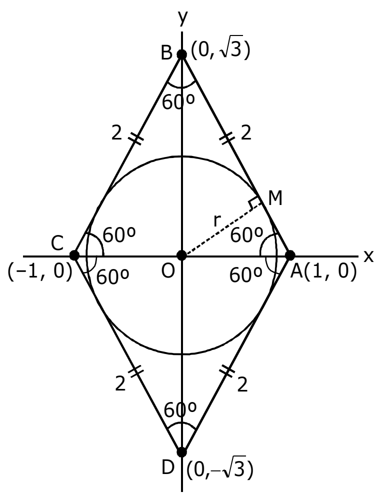
AC = 2 = AB = BC = CA = AD

$OB = \sqrt{2^2 - 1} = \sqrt{3}$

In ΔOAM,

$\frac{r}{OA}$ sin 60º

$\Rightarrow r = \dfrac{\sqrt{3}}{2}$



Any point
on the circle

$P\left(\dfrac{\sqrt{3}}{2}\cos\theta, \dfrac{\sqrt{3}}{2}\sin\theta\right)$

$|PA|^2 = \left(\dfrac{\sqrt{3}}{2}\cos\theta - 1\right)^2 + \left(\dfrac{\sqrt{3}}{2}\sin\theta\right)^2 = \dfrac{3}{4} + 1 - \cos\theta$

$|PB|^2 = \left(\dfrac{\sqrt{3}}{2}\cos\theta\right)^2 + \left(\dfrac{\sqrt{3}}{2}\sin\theta - \sqrt{3}\right)^2 = \dfrac{3}{4} + 3 - 3$

$\sin\theta$

$|PC|^2 = \left(\dfrac{\sqrt{3}}{2}\cos\theta + 1\right)^2 + \left(\dfrac{\sqrt{3}}{2}\sin\theta\right)^2 = \dfrac{3}{4} + 1 + \sqrt{3}$

$\cos\theta$

$|PD|^2 = \left(\dfrac{\sqrt{3}}{2}\cos\theta\right)^2 + \left(\dfrac{\sqrt{3}}{2}\sin\theta + \sqrt{3}\right)^2 = \dfrac{3}{4} + 3 + 3$

$\sin\theta \Rightarrow$ sum $= 4 \cdot \dfrac{3}{4} + 8 = 11$

Q.14 (A)

$x^2 + y^2 < 25$

on x-axis & y-axis $4 \times 4 + 1 = 17$

$x = 1, y = 1, 2, 3, 4$

$x = 2, y = 1, 2, 3, 4$

$x = 3, y = 1, 2, 3$

$x = 4, y = 1, 2$

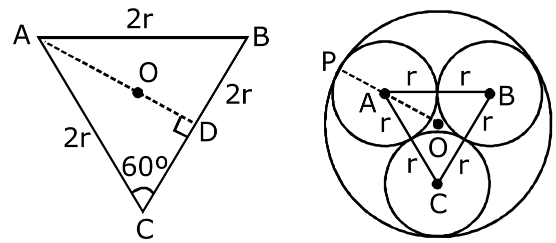
In I^s quadrant 13

In all quadrant $= 13 \times 4 = 52$

No. of points $= 52 + 17 = 69$

Q.15 (B)

$AD = 2r \sin 60^\circ = 2r\,\dfrac{\sqrt{3}}{2} = \sqrt{3}\,r$

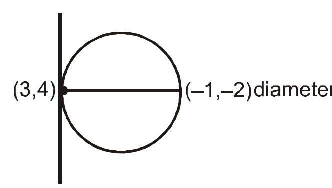


$AO = \sqrt{3}r \times \dfrac{2}{3} = \dfrac{2r}{\sqrt{3}}$

$OP = OA + AP$

$= \dfrac{2r}{\sqrt{3}} + r = \dfrac{(2+\sqrt{3})r}{\sqrt{3}}$

Q.16 (B)



$(x - 3)(x + 1) + (y - 4)(y + 2) = 0$

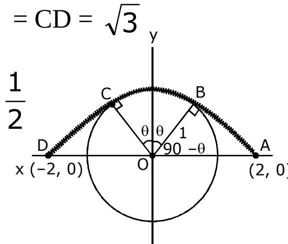
Equation $x^2 + y^2 - 2x - 2y - 11 = 0$

Q.17 (C)

$r = 1$

$AB = \sqrt{2^2 - 1} = CD = \sqrt{3}$

$\cos(90 - \theta) = \dfrac{1}{2}$



$\sin\theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{6} \Rightarrow 2\theta = \dfrac{\pi}{3}$

arc $BC = \ell(\widehat{BC}) = \dfrac{2\pi.1}{6} = \dfrac{\pi}{3}$

Shortest path is $= 2\sqrt{3} + \dfrac{\pi}{3}$

Q.18 (C)

as we know $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2} = 7$

$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} = 11$

squaring & subtact $r_1r_2 = 18$

Q.19 (A)

Let any point $P(x_1, y_1)$ to the circle $x^2 + y^2 - \frac{16x}{5} + \frac{64y}{15} = 0$

$x_1^2 + y_1^2 - \frac{16}{5}x_1 + \frac{64}{15}y_1 = 0$

Length of tangent from $P(x_1, y_1)$ to the circle are in ration

$$\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{\sqrt{x_1^2 + y_1^2 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}}{\sqrt{x_1^2 + y_1^2 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{-96x_1 + 128y_1 + 900}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{4(-24x_1 + 32y_1 + 225)}} = \frac{1}{2}$$

Q.20 (A)

Standard result $= \frac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2} = \frac{3(25-9)^{3/2}}{25}$

$= \frac{3 \times 16 \times 4}{25} = \frac{192}{25}$

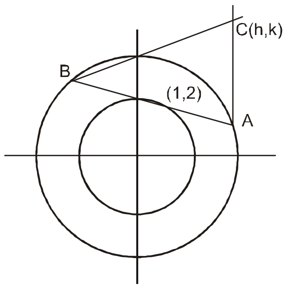
Q.21 (D)

Tangent at (1, 2) to the circle $x^2 + y^2 = 5$

$x + 2y - 5 = 0$

chord of contact from C(h, k) to $x^2 + y^2 = 9$

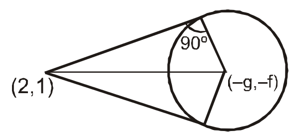
$hx + ky - 9 = 0$



compare both equations $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

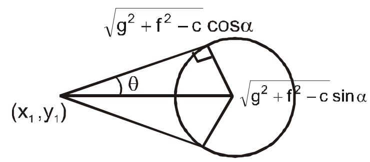
$(h, k) \equiv \left(\frac{9}{5}, \frac{18}{5}\right)$

Q.22 (A)



$(x + g)(x - 2) + (y + f)(y - 1) = 0$

Q.23 (B)



$\tan\theta = \tan\alpha \Rightarrow \theta = \alpha$

angle $= 2\alpha$

Q.24 (B, C)

$(x - 4)^2 + (y - 8)^2 = 20$

$x^2 + y^2 - 8x - 16y + 60 = 0$

C.O.C.

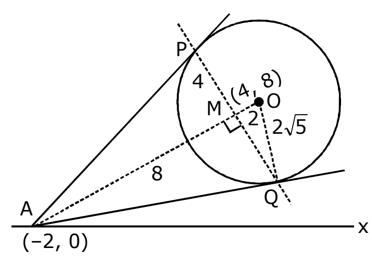
$-2x - 4(x - 2)\ (x - 2) - 8\ (y + 0) + 60 = 0$

$-6x - 8y + 68 = 0$

$\Rightarrow 3x + 4y - 34 = 0$

$AO = \sqrt{6^2 + 8^2} = 10$

$OM = \frac{12x + 32 - 34}{\sqrt{3^2 + 4^2}} = \frac{10}{5} = 2$



$M\left(\frac{14}{5}, \frac{32}{5}\right)$

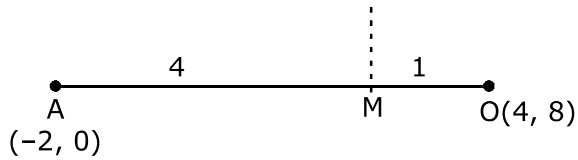
$PM = \sqrt{20 - 4} = \sqrt{16} = 4$

C.O.C $= \tan\theta = \frac{-3}{4}$

$\Rightarrow \sin\theta = \frac{3}{5}, \cos\theta = \frac{-4}{5}$

in parametric form

$$\frac{x-\frac{14}{5}}{-\frac{4}{5}} = \frac{y-\frac{32}{5}}{\frac{3}{5}} = \pm 4$$



$\Rightarrow \frac{5x-14}{-4} = \frac{5y-32}{3} = \pm 4$

$\Rightarrow$ 5x = 14 – 16, 5y = 32 + 12

$x = -\frac{2}{5}, y = \frac{44}{5}$

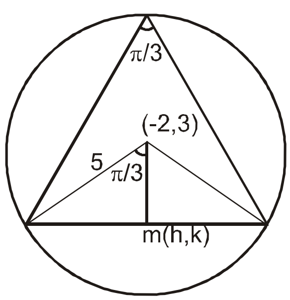
$\left(\frac{-2}{5}, \frac{44}{5}\right)$

5x = 14 + 16 , 5y = 32 – 12
x = 6, y = 4
(6, 4)

Q.25 (B)

$\cos \pi/3 = \frac{\sqrt{(h+2)^2+(k-3)^2}}{5}$

Locus $(x + 2)^2 + (y - 3)^2 = 6.25$

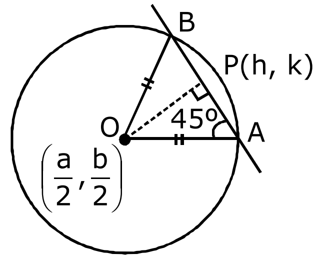


Q.26 (C)
Given $x^2 + y^2 - ax - by = 0$

Centre $\equiv \left(\frac{a}{2}, \frac{b}{2}\right)$, $r = \frac{\sqrt{a^2+b^2}}{2}$

In ΔOPA,

$\Rightarrow \frac{OP}{OA} = \sin 45^o$



$\Rightarrow OP = \frac{OA}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{a^2+b^2}}{2\sqrt{2}} = \sqrt{\left(h-\frac{a}{2}\right)^2 + \left(k-\frac{b}{2}\right)^2}$

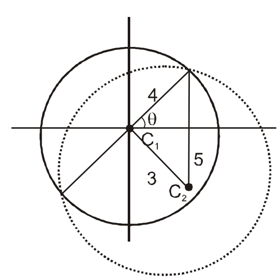
$\Rightarrow \frac{a^2+b^2}{8} = h^2 - ah + \frac{a^2}{4} + k^2 - bk + \frac{b^2}{4}$

$\Rightarrow h^2 + k^2 - ah - bk + \frac{a^2+b^2}{8} = 0$

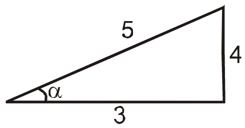
$\Rightarrow x^2 + y^2 - ax - by + \frac{a^2+b^2}{8} = 0$

Q.27 (C)
Pair of tangents from (0, 0) on
$x^2 + y^2 + 20 (x + y) + 20 = 0$
$T^2 = SS_1$
$(0 + 20 (x + y) + 20)^2$
$= (x^2 + y^2 + 20x + 20y + 20) (20)$
$(x + y)^2 + 400 (x + y) + 400$
$= 20 (x^2 + y^2) + 400 (x + y) + 400$
$5(x + y)^2 = x^2 + y^2$
$4x^2 + 4y^2 + 10xy = 0$
$2x^2 + 5xy + 2y^2 = 0$
M-II C.O.C from (0, 0) 8 honoziniation to circle and get pair to tangents.

Q.28 (B)



slope of C_1C_2 is $\tan\alpha = -\frac{4}{3}$



By using parametric coordinates

C_2 (± 3 cos α , ± 3 sin α)

C_2 (± 3 (–3/5) , ± 3 (4/5))

C_2 (± 9/5 , ∓ 12/5)

Q.29 (B)

If two circles touch each other, then

$C_1C_2 = r_1 + r_2$

$$\sqrt{(-g_1+g_2)^2+(-f_1+f_2)^2} = \sqrt{g_1^2+f_1^2} + \sqrt{g_2^2+f_2^2}$$

squaring both sides

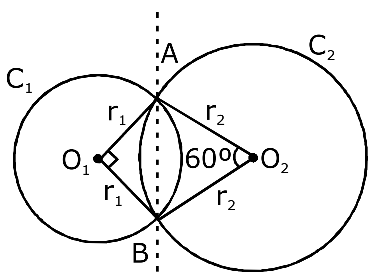
$$-2g_1g_2 - 2f_1f_2 = 2\sqrt{(g_1^2+f_1^2)(g_2^2+f_2^2)}$$

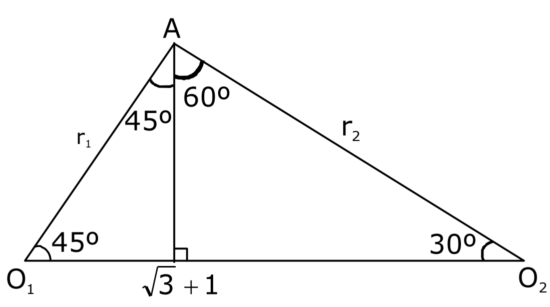
$$\Rightarrow (g_1 f_2)^2 + (g_2 f_1)^2 - 2g_1g_2f_1f_2 = 0 \Rightarrow \frac{g_1}{g_2} = \frac{f_1}{f_2}$$

Q.30 (C)

$O_1O_2 = \sqrt{3} + 1$

Sine rule in AO_1O_2





$$\frac{\sqrt{3}+1}{\sin 105^\circ} = \frac{r_1}{\sin 30^\circ} = \frac{r_2}{\sin 45^\circ}$$

$$r_1 = \frac{\sqrt{3}+1}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)} \times \frac{1}{2} = \sqrt{2}$$

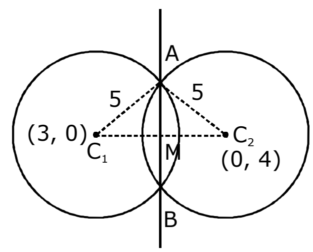
$r_2 = 2$

Q.31 (B)

Common chord $r_1 = 5 = r_2$

$-6x + 8y - 7 = 0$

$\Rightarrow 6x - 8y + 7 = 0$



$$C_1M = \left|\frac{18-0+7}{\sqrt{6^2+8^2}}\right| = \frac{25}{10} = \frac{5}{2}$$

$$AM = \sqrt{25-\frac{25}{4}} = \sqrt{\frac{75}{4}} = \frac{5}{2}\sqrt{3}$$

$AB = 2AM = 5\sqrt{3}$

Aliter :

$r_1 = r_2 = 5$

$AC_1 = AC_2 = C_1C_2 = 5$

$\Rightarrow \Delta AC_1C_2$ equilateral

$$AM = 5\sin 60^\circ = \frac{5\sqrt{3}}{2} \Rightarrow AB = 5\sqrt{3}$$

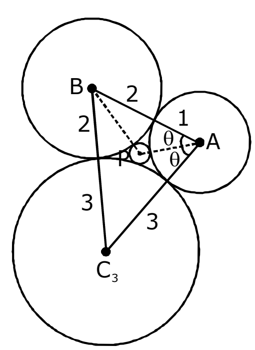
Q.32 (C)

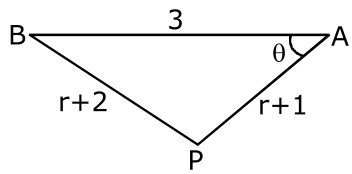
(4) a = 5, b = 4, c = 3

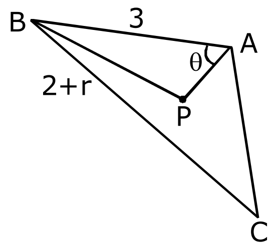
which is right angled Δ at A

∠PAB = θ, ∠PAC = α, θ + α = 90°

In ΔABP







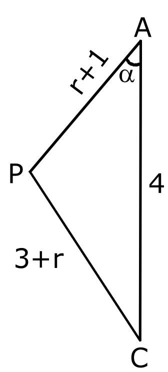
$$\cos\theta = \frac{9+(r+1)^2-(r+2)^2}{2.3.(r+1)}$$

$$= \frac{9+r^2+2r+1-r^2-4r-4}{6(r+1)} = \frac{6-2r}{6(r+1)}$$

$\Rightarrow \cos\theta = \dfrac{3-r}{3(1+r)}$ (A)

In ΔACP

$$\cos\alpha = \frac{16+(r+1)^2-(3+r)^2}{2.4.(r+1)}$$



$$= \frac{16+r^2+2r-1-9-6r-r^2}{2.4(r+1)}$$

$$= \frac{8-4r}{8(r+1)} = \frac{(2-r)}{2(1+r)}$$

$\theta + \alpha = 90^o$

$\theta = 90 - a \Rightarrow \cos\theta = \sin\alpha$

$\Rightarrow \cos^2\theta = \sin^2\alpha$

$$= \frac{(3-r)^2}{9(1+r)^2} = \frac{4(1+r)^2-(2-r)^2}{4(1+r)^2}$$

$\Rightarrow$ $4(9 - 6r + r^2) = 9\,[4 + 8r + 4r^2 + 4r - r^2]$

$\Rightarrow$ $36 - 24r + 4r^2 = 108r + 27r^2$

$\Rightarrow$ $23r^2 + 132\,r - 36 = 0$

$\Rightarrow$ $(r + 6)\,(23r - 6) = 0$

$\Rightarrow$ $r = \dfrac{6}{23}$

$\because$ $r + 6 \neq 0$

Q.33 (C)

$x^2 + y^2 = 1$, $C_1(0, 0)$, $r_1 = 1$

$x^2 + y^2 - 2x - 6y + 6 = 0$, $C_2\,(1, 3)$, $r_2 = 2$

$$\frac{C_1P}{C_2P} = \frac{1}{2}$$

O is mid point of PC_2

$P(-1, -3)$

D.C.T.

$y + 3 = m(x + 1) \Rightarrow mx - y + m - 3 = 0$

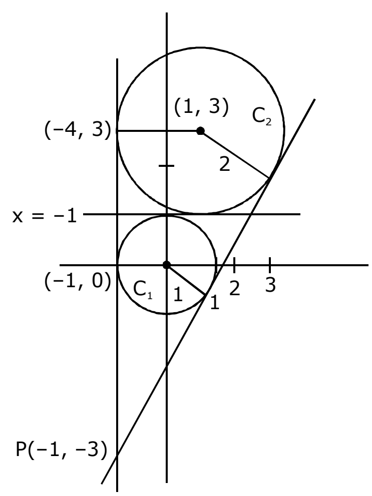
$$1 = \frac{|m-3|}{\sqrt{m^2+1}}$$

$\Rightarrow$ $m^2 + 1 = m^2 + 1 = m^2 - 6m + 9$

$m = \dfrac{4}{3}$ & $m = \infty$

$x = -1$ & $4x - 3y - 5$

Q. $\left(\dfrac{1.1+2.0}{3}, \dfrac{3.1+2.0}{3}\right) \equiv \left(\dfrac{1}{3}, 1\right)$



T.C.T.

$$y - 1 = m\left(x - \frac{1}{3}\right)$$

$\Rightarrow$ $3mx - 3y + 3 - m = 0$

$$1 = \frac{|3-m|}{\sqrt{9m^2+9}}$$

$\Rightarrow$ $9m^2 + 9 = m^2 - 6m + 9$

$\Rightarrow$ $8m^2 + 6m = 0$

$m = 0, m = -\frac{3}{4}$

y = 1 & 3x + 4y – 5 = 0

Q.34 (A)

Common chord of given circle

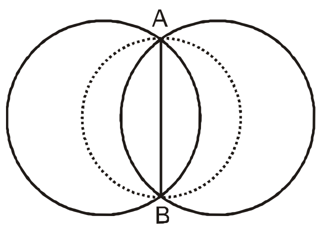
2x + 3y – 1 = 0

family of circle passing through point of intersection of given circle

$(x^2 + y^2 + 2x + 3y - 5) + \lambda(x^2 + y^2 - 4) = 0$

$(\lambda + 1)x^2 + (\lambda + 1)y^2 + 2x + 3y - (4\lambda + 5) = 0$

$$x^2 + y^2 + \frac{2x}{\lambda+1} + \frac{3}{\lambda+1}y - \frac{(4\lambda+5)}{\lambda+1} = 0$$



$$\text{centre } \left(-\frac{1}{\lambda+1}, \frac{-3}{2(\lambda+1)}\right)$$

This centre lies on AB

$$2\left(-\frac{1}{\lambda+1}\right) + 3\left(\frac{-3}{2(\lambda+1)}\right) - 1 = 0$$

$-4 - 9 - 2\lambda - 2 = 0$

$\Rightarrow 2\lambda = -15$

$\Rightarrow \lambda = -15/2$

$$\left(-\frac{15}{2}+1\right)x^2 + \left(-\frac{15}{2}+1\right)y^2 + 2x + 3y - \left(-4\times\frac{15}{2}+5\right) = 0$$

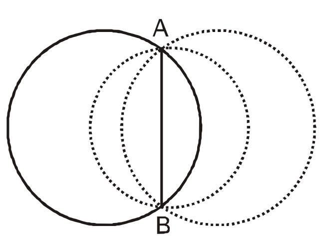
$$\Rightarrow -\frac{13x^2}{2} - \frac{13y^2}{2} + 2x + 3y + 25 = 0$$

$\Rightarrow 13(x^2 + y^2) - 4x - 6y - 50 = 0$

Q.35 (B)

$(x^2 + y^2 - 6x - 4y - 12) + \lambda(4x + 3y - 6) = 0$

This is family of circle passing through points of intersection of circle



$x^2 + y^2 - 6x - 4y - 12 = 0$ and line 4x + 3y – 6 = 0

other family will cut this family at A & B.

Hence locus of centre of circle of other family is this common chord 4x + 3y – 6 = 0

Q.36 (A)

Let required equation of circle is $x^2 + y^2 + 2gx + 2gx + 2fy + c = 0$

it cuts the circle $x^2 + y^2 - 9 = 0$ orthogonally

$\therefore\ 2g(0) + 2f(0) = c - 9 \Rightarrow c = 9$

It also touches straight line $\ell x + my + n = 0$

$$\therefore \left|\frac{\ell(-g)+m(-f)+n}{\sqrt{\ell^2+m^2}}\right| = \sqrt{g^2+f^2-9}$$

Locus of centre (–g, –f) is $(\ell x + my + n)^2 = (x^2 + y^2 - 9)(\ell^2 + m^2)$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, D)

$$\left|\frac{4C+3C-12}{5}\right| = C \quad \Rightarrow \quad C = 1, 6$$

Q.2 (B, C)

Let equation of required circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

it passes through (1, –2) & (3, –4)

2g – 4f + c = –5

6g – 8f + c = –25

4g – 8f + 2c = –10

6g – 8f + c = –25

–2g + c = 15

circle touches x-axis $g^2 = c \Rightarrow g^2 - 2g - 15 = 0$

g = 5, – 3

g = 5, c = 25, f = 10 $\Rightarrow x^2 + y^2 + 10x + 20y + 25 = 0$

g = –3, c = 9, f = 2 $\Rightarrow x^2 + y^2 - 6x + 4y + 9 = 0$

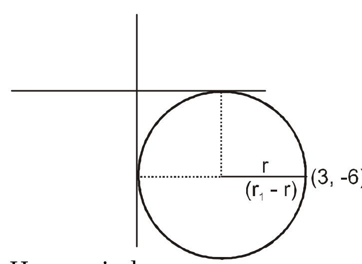
Q.3 (A, D)

Now

$(r - 3)^2 + (-r + 6)^2 = r^2$

$r^2 - 18r + 45 = 0$

$\Rightarrow r = 3, 15$



Hence circle

$(x - 3)^2 + (y + 3)^2 = 3^2$

$x^2 + y^2 - 6x + 6y + 9 = 0$

$(x - 15)^2 + (y + 15)^2 = (15)^2$

$\Rightarrow x^2 + y^2 - 30x + 30y + 225 = 0$

Q.4 (A, D)

Two fixed pts. are point of intersection of

$x^2 + y^2 - 2x - 2 = 0$ & $y = 0$

Point $x^2 - 2x - 2 = 0$

$(x - 1)^2 - 3 = 0$

$\Rightarrow x - 1 = \sqrt{3}$, $x - 1 = -\sqrt{3}$

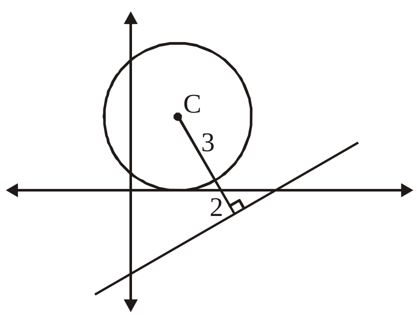
$(1+\sqrt{3}, 0)$ $(1-\sqrt{3}, 0)$

Q.5 (C,D)

$r = \sqrt{2^2 + 3^2 - 4} = 3 \Rightarrow CP = 5$

$$\frac{|2a + 9 + 8|}{\sqrt{a^2 + 9}} = 5$$

$|2a + 17| = 5\sqrt{a^2 + 9}$



$4a^2 + 289 + 68a = 25a^2 + 225$

$21a^2 - 68a - 64 = 0$

$$S = \frac{68}{21}$$

$\Rightarrow [S] = 3$

Q.6 (B, C)

$(x - r)^2 + y^2 = r^2$

$\Rightarrow x^2 + y^2 - 2xr = 0$

8 tangent at (x_1, y_1)

$xx_1 + yy_1 - r(x + x_1) = 0$

$(x_1 - r)x + yy_1 - rx_1 = 0$

slope $m_T = \frac{r - x_1}{y_1} = \frac{r - x}{y}$

(B)

$$\frac{r - x}{y} = \frac{2xr - 2x^2}{2xy}$$

$$= \frac{x^2 + y^2 - 2x^2}{2xy} = \frac{y^2 - x^2}{2xy}$$

(C)

Q.7 (A,C)

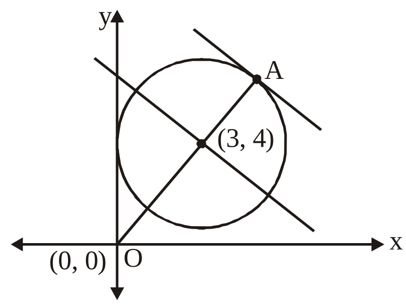
Point A is on the circle which is farthest from the origin

$\therefore$ Equation of tangent at A

$3x + 4y = \lambda$

Applying $p = r$

$$\left|\frac{9 + 16 - \lambda}{5}\right| = 3$$



$\Rightarrow 25 - \lambda = \pm 15$

$\Rightarrow \lambda = 40$ or 10

Required tangent is $3x + 4y = 40$

Normal to the circle which is forthest from the origin is, straight line perpendicular to OA passing through the centre

$\therefore 3x + 4y - 25 = 0$

Q.8 (A,B,C)

$(x - 3)^2 + (y - a)^2 = a^2 - 8$

Equation of director circle $(x - 3)^2 + (y - a)^2 = 2(a^2 - 8)$

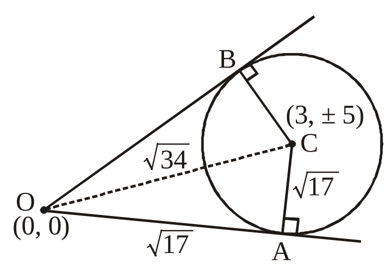
passes (0, 0), $9 + a^2 = 2a^2 - 16$

$a^2 = 25 \Rightarrow a = -5, 5$

$\Rightarrow$ S : $(x - 3)^2 + (y - 5)^2 = 17$

OR

$(x - 3)^2 + (y + 5)^2 = 17$



area of □OACB = 17

chord of contact AB : $-3(x + 0) \pm 5(y) + 17 = 0$

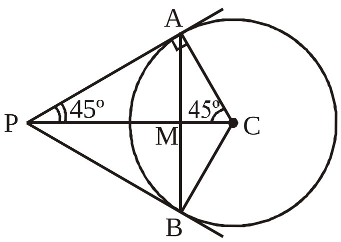
$3x \mp 5y = 17$]

Q.9 (B,C)

$\because$ Pair of tangents are perpendicular to each other

$\therefore$ PA = radius = 5

$$AM = PA \sin 45^\circ = \frac{5}{\sqrt{2}}$$



$\therefore$ length of AB = $5\sqrt{2}$

area of quadrilateral = 2 × area of ΔPAC = 2 × $\frac{1}{2}$ × 5 × 5 = 25

Circumcircle of ΔPAB will circle with PC as diameter

length of PC = $5\sqrt{2}$

$\therefore$ radius = $\frac{5}{\sqrt{2}}$ **Ans.**]

Q.10 (A,C)

$S_1 \equiv x^2 + y^2 + 6x = 0$

$\Rightarrow C_1(-3, 0), r_1 = 3$

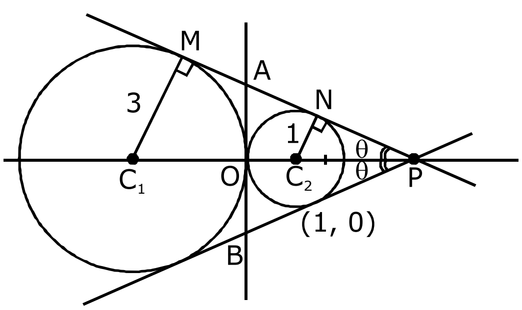
$S_2 \equiv x^2 + y^2 - 2x = 0$

$\Rightarrow C_2 (1, 0), r_2 = 1$

$C_1C_2 = 4$ $\quad r_1 + r_2 = 4$

$C_1C_2 = r_1 + r_2$ $\quad$ (A)

S_1 & S_2 touch each other externally



$\frac{PC_1}{PC_2} = \frac{3}{1}$

PO $\left(\frac{(-3)1-(1)3}{1-3}, 0\right) \equiv$ P (3, 0)

$OP = 3, OC_2 = 1, C_2P = 2$

In $\Delta C_2NP \Rightarrow \frac{1}{2} = \sin\theta \Rightarrow \theta = 30^\circ$

$\frac{OA}{OP} = \tan 30^\circ$

$\Rightarrow OA = \frac{3}{\sqrt{3}} \Rightarrow OA = \sqrt{3}$

Area of ΔPAB= $\frac{1}{2}$ AB×OP

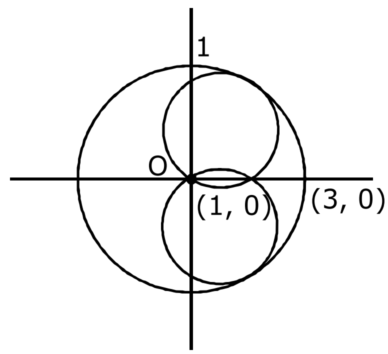
$= \frac{1}{2} \times 2\sqrt{3} \times 3 = 3\sqrt{3}$ (C)

Q.11 (C, D)

Let circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

passing (0, 0) & (1, 0)



$C = 0 \quad 1 + 2q = 0 \Rightarrow g = -\frac{1}{2}$

Circle will be

$x^2 + y^2 - x + 2fy = 0$

$\left(\frac{1}{2}, -f\right)$, $r_1 = \sqrt{f^2 + \frac{1}{4}}$

touches internally

$x^2 + y^2 = 9, (0, 0), r_2 = 3$

$\sqrt{\left(\frac{1}{2}\right)^2 + f^2} = \left|3 - \sqrt{f^2 + \frac{1}{4}}\right| \quad \left\{\because 3 > \sqrt{f^2 + \frac{1}{4}}\right.$

$\frac{1}{4} + f^2 = \left(3 - \sqrt{f^2 + \frac{1}{4}}\right)^2$

$\Rightarrow \frac{1}{4} + f^2 = 9 + f^2 + \frac{1}{4} - 6\sqrt{f^2 + \frac{1}{4}}$

$\Rightarrow \sqrt{f^2 + \frac{1}{4}} = \frac{3}{2} \Rightarrow f^2 + \frac{1}{4} = \frac{9}{4}$

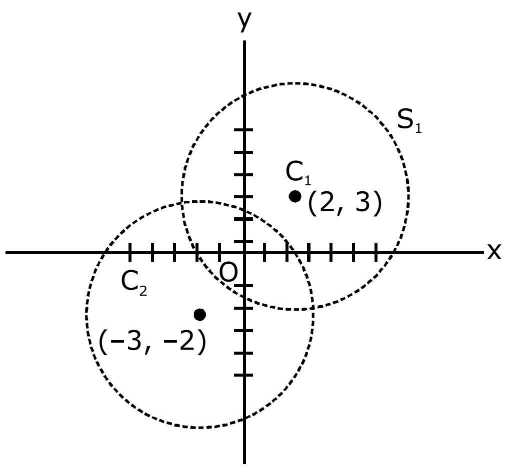
$\Rightarrow f^2 = 2 \Rightarrow f = \pm\sqrt{2}$

Centres are $\left(\frac{1}{2}, \pm\sqrt{2}\right)$

Q.12 (B,C,D)

$S_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$

$\Rightarrow C_1 (2, 3), r = 5$



$S_2 \equiv x^2 + y^2 + 6x + 4y - 12 = 0$
C_2 (–3, –2), r = 5
L = x + y = 0
$S_1 - S_2 = 0$
– 10x – 10y = 0
$\Rightarrow$ x + y = 0
(A) Origin inside both cirlce
(B) L is common chord
(C) L is radical Axis

(D) $m_{C_1C_2} = \dfrac{5}{5} = 1$ & $m_L = -1$

$C_1C_2 \perp L$

Q.13 (A,B,C)
$\because$ Centre of S_1 = (5, 0) and radius $r_1 = 3$
$\therefore$ Centre of S_2 = (0, 5) and radius $r_2 = 3$
and Centre of S_3 = (0, –5) and radius $r_3 = 3$
$\therefore$ Radical centre of S_1, S_2 and S_3 will be (0, 0)
Length of tangent from (0, 0) upon S_1 or S_2 or S_3 = 4
$\therefore$ Equation of S' will be $\Rightarrow$ $x^2 + y^2 = 16$ and radius = 4.

Q.14 (A,B,C,D)
Equation of required circle is S + λS' = 0,
where $S \equiv x^2 + y^2 + 3x + 7y + 2k - 5 = 0$ and $S' \equiv x^2 + y^2 + 2x + 2y - k^2 = 0$.
As, it passes through (1, 1)

So, the value of $\lambda = \dfrac{-(7+2k)}{(6-k^2)}$.

If 7 + 2k = 0, it becomes second circle.
$\therefore$ It is true for all values of k. **Ans.**]

Q.15 (A, D)
Two fixed pts. are point of intersection of
$x^2 + y^2 - 2x - 2 = 0$ & y = 0
Point $x^2 - 2x - 2 = 0$
$(x - 1)^2 - 3 = 0$
$\Rightarrow$ $x - 1 = \sqrt{3}$, $x - 1 = -\sqrt{3}$

$(1+\sqrt{3},\ 0)$ $(1-\sqrt{3},\ 0)$

Q.16 (B,C)
C : $x^2 + y^2 + 2gx + 2fy + c = 0$
$x^2 + y^2 = 4$
$2(g_1g_2 + f_1f_2) = C_1 + C_2$
$2(0 + 0) = C - 4 \Rightarrow C = 4$
also 2x – 2y+ 9 = 0
2(– g) – 2(– f) + 9 = 0
2f = 2g – 9
$\therefore$ $x^2 + y^2 + 2gx + (2g - 9)y + 4 = 0$
$\therefore$ $(x^2 + y^2 - 9y + 4) + 2g(x + y) = 0$
$\therefore$ $x^2 + y^2 - 9y + 4 = 0$ and x + y = 0
$\therefore$ $x^2 + x^2 + 9x + 4 = 0 \Rightarrow 2x^2 + 9x + 4 = 0 \Rightarrow$

$(2x + 1)(x + 4) = 0 \Rightarrow x = \dfrac{-1}{2}, -4.$

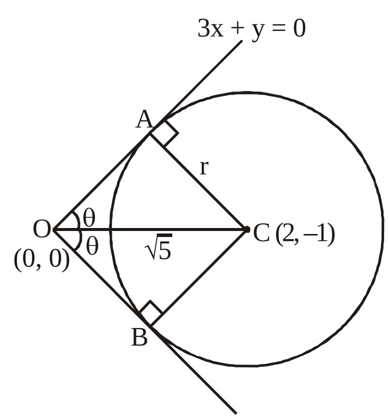
$\therefore$ Point $\left(\dfrac{-1}{2}, \dfrac{1}{2}\right)$, (– 4, 4). **Ans.**]

Comprehenssion # 1 (Q. No. 17 to 19)

Q.17 (D)
Q.18 (A)
Q.19 (C)

. $r = \left|\dfrac{6-1}{\sqrt{10}}\right| = \dfrac{5}{\sqrt{10}} = \sqrt{\dfrac{5}{2}}$

Here $\sin\theta = \dfrac{r}{\sqrt{5}} = \dfrac{\sqrt{5}}{\sqrt{2}\cdot\sqrt{5}} = \dfrac{1}{\sqrt{2}}$



$\theta = \dfrac{\pi}{4}$

$\therefore$ ∠AOB = 90°
Hence 'O' lies on the director circle of S = 0.
$\therefore$ equation of the director circle is

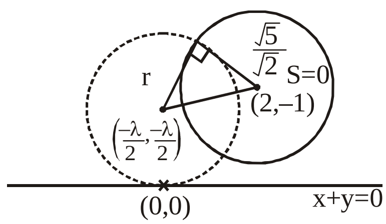
$$(x-2)^2 + (y+1)^2 = \left(\frac{\sqrt{5}}{\sqrt{2}}\cdot\sqrt{2}\right)^2 = 5$$

(ii) $x^2 + y^2 - 4x + 2y = 0$ **Ans.(ii)**

(i) Equation of the other tangent OB = $x - 3y = 0$ **Ans.(i)**

(iii) Let the required circle, is
$x^2 + y^2 + \lambda(x + y) = 0$

Also, S = 0 is, $(x-2)^2 + (y+1)^2 = \frac{5}{2}$.



or, $x^2 + y^2 - 4x + 2y + \frac{5}{2} = 0$

Clearly, $2\left[\frac{\lambda}{2}(-2) + \frac{\lambda}{2}(1)\right] = 0 + \frac{5}{2} \Rightarrow -2\lambda +$

$\lambda = \frac{5}{2} \Rightarrow \lambda = \frac{-5}{2} \Rightarrow x^2 + y^2 - \frac{5x}{2} - \frac{5y}{2} = 0$

So, radius $= \sqrt{\frac{25}{16} + \frac{25}{16}} = \sqrt{\frac{50}{16}} = \frac{5\sqrt{2}}{4}$.

Ans.(iii)]

Comprehenssion # 2 (Q. No. 20 to 22)

Q.20 (C)

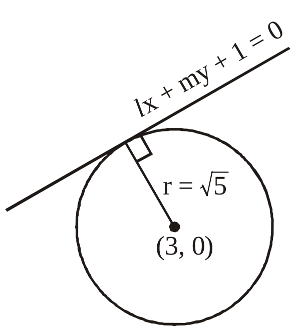
Q.21 (B)

Q.22 (A)

Given $4l^2 - 5m^2 + 6l + 1 = 0$
$(l, m \in R)$

$\Rightarrow (3l + 1)^2 = 5(l^2 + m^2)$

$\Rightarrow \frac{|3l+1|}{\sqrt{l^2+m^2}} = \sqrt{5}$,



So, clearly the line $lx + my + 1 = 0$
is tangent to a fired cirlcle S = 0

i.e., $(x-3)^2 + (y-0)^2 = \left(\sqrt{5}\right)^2$, whose centre is (3, 0) and $r = \sqrt{5}$

(i) $\Rightarrow$ Circle is $x^2 + y^2 - 6x + 4 = 0$ ……(1)

(ii) Any point on line $x + y - 1 = 0$ is $(t, 1 - t)$, $t \in R$.
$\therefore$ The equation of chord of contact for the circle (1) w.r.t. $(t, 1 - t)$ is
$tx + (1 - t)y - 3(t + x) + 4 = 0$
i.e. $t(x - y - 3) + (-3x + y + 4) = 0$, which passes through $\left(\frac{1}{2}, \frac{-5}{2}\right)$

(iii) As line $x - 2y + c = 0$ intersects the circle S orthogonally so the line must passes through centre of circle S.
$\Rightarrow 3 - 2(0) + c = 0 \Rightarrow c = -3$ **Ans.**

Alternative :

Let the required equation of circle S be
$x^2 + y^2 + 2gx + 2fy + c = 0$ ……(1)
As line
$lx + my + 1 = 0$ ……(2)
is tangent to circle (1), so

$$\frac{|-gl - mf + 1|}{\sqrt{l^2+m^2}} = \sqrt{g^2+f^2-c}$$

$\Rightarrow (gl + mf - 1)^2 = (l^2 + m^2)(g^2 + f^2 - c)$
$\Rightarrow (c - f^2)l^2 + (c - g^2)m^2 - 2g\cdot l - 2f\cdot m + 2gf \cdot lm + 1 = 0$ ……(3)
But, we are given
$4l^2 - 5m^2 + 6l + 1 = 0$ ……(4)
$\therefore$ On comparing (3) and (4), we get

$$\frac{c-f^2}{4} = \frac{c-g^2}{-5} = \frac{-2g}{6} = \frac{-2f}{0} = \frac{2fg}{0} = \frac{1}{1}$$

$\Rightarrow g = -3,\ f = 0,\ c = -5 + g^2 = 4$
$\Rightarrow$ The equation of fixed circles $x^2 + y^2 - 6x + 4 = 0$]

Comprehenssion # 3 (Q. No. 23 to 25)

Q.23 (D)

Q.24 (D)

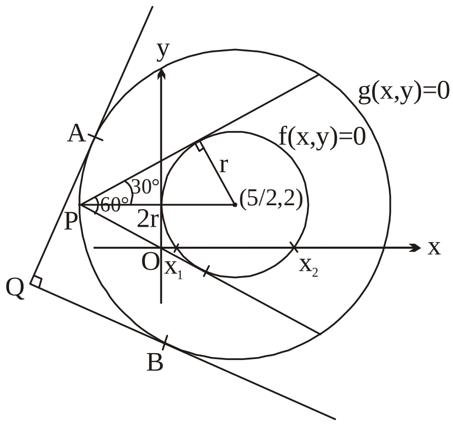
Q.25 (D)

Given f (x, y) = 0 is circle. As f (0, y) has equal roots hence f (x, y) = 0 touches the y-axis and as f (x, 0) = 0 has two distinct real roots hence f (x, y) = 0 cuts the x-axis in two distinct points. Hence f (x, y) = 0 will be as shown
now, given $g(x, y) = x^2 + y^2 - 5x - 4y + c$

centre $= \left(\frac{5}{2}, 2\right)$; radius $= \sqrt{\frac{25}{4} + 4 - c}$

Note that radius of g (x, y) = twice the radius of f (x, y) = 0

but as it is clear from the adjacent figure r = $\frac{5}{2}$



$\therefore$ radius of g (x, y) = 5

hence $\frac{25}{4} + 4 - c$ = 25 $\Rightarrow$ c = $-\frac{59}{4}$

$\therefore$ equation of g (x, y) is

$x^2 + y^2 - 5x - 4y - \frac{59}{4} = 0$

equation of $f(x, y) = 0$

$$\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \frac{25}{4}$$

$$y = 0, \left(x - \frac{5}{2}\right)^2 = \frac{25}{4} - 4 = \frac{9}{4}$$

$$x - \frac{5}{2} = \frac{3}{2} \text{ or } -\frac{3}{2} \Rightarrow x = 4 \text{ or } x = 1$$

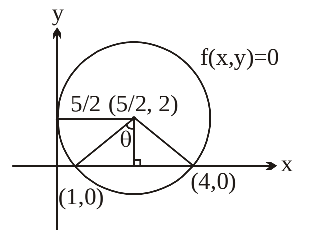
(a) Area of Δ QAB = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$

(b) $\theta = \tan^{-1}\frac{3}{4}$

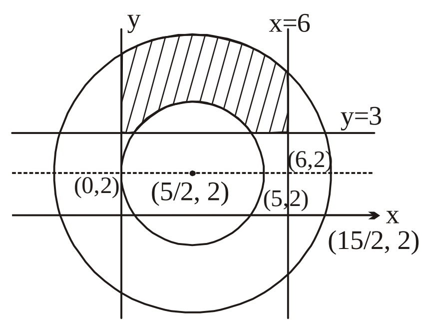
$$2\theta = \tan^{-1}\left(\frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}}\right) = \tan^{-1}\left(\frac{24}{7}\right)$$

Area of region inside f(x, y) = 0 above the x-axis is

$$x\text{–axis} = \frac{1}{2}\left(\frac{5}{2}\right)^2\left(2\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) + \frac{1}{2} \times 3 \times 2$$



$$= 3 + \frac{25}{8}\left(2\pi - \tan^{-1}\left(\frac{24}{7}\right)\right)$$



(c) Points satisfying the conditions are (1, 5) (1, 6), (2, 5), (2, 6) (3, 5), (3, 6) (4, 5), (4, 6), (5, 4), (5, 5), (5, 6).

Q.26 (A) $\rightarrow$ (q), (B) $\rightarrow$ (p), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)

(A) $S_1 - S_2 = 0$ is the required common chord i.e 2x = a

Make homogeneous, we get $x^2 + y^2 - 8.4\frac{x^2}{a^2} = 0$

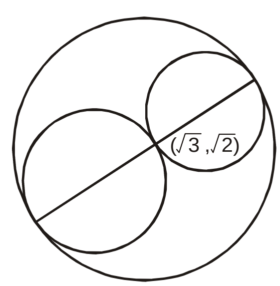
As pair of lines substending angle of 90° at origin

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ a = ± 4

(B) $y = 22\sqrt{3}\,(x - 1)$ passes through centre (1, 0) of circle

(C) Three lines are parallel



(D) $2(r_1 + r_2) = 4$

$r_1 + r_2 = 2$

$\frac{r_1 + r_2}{2} = 1$

Q.27 (A) → (p,q,r,s) (B) → (p,q,r,s,t) (C) → (r,s)

(A) Distance from centre (0, 10) to the line (y – mx = 0)

$= \frac{10}{\sqrt{(1+m^2)}} \geq$ radius

$= \sqrt{10}$ or $\frac{10}{\sqrt{(1+m^2)}} \geq \sqrt{10}$

$\Rightarrow \sqrt{10} \geq \sqrt{1+m^2}$

$\Rightarrow m^2 \leq 9$

$\therefore -3 \leq m \leq 3$

Then $0 \leq |m| \leq 3$

$\therefore |m| = 0, 1, 2, 3$ (p, q, r, s)

(B) Distance from the centre (2, 4) to the line

$(3x - 4y - 5k = 0) = \frac{|6 - 16 - 5k|}{5} \leq$ radius = 5

$\Rightarrow |10 + 5k| \leq 25$

$\Rightarrow 0 \leq |2 + k| \leq 5$

$\therefore |2 + k| = 0, 1, 2, 3, 4, 5$ (p, q, r, s, t)

(C) The given circles will cut orthogoally, if

$2\left(\frac{1}{2}\right)(-5) + 2\left(\frac{p}{2}\right)(p) = -7 + 1$

$\Rightarrow -5p + p^2 = -6$

$\Rightarrow p^2 - 5p + 6 = 0$

$\Rightarrow (p - 2)(p - 3) = 0$

$\therefore p = 2, 3$ (r, s)

Q.28 (A) → (r), (B) → (s), (C) → (p), (D) → (q)

(A) C_1 (1, 0), $r_1 = 1$ and C_2 (–3, 3), $r_2 = 4$

distance between centres C_1 and C_2 = d = 5

$d = r_1 + r_2 = 5$ $\Rightarrow$ 3 common tangents

(B) C_1 (2, 5), $r_1 = 5$ and C_2 (3, 6), $r_2 = 10$

distance between centres C_1 and C_2 = d = $\sqrt{2}$

$d < |r_1 - r_2|$

$\Rightarrow$ no common tangent

(C) C_1 (1, 2), $r_1 = \sqrt{5}$ and C_2 (0, 4), $r_2 = 2\sqrt{5}$

distance between centres C_1 and C_2 = d = $\sqrt{5}$

$|r_1 - r_2| = d$

number of common tangents is 1

(D) C_1 (–1, 4), $r_1 = 2$ and C_2 (3, 1), $r_2 = 2$

distance between centres C_1 and C_2 = d = 5

$d > r_1 + r_2$

$\Rightarrow$ number of direct common tangents is 2

NUMERICAL VALUE BASED

Q.1 **(1)**

Let equation of circle is $(x - \sqrt{2})^2 + (y - \sqrt{3}) = r^2$,

(x_1, y_1) & (x_2, y_2) are integer points on circle

$(x_1 - \sqrt{2})^2 + (y_1 - \sqrt{3})^2 = (x_2 - \sqrt{2})^2 + (y_2 - \sqrt{3})^2 = r^2$

$(x_2 - x_1)(x_2 + x_1 - 2\sqrt{2}) + (y_2 - y_1)(y_2 + y_1 - 2\sqrt{3}) = 0$

$(x_2^2 - x_1^2) + (y_2^2 - y_1^2) = 2\sqrt{3}(y_2 - y_1) + 2\sqrt{2}(x_2 - x_1)$ $A = \sqrt{3}B + \sqrt{2}C$

Therefore A = B = C = 0

$x_1 = x_2$ & $y_1 = y_2$

So, no distinct points are possible.

Q.2 **(49)**

$x^2 + y^2 - 5x + 2y - 5 = 0$

$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y + 1)^2 - 5 - \frac{25}{4} - 1 = 0$

$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y + 1)^2 = \frac{49}{4}$

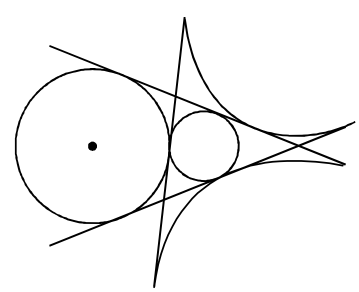
$\Rightarrow$ So the axes are shifted to $\left(\frac{5}{2}, -1\right)$

New equation of circle must be $x^2 + y^2 = \frac{49}{4}$

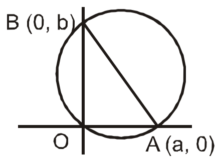
Q.3 **(4)**

Four circles

{one incircle & three excircles}



Q.4 **(2)**



Equation of circum circle of triangle OAB $x^2 + y^2 - ax - by = 0$.

Equation of tangent at origin $ax + by = 0$.

$d_1 = \frac{|a^2|}{\sqrt{a^2+b^2}}$ and $d_2 = \frac{|b^2|}{\sqrt{a^2+b^2}}$

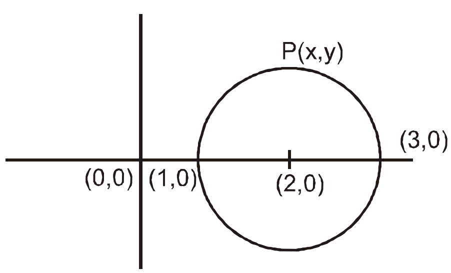
$\Rightarrow \quad d_1 + d_2 = \sqrt{a^2+b^2}$ = diameter

Q.5 (8)

$x^2 + y^2 - 4x + 3 = 0$

$\sqrt{x^2+y^2}$ represents distance of p from origin

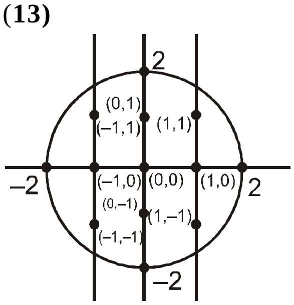
Hence $M = 3^2 + 0^2$



$M = 1^2 + 0^2$

$M - m = 8$

Q.6 **(13)**

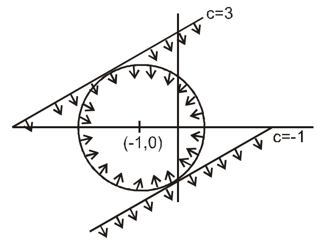


Q.7 **(1)**

$\left|\frac{-1-0+c}{\sqrt{2}}\right| = \sqrt{2} \Rightarrow c - 1 = \pm 2 \Rightarrow c = -1, 3$

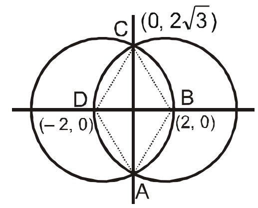
But c = –1 common point is one

c = 3 common point is infinite



Hence c = –1 is Answer.

Q.8 **(8)**

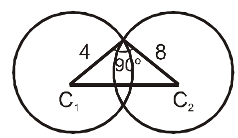


Area of ABCD = $4\left(\frac{1}{2}.2.2\sqrt{3}\right)$.

Q.9 **(16)**

$C_1C_2 = \sqrt{80}$

Area = $\frac{1}{2} \times 4 \times 8 = \frac{1}{2} \times \sqrt{80} \times \frac{\ell}{2}$



$\ell = \frac{64}{\sqrt{80}} = \frac{16}{\sqrt{5}}$

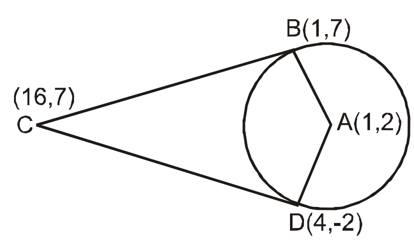
Q.10 **(75)**

Given circle $x^2 + y^2 - 2x - 4y - 20 = 0$

Tangents at B(1, 7) is

$x + 7y - (x + 1) - 2(y + 7) - 20 = 0$

$5y - 35 = 0 \Rightarrow y = 7$



at D (4, –2)

$4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$

$3x - 4y = 20$

Hence c(16, 7)

Area of quadrilateral ABCD = AB × BC = 5 × 15 = 75 square units.

Q.11 **(0)**

Let $S_1 : x^2 + y^2 + 2ax + cy + a = 0$

$S_1 : x^2 + y^2 - 3ax + dy - 1 = 0$

common chord $S_1 - S_2 = 0 \Rightarrow 5ax + y(c - d) + (a + 1)$

= 0
given line is 5x + by – a = 0

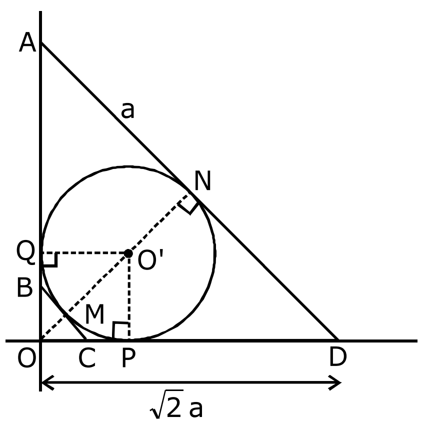
compare both $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$

$\Rightarrow \quad a = \frac{c-d}{b} = -1 - \frac{1}{a}$

(i) (ii) (iii)
From (i) & (iii) $a^2 + a + 1 = 0$
$\Rightarrow a = \omega, \omega^2$ no real a.

Q.12 **(15)**

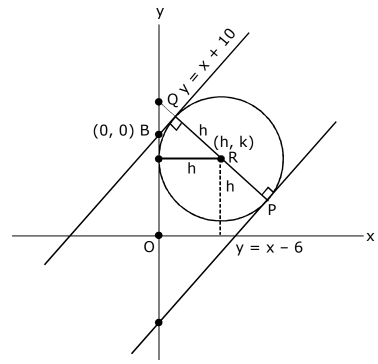
area ABCD = $900\sqrt{2}$ sq. units
ON = ND = NA = a (let)
area $\Delta OAD = a^2$
OD = OA = $\sqrt{2}$ a
OP = $\sqrt{2}$ a – a
= a($\sqrt{2}$ – 1) = radius



OM = ON – 2r
= a – 2a ($\sqrt{2}$ – 1) = a(3 – 2$\sqrt{2}$)
area $\Delta OBC = (OH)^2 = a^2 (3 - 2\sqrt{2})^2$
$a^2 - a^2 (3 - 2\sqrt{2}) = 900\sqrt{2}$
$\Rightarrow a^2 [1 - (3 - 2\sqrt{2})^2] = 900\sqrt{2}$
$\Rightarrow a^2 = \frac{900\sqrt{2}}{(1+3-2\sqrt{2})(1-3+2\sqrt{2})}$
$\Rightarrow = \frac{900\sqrt{2}}{2\sqrt{2}(\sqrt{2}-1)2(\sqrt{2}-1)}$
$\Rightarrow a^2 = \frac{225}{(\sqrt{2}-1)^2} \Rightarrow a = \frac{15}{(\sqrt{2}-1)}$
$\Rightarrow a(\sqrt{2} - 1) = 15 = r$

Q.13 (10**)**

y = x + 10
y = x – 6
$2r = 2h = \frac{10+6}{\sqrt{2}} = \frac{16}{\sqrt{2}} = 8\sqrt{2}$
$2h = 8\sqrt{2}$



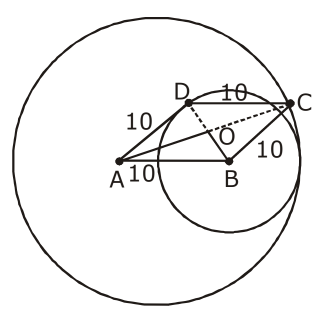
$h = 4\sqrt{2}$
⊥ distance equal to h = $4\sqrt{2}$ from ($4\sqrt{2}$, k)

$4\sqrt{2} = \frac{|4\sqrt{2} - k + 10|}{\sqrt{1^2+1^2}} \quad \Rightarrow \quad 8 = |4\sqrt{2} - k + 10|$

{geometrically k < 10}
$8 = 4\sqrt{2} - k + 10$
$k = 10 - 8 + 4\sqrt{2}$
$k = 2 + 4\sqrt{2}$
$h + k = 2 + 8\sqrt{2}$
$h + k = 2 + 8\sqrt{2}$
$= a + b\sqrt{2}$ a = 2, b = 8
∴ a + b = 10

Q.14 (400)

BD = r_2
AC = r_1
$r_1 - r_2 = 10$
$\Rightarrow (r_1 - r_2)^2 - 2r_1r_2 = 100$
$\Rightarrow 2r_1r_2 = 400 - 100$



$\frac{r_1 r_2}{2} = \frac{300}{4}$ =75 sq. units

In ΔOAB

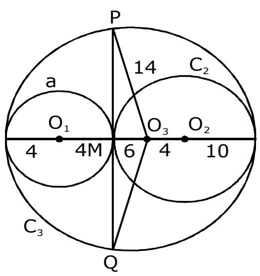
$$\left(\frac{r_1}{2}\right)^2 + \left(\frac{r_2}{2}\right)^2 = 10^2$$

$r_1^2 + r_2^2 = 400$

Q.15 (19)

$r_1 = 4$, $r_2 = 10$

$$r_3 = \frac{2(r_1 + r_2)}{2}$$



$r_3 = 14$

In ΔO_3MP

$O_3M = 6$

$PM = \sqrt{14^2 - 6^2} = \sqrt{160} = 4\sqrt{10}$

PQ = 2PM

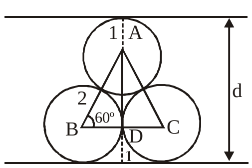
$$= \frac{8\sqrt{10}}{1} = \frac{m\sqrt{n}}{p}$$

$\Rightarrow$ m + n + p = 8 + 10 + 1 = 19

KVPY

PREVIOUS YEAR'S

Q.1 (A)



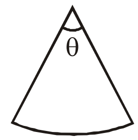
$$\sin 60^\circ = \frac{AD}{2}$$

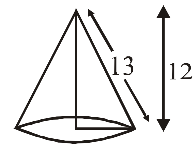
$$AD = 2 \sin 60^\circ = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

d = 1 + AD + 1

$d = 2 + \sqrt{3}$

Q.2 (A)





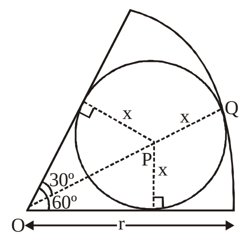
Slant height = 13

$$\theta = \frac{S}{r}$$

$\Rightarrow S = r\theta$ $\Rightarrow 2\pi(5) = 13\theta$

$$\Rightarrow \theta = \frac{10\pi}{13}$$

Q.3 (B)



Say the radius of smaller circle is x

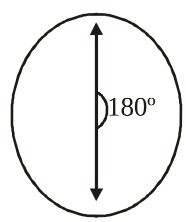
Here OP = x cosec 30º

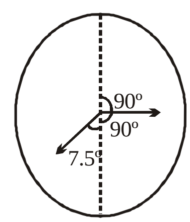
while OQ = r = x + x cosec 30º

$$x = \frac{r}{3}$$

Q.4 (A)

We want to find here angle between minute hand and hour hand at 6 : 15





Hour hand covers 30° in 60 minute.

Then in 15 minute it covers = 7.5°

So angle between both hand at 6 : 15 is 90° + 7.5 = 97.5° Another angle is 360° - 97.5° = 262.5°

Hence difference is 262.5° - 97.5° = 165°

Q.5 (3)

$(x - 3)^2 + (y - p)^2 = 9 - 17 + p^2$

Director circle is

$(x - 3)^2 + (y - p)^2 = 2(p^2 - 8)$

Passes through (0, 0)

$9 + p^2 = 2p^2 - 16$

$p^2 = 25 \Rightarrow p = \pm 5 \geq |p| = 5$

Q.6 (B)

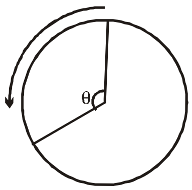
$$2\sqrt{g^2 - c} = a$$

$$2\sqrt{f^2 - c} = b$$

Polar coordinates of centre of circle be $(r\cos\theta, r\sin\theta)$

$g = -r\cos\theta$ and $g^2 - f^2 = \dfrac{a^2 - b^2}{4}$

Q.7 (B)

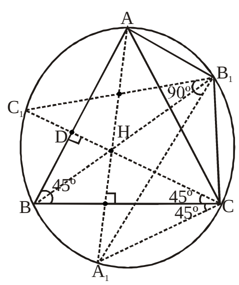


$$\theta = \frac{2\pi}{40} \times 15 = 2\pi - \frac{2\pi}{n} \times 15$$

$$\therefore \frac{3}{8} = 1 - \frac{15}{n}$$

$\Rightarrow n = 24$

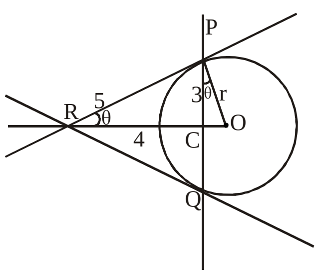
Q.8 (C)



$\angle BCH = 45^\circ = \angle BCA_1$

$\angle C_1CA_1 = \angle C_1B_1A_1 = 90^\circ$

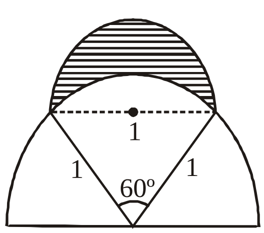
Q.9 (C)



In $\Delta RCP \Rightarrow \cos\theta = \dfrac{4}{5}$

In $\Delta PCO \Rightarrow \cos\theta = \dfrac{3}{r}$

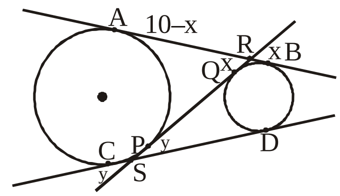
Q.10 (B)



$$\text{Required area} = \frac{\pi\left(\frac{1}{2}\right)^2}{2} - \left(\frac{60^\circ}{360^\circ} \times \pi(1)^2 - \frac{\sqrt{3}}{4} \times 1^2\right)$$

$$= \frac{\pi}{8} - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{4} - \frac{\pi}{24}$$

Q.11 (C)



$AR = PR = 10 - x$

$PQ = 10 - 2x$

$AB = CD = 10$

$CD = CS + SD = y + SD$

$= y + SP + PQ$

$10 = y + y + 10 - 2x$

$\Rightarrow y = x$

Now $RS = SP + PQ + QR$

$= y + 10 - 2x + x$

$= 10 + y - x = 10$

Q.12 (B)

$x^2 + y^2 = 1$

$$L_t : \frac{x}{t} + \frac{y}{1} = 1$$

$$y = 1 - \frac{x}{t}$$

$$x^2 + 1 + \frac{x^2}{t^2} - \frac{2x}{t} = 1$$

$$x^2\left(1 + \frac{1}{t^2}\right) - \frac{2x}{t} = 0$$

$x = 0,$

$y = 1$

$(0, 1)$

$$x\left(1 + \frac{1}{t^2}\right) = \frac{2}{t}$$

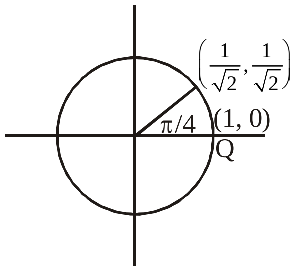
$$x = \frac{2t}{t^2+1};\ y = 1 - \frac{2}{t^2+1}$$

$$y = \frac{t^2-1}{t^2+1}$$

$$Q_t\left(\frac{2t}{1+t^2}, \frac{t^2-1}{t^2+1}\right)$$

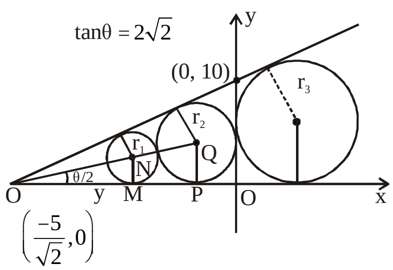
$1 \le t \le 1 + \sqrt{2}$ $\quad$ $t = \tan\theta$ $\quad$ $Q_t(\sin 2\theta, -\cos 2\theta)$

$\theta \in \left(45^\circ, 67\frac{1^\circ}{2}\right)$ $\quad$ lies on circle C



so angle at centre $=\frac{\pi}{4}$

Q.13 (D)



$$\tan\theta=\frac{2\tan\theta/2}{1-\tan^2\theta/2}$$

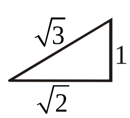
$$2\sqrt{2}=\frac{2\tan\theta/2}{1-\tan^2\theta/2}$$

$$\sqrt{2}\tan^2\theta/2+\tan\theta-\sqrt{2}=0$$

$$\tan\theta/2=\frac{-1\pm\sqrt{1+8}}{2\sqrt{2}}$$

$$=\frac{-1\pm 3}{2\sqrt{2}}=\frac{1}{\sqrt{2}}\quad\text{or}-\sqrt{2}$$

$$\therefore\ \tan\theta/2=\frac{1}{\sqrt{2}}$$



In ΔOMN $\sin\frac{\theta}{2}=\frac{r_1}{ON}$ $\quad\sin\frac{\theta}{2}=\frac{1}{\sqrt{3}}$

$ON=\sqrt{3}r_1$

In ΔOPQ $\sin\frac{\theta}{2}=\frac{r_2}{ON+r_1+r_2}\Rightarrow\frac{1}{\sqrt{3}}=\frac{r_2}{\sqrt{3}r_1+r_1+r_2}$

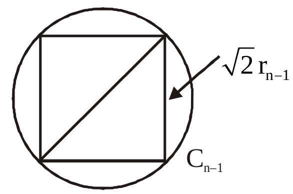
$$\sqrt{3}r_1+r_1+r_2=\sqrt{3}r_2$$

$$r_1\left(\sqrt{3}+1\right)=r_2\left(\sqrt{3}-1\right)$$

$$\frac{r_2}{r_1}=\frac{\sqrt{3}+1}{\sqrt{3}-1}=\frac{\left(\sqrt{3}+1\right)^2}{2}=2+\sqrt{3}$$

Q.14 (D)

$$\sum_{i-0}^{\infty}\text{Area}\,(C_i)=\pi r_0^{\ 2}+\pi r_1^{\ 2}+\pi r_2^{\ 2}+\pi r_3^{\ 2}+....\infty$$



Area of $C_n=\pi r_n^{\ 2}=\left(\sqrt{2}r_{n-1}\right)^2$

$$r_n^2=\frac{2}{\pi}r_{n-1}^2$$

$$\text{so } r_1^2=\frac{2}{\pi}r_0^{\ 2},\ r_2^{\ 2}=\frac{2}{\pi}r_1^{\ 2}$$

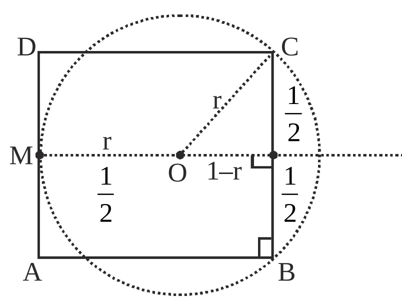
$$=\frac{2}{\pi}\left(\frac{2}{\pi}r_0^{\ 2}\right)$$

$$r_2^{\ 3}=\frac{2}{\pi}\left(r_2^{\ 2}\right)=\frac{2}{\pi}\left(\frac{2}{\pi}\frac{2}{\pi}r_0^{\ 2}\right)$$

$$\text{So}\sum_{i=0}^{\infty}\text{Area}\left(C_i\right)=\pi\left[r_0^{\ 2}+\frac{2}{\pi}r_0^{\ 2}+\frac{2}{\pi}.\frac{2}{\pi}r_0^{\ 2}....\infty\right]$$

$$=\frac{\pi r_0^{\ 2}}{1-\frac{2}{\pi}}=\frac{\pi^2 r_0^{\ 2}}{\pi-2}\ \forall\ r_0=1\quad=\frac{\pi^2}{\pi-2}$$

Q.15 (4)



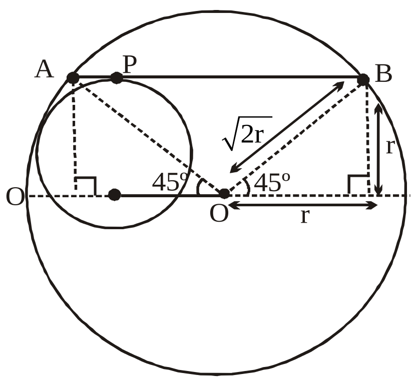
Let O be centre of circle.
OM = radius = r

$$\therefore\ r^2=(1-r)^2+\left(\frac{1}{2}\right)^2$$

$$\Rightarrow\ 2r-1=\frac{1}{4}\Rightarrow\ 2r=\frac{5}{4}$$

$$\Rightarrow\ r=\frac{5}{8}$$

Q.16 (B)

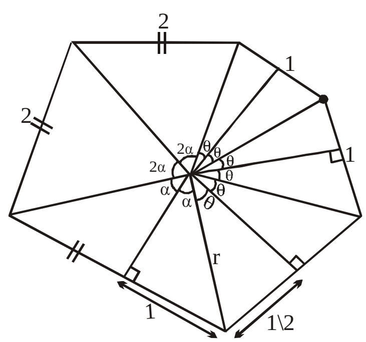


Chose AB subtend 90º at centre.
so that AB subtend 45º at O(circumference of circle)

Q.17 (B)

Sphere $x^2 + y^2 + z^2 - 4x - 6x - 12z + 48 = 0$
Centre (2, 3, 6)
radius $= \sqrt{4+9+36-48} = 1$
distance between centre and origin $= \sqrt{4+9+36} = 7$
shortest distance = 7 – 1 = 6(Origin lies outside the sphere)

Q.18 (B)



From the figure

$\sin\theta = \frac{1}{2r}$ & $\sin\alpha = \frac{1}{r}$

$3 \times (2\theta) + (2\alpha) \times 3 = 360°$

$\theta + \alpha = 60°$

Now, $\cos(\theta + \alpha) = \frac{1}{2}$

$\Rightarrow \cos\theta.\cos\alpha - \sin\theta.\sin\alpha = \frac{1}{2}$

$\Rightarrow \sqrt{1-\frac{1}{4r^2}}\sqrt{1-\frac{1}{r^2}} - \frac{1}{2r}\cdot\frac{1}{r} = \frac{1}{2}$

$\Rightarrow \sqrt{4r^2-1}\ \sqrt{r^2-1} - 1 = r^2$

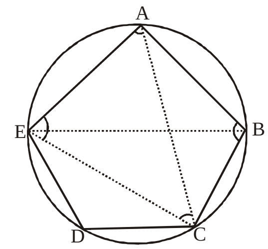
$\Rightarrow (4r^2 - 1)(r^2 - 1) = (r^2 + 1)^2$

$\Rightarrow 4r^4 - 5r^2 + 1 = r^4 + 2r^2 + 1$

$\Rightarrow 3r^4 = 7r^2$

$\Rightarrow r^2 = \frac{7}{3} \quad \Rightarrow r = \sqrt{\frac{7}{3}}$

Q.19 (2)



Q.20 (A)

circle is $x^2 + y^2 = 1$

$y = \pm\sqrt{1-\frac{a^2}{b^2}} \qquad \left(\because x = \frac{a}{b}\right)$

$y = \pm\frac{1}{6}.\sqrt{b^2 - a^2}$

As y is retional so

$b^2 - a^2 = p^2$
↓ ↓ ↓
even odd odd

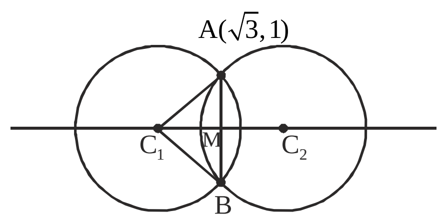
$b^2 = a^2 + p^2$
$= (2k + 1)^2 + (2\lambda + 1)^2$
$= 4k^2 + 4k + 1 + 4\lambda^2 + 4\lambda + 1$
$b^2 = 4(k^2 + \lambda^2 + k + \lambda) + 2$ impossible
as L.H.S is multiple of 4 but R.H.S is not multiple of 4

Q.21 (C)

Let two circles are

$x^2 + y^2 = 4$ & $\left(x - 2\sqrt{3}\right)^2 + y^2 = 4$

$\therefore$ equation of common chord is $x = \sqrt{3}$



$\therefore\ A(\sqrt{3}, 1), B(\sqrt{3}, -1)$

So $\angle AC_1B = 60°$

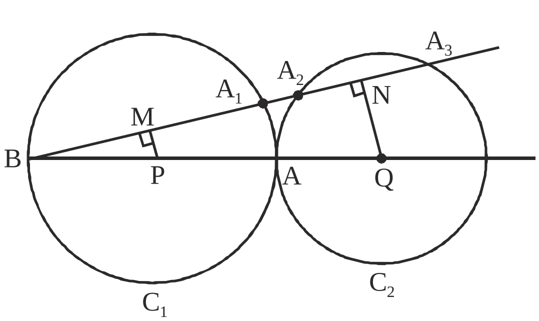
$AB = 2$ & $MC_1 = \sqrt{3}$

Required area = 2[area of sector C_1AB – ar ΔC_1AB]

$= 2\left[\frac{1}{2}\times 2^2 \times \frac{\pi}{3} - \frac{1}{2}\times 2 \times \sqrt{3}\right]$

= .723

Q.22 (A)



$BM = A_1M = 1$
$A_1A_2 = 1$

$A_2N = A_3N = \frac{1}{2}$

Let radius of C_1 is r_1
Let radius of C_2 is r_2

$PM = \sqrt{r_1^2 - 1}$, $QN = \sqrt{r_2^2 - \frac{1}{4}}$

$\because\ \Delta QNB \sim \Delta PMB$

$$\therefore\ \frac{\sqrt{r_2^2 - \frac{1}{4}}}{\sqrt{r_1^2 - 1}} = \frac{BN}{BM} = \frac{7/2}{1}$$

$\Rightarrow\ 4r_2^2 = 49r_1^2 - 48$ ......(i)

Also, in ΔQNB
$BQ^2 = BN^2 + NQ^2$

$(2r_1 + r_2)^2 = \frac{49}{4} + r_2^2 - \frac{1}{4}$

$\Rightarrow\ r_1^2 + r_1r_2 = 3$
.......(ii)

Solve (i) & (ii)

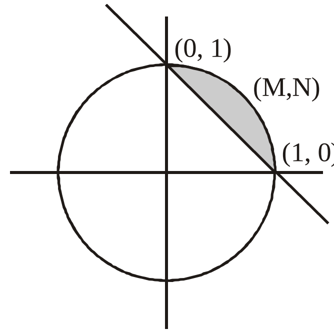
$r_1 = \sqrt{\frac{6}{5}} = \frac{\sqrt{30}}{5}$ & $r_2 = \frac{3\sqrt{30}}{10}$

Q.23 (A)

Required equation of circle
$(x - h)^2 + (y - h)^2 = h^2$
Both circle touch internally
$C_1C_2 = |r_1 - r_2|$
$\sqrt{h^2 + h^2} = |h - 1|$
Solve this $h = \sqrt{2} - 1$
Area $\pi(\sqrt{2} - 1)^2 = \pi(3 - 2\sqrt{2})$

Q.24 (D)

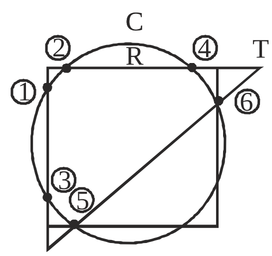
Let $a^2 = m$ & $b^2 = N$ then $m > 0$ and $N > 0$
Now given condition is $M + N > 1$ and $M^2 + N^2 < 1$



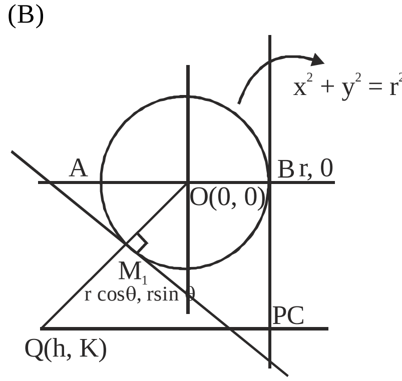
(MN) lies inside circle $x^2 + y^2 < 1$ and above line $x + y > 1$

$\Rightarrow$ (M,N) lies in shaded region and number of points in shaded region are infinite, so number of pair (a,b) are also infinite.

Q.25 (D)



Q.26 (B)



Equation of tangent at M, $x\cos\theta + y\sin\theta = r$
put X = r, to get y-coordinate of point P.
$r\cos\theta + y\sin\theta = r$

$$\Rightarrow\ y = \frac{1(1-\cos\theta)}{\sin\theta} = \frac{r.2.\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}} = r\tan\frac{\theta}{2}$$

$$\therefore\ P \equiv \left(r,\ \ r\tan\frac{\theta}{2}\right)$$

$\therefore$ Q has y - coodinate same as point P

$\therefore\ K = r\tan\frac{\theta}{2}$ $\quad\Rightarrow\ \tan\frac{\theta}{2} = \frac{K}{r}$

Slope of tangent at M = $-\cot\theta$

Slope of OQ = $\frac{K}{h}$

$$\therefore \frac{K}{h},(-\cot\theta) = -1 \Rightarrow \tan\theta = \frac{K}{h}$$

$$\Rightarrow \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{K}{h} \Rightarrow \frac{2.\frac{K}{r}}{1-\frac{K^2}{r^2}} = \frac{K}{h}$$

$$\Rightarrow \frac{2h}{r} = 1-\frac{K^2}{r^2} \Rightarrow \frac{2h}{r} = \frac{r^2-K^2}{r^2}$$

$\Rightarrow$ $2hr = r^2 - K^2$

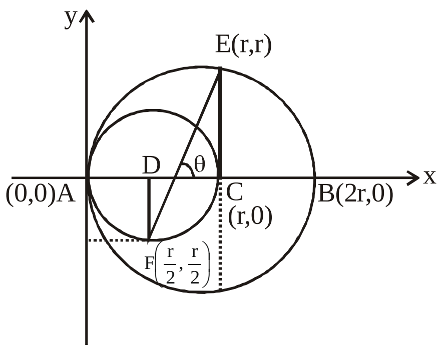
$\Rightarrow$ $y^2 = r^2 - 2Kr$

$y^2 = 2r(x - r/2)$

$\therefore$ Parabola

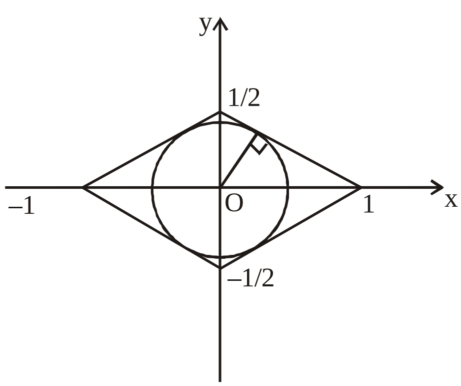
Q.27 (A)

$\tan\theta$ = slope of FE = 3



$$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}} \Rightarrow \sin(90° - \theta) = \frac{1}{\sqrt{10}}$$

Q.28 (B)

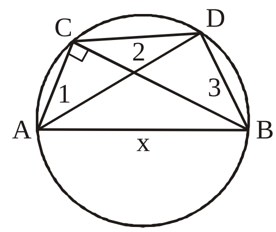


$$\frac{1}{2}\times\frac{\sqrt{5}}{2}\times r = \frac{1}{2}\times 1\times\frac{1}{2}$$

$$\Rightarrow r = \frac{1}{\sqrt{5}}$$

Q.29 (B)

$$BC = \sqrt{x^2-1},\ AD = \sqrt{x^2-9}$$



by Ptolemy's theorem

$AB.CD + AC.BD = AD.BC$

$$\Rightarrow 2x+3 = \sqrt{x^2-9}\sqrt{x^2-1}$$

$\Rightarrow$ $4x^2 + 12x + 9 = x^4 - 10x^2 + 9$

$\Rightarrow$ $x^4 - 14x^2 - 12x = 0 \Rightarrow x^3 - 14x - 12 = 0$

Let $f(x) = x^3 - 14x - 12$

$\Rightarrow$ $f'(x) = 3x^2 - 14$ $\Rightarrow$ f(x) has only one

positive root $\in \left(0, \sqrt{\frac{14}{3}}\right)$

$f(4, 1) < 0$ and $f(4, 2) > 0$ $\Rightarrow x \in (4.1, 4.2)$

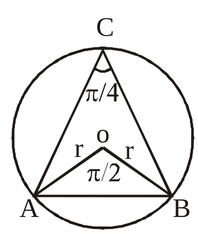
Q.30 **(A)**

$$\text{Area (C)} = \pi\left(\frac{\ell}{2\pi}\right)^2 = \frac{\ell^2}{4\pi}$$

$$\text{Area}(T) \le \frac{\sqrt{3}}{4}\left(\frac{\ell}{3}\right)^2 = \frac{\ell^2}{12\sqrt{3}} \Rightarrow (A)$$

$$\text{Hence } \frac{\text{Area}(c)}{\text{Area}(\tau)} \ge \frac{3\sqrt{3}}{\pi}$$

Q.31 (D)



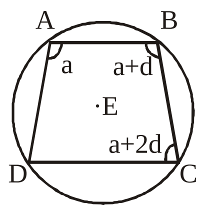
Let O be the centre of the circle

In ΔOAB

$AB = \sqrt{2}\,r$ and $r = 1$

$\Rightarrow AB = \sqrt{2}$

Q.32 (D)



AE = BE = CE = DE

$\angle DAB, \angle ABC, \angle BCD \rightarrow AP$

Let $\angle DAB = a$

$\angle ABC = a + d$

$\angle BCD = a + 2d$

Since AE = BE = CE = DE so ABCD is cyclic quadrilateral

Hence $\angle DAB + \angle DCB = 180^\circ$

$2a + 2d = 180^\circ \Rightarrow a + d = 90^\circ$

so median of $\{a, a + d, a + 2d\}$ is $a + d = 90^\circ$

Q.33 (D)

JEE MIAN
PREVIOUS YEAR'S

Q.1 56.25

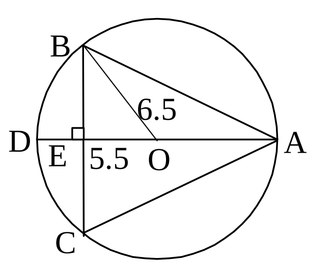
Internal point which divide (5,0) & (–5,0) in the ratio 3 : 1 is $\left(\frac{-5}{2}, 0\right)$ External point which divide (5,0) & (–5,0) in the ratio 3 : 1 is (–10,0)

$$2r = \left(\frac{-5}{2} + 10\right) = \frac{15}{2} = 7.5$$

$(2r)^2 = 56.25$

Q.2 41.568

Let O be mid-point of AD, now perpendicular from C to BC bisects chord BC, (ΔACE and ΔABE are congruent). Hence AD is diameter and O is centre of circle.

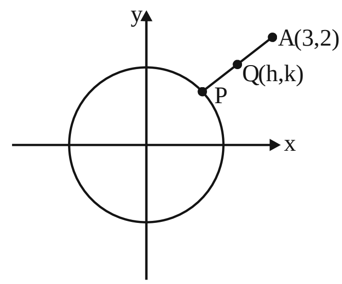


So BE = $\sqrt{(6.5)^2 - (5.5)^2}$

$= \sqrt{12}$

Hencce are = $\frac{1}{2} \cdot 12.2\sqrt{12} = 24\sqrt{3}$

Q.3 (2)

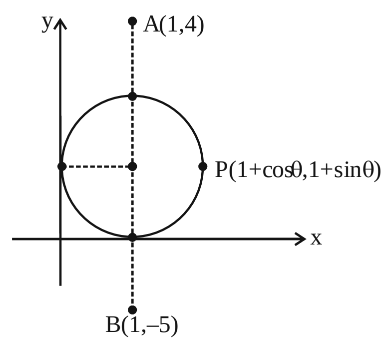


$\therefore$ P≡(2h–3, 2k–2)→on circle

$$\therefore \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4}$$

$\Rightarrow$ radius = $\frac{1}{2}$

Q.4 (3)



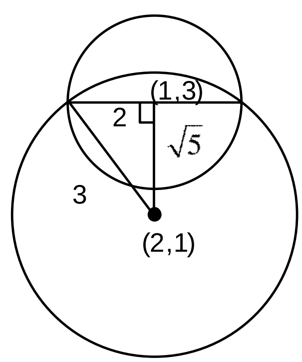
$\therefore PA^2 = \cos^2\theta + (\sin\theta - 3)^2 = 10 - 6\sin\theta$

$PB^2 = \cos^2\theta + (\sin\theta - 6)^2 = 37 - 12\sin\theta$

$PA^2 + PB^2 = 47 - 18\ \sin\theta|_{max} \Rightarrow \theta = \frac{3\pi}{2}$

$\therefore$ P,A,B lie on a line x=1

Q.5 (3)

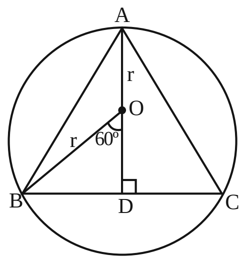


distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

Q.6 (3)



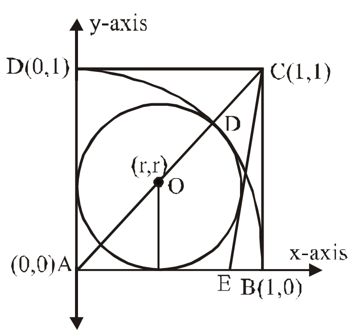
$OD = r\cos 60^\circ = \dfrac{r}{2}$

Height = $AD = \dfrac{3r}{2}$

Now $\sin 60^\circ = \dfrac{3\frac{r}{2}}{AB}$

$\Rightarrow AB = \sqrt{3}r$

Q.7 **(1)**



Here AO + OD = 1 or $(\sqrt{2}+1)$ r = 1

$\Rightarrow r = \sqrt{2-1}$

equation of circle $(x-r)^2 + + (y-r)^2 = r^2$

Equation of CE

$y - 1 = m(x - 1)$

$mx - y + 1 - M = 0$

It is tangent to circle

$\therefore \left|\dfrac{mr - r + 1 - m}{\sqrt{m^2+1}}\right| = r$

$\left|\dfrac{(m-1)r + 1 - m}{\sqrt{m^2+1}}\right| = r$

$\dfrac{(m-1)^2 (r-1)^2}{m^2+1} = r^2$

Put $r = \sqrt{2} - 1$

On solving $m = 2 - \sqrt{3}, 2 + \sqrt{3}$

Taking greater slope of CE as

$2 + \sqrt{3}$

$y - 1 = (2 + \sqrt{3})(x - 1)$

Put y = 0

$-1 = (2 + \sqrt{3})(x - 1)$

$$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x - 1$$

$x - 1 = \sqrt{3} - 1$

$EB = 1 - x = 1 - (\sqrt{3} - 1)$

$EB = 2 - \sqrt{3}$

Q.8 **(3)**

$x2 + y2 + ax + 2ay + c = 0$

$$2\sqrt{g^2 - c} = 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$$

$\Rightarrow \dfrac{a^2}{4} - c = 2$ ...(1)

$$2\sqrt{f^2 - c} = 2\sqrt{a^2 - c} = 2\sqrt{5}$$

$\Rightarrow a2 - c = 5$ ...(2)

(1) & (2)

$\dfrac{a^2}{3} = 3 \Rightarrow a = -2\ (a < 0)$

$\therefore c = -1$

Circle $\Rightarrow x2 + y2 - 2x - 4y - 1 = 0$

$\Rightarrow (x - 1)2 + (y - 2)2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\dfrac{1}{2}$

$m_{tangent} = 2$

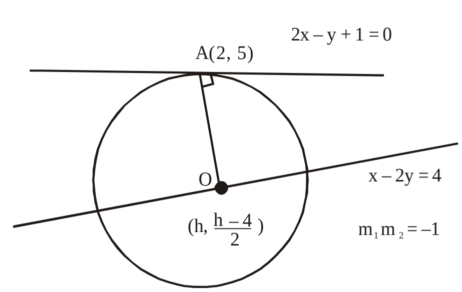
Equation of tangent

$\Rightarrow (y - 2) = 2(x - 1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from (0, 0) = $\left|\dfrac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

Q.9 (1)



$$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2)=-1$$

h = 8

center (8, 2)

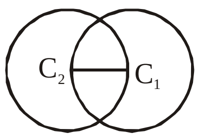
Radius = $\sqrt{(8\quad 2)^2\quad (2\quad 5)^2}\quad 3\sqrt{5}$)

Q.10 (2)

$r_1 = 3$, c_1 (5, 5)

$r_2 = 3$, c_2 (8, 5)

$C_1C_2 = 3$, $r_1 = 3$, $r_2 = 3$

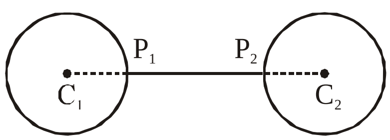


Q.11 (1)

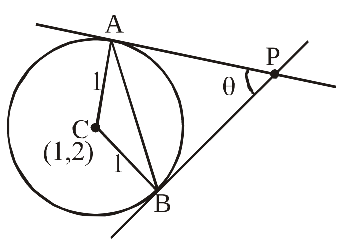
Given $C_1(5, 5)$, $r_1 = 3$ and C_2 (12, 5), $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, (P_1P_2)min = 7 – 6 = 1



Q.12 (2)



$$\tan\theta=\frac{12}{5}$$

$$PA=\cot\frac{\theta}{2}$$

$\therefore$ area of $\Delta PAB = \frac{1}{2}$ (PA) $\sin\theta\ \ \frac{1}{2}\cot^2\frac{\theta}{2}\sin\theta$

$$=\frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta$$

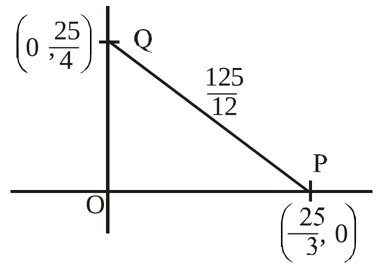
$$=\frac{1}{2}\left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)\left(\frac{12}{13}\right)=\frac{1}{2}\frac{18}{18}\times\frac{2}{13}=\frac{27}{26}$$

area of $\Delta CAB = \frac{1}{2}\sin\theta = \frac{1}{2}\left(\frac{12}{13}\right)\ \ \frac{6}{13}$

$$\therefore\ \frac{\text{area of }\Delta PAB}{\text{area of }\Delta CAB}=\frac{9}{4}$$ **Option (2)**

Q.13 (3)

Tangent to circle 3x + 4y = 25



OP + OQ + OR = 25

$$\text{Incentre}=\left(\frac{\frac{25}{4}\times\frac{25}{3}}{25},\frac{\frac{25}{4}\times\frac{25}{3}}{25}\right)$$

$$=\left(\frac{25}{12},\frac{25}{12}\right)$$

$$\therefore\ r^2=2\left(\frac{25}{12}\right)^2=2\times\frac{625}{144}=\frac{625}{72}$$

Option (3)

Q.14 (3)

$x^2 + y^2 - 10x - 10y + 41 = 0$

A(5,5), $R_1 = 3$

$x^2 + y^2 - 22x$. $10y + 137 = 0$

B(11,5), $R_2 = 3$

AB = 6 = $R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point.

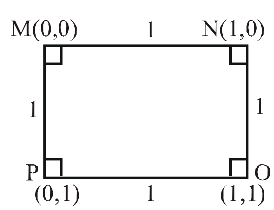
Q.15 (2)

M : $x^2 + y^2 = 1$ (0,0)

N : $x^2 + y^2 - 2x = 0$ (1,0)

O : $x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

P : $x^2 + y^2 - 2y = 0$ (0,1)

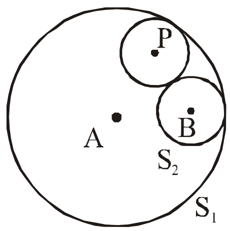


Q.16 (3)

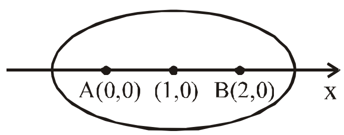
$S_1 : x^2 + y^2 = 9$ — $r_1 = 3$, A(0, 0)

$S_2 : (x - 2)^2 + y^2 = 1$ — $r_2 = 1$, B(2, 0)

Q $c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally
Let a veriable circle with centre P and radius r
$\Rightarrow$ PA = r_1 – r and PB = r_2 + r
$\Rightarrow$ PA + PB = r_1 + r_2
$\Rightarrow$ PA + PB = 4 (> AB)
$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is 2a = 4, $e = \frac{1}{2}$
$\Rightarrow$ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$
if x-ellipse



$\Rightarrow$ E: $\frac{(x-1)^2}{4} + \frac{y^2}{3} =$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

Q.17 **(4)**
Q.18 (3)
Q.19 (4)
Q.20 (3)
Q.21 (2)
Q.22 (3)
Q.23 **(3)**
Q.24 (3)
Q.25 **(18)**
Q.26 [165]

Q.27 (1)
Q.28 (4)
Q.29 [30]
Q.30 **(1)**
Q.31 **[13]**

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 **(D)**
Let equation of circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$
as it passes through (-1,0) & (0,2)
$\therefore$ 1 – 2g + c = 0 and 4 + 4f + c = 0
also $f^2 = c$

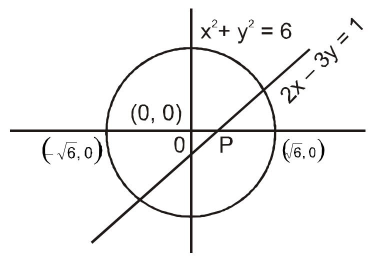
$\Rightarrow$ f = –2, c = 4 ; $g = \frac{5}{2}$

$\therefore$ equation of circle is $x^2 + y^2 + 5x - 4y + 4 = 0$
which passes through (–4, 0)

Q.2 (2)
$2x - 3y = 1$, $x^2 + y^2 \le 6$

$S \equiv \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$
(I) (II) (III) (IV)

Plot the two curves



I, III, IV will lie inside the circle and point (I, III, IV) will lie on the P region
if (0, 0) and the given point will lie opposite to the line 2x – 3y – 1 = 0

P(0, 0) = negative, $P\left(2, \frac{3}{4}\right)$ = positive, $P\left(\frac{1}{4}, -\frac{1}{4}\right)$

= positive $P\left(\frac{1}{8}, \frac{1}{4}\right)$ = negative

$P\left(\frac{5}{2}, \frac{3}{4}\right)$ = positive , but it will not lie in the given circle

so point $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ will lie on the opp side of the line

so two point $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$

Further $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ satisfy $S_1 < 0$

Q.3 **(A)**
Circle $x^2 + y^2 = 9$; line $4x - 5y = 20$,

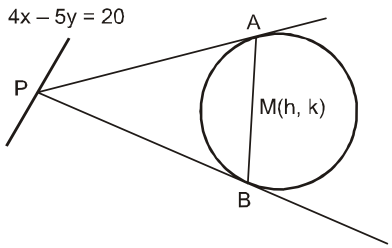
$P\left(t, \frac{4t-20}{5}\right)$

equation of chord AB whose mid point is M (h, k)
T = S_1
$\Rightarrow$ $hx + ky = h^2 + k^2$ ....(1)
equation of chord of contact AB with respect to P.
T = 0

$\Rightarrow tx + \left(\dfrac{4t-20}{5}\right)y = 9$ ....(2)

comparing equation (1) and (2)

$$\frac{h}{t} = \frac{5k}{4t-20} = \frac{h^2+k^2}{9}$$



on solving

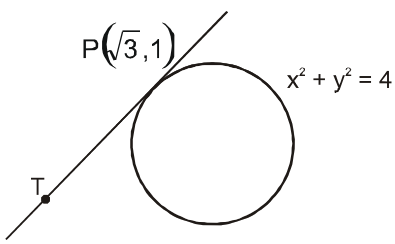
$45k = 36h - 20h^2 - 20k^2$

$\Rightarrow$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

Comprehension # 1 (Q. No.4 & 5)

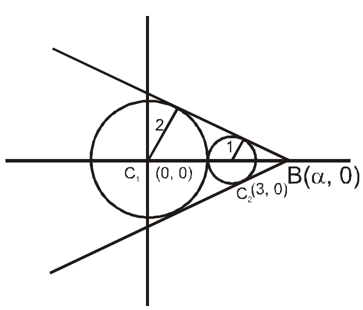
Q.4 **(D)**

Q.5 **(A)**



Equation of tangent at $(\sqrt{3}, 1)$

$\Rightarrow \sqrt{3}x + y = 4$



B divides $C_1 C_2$ in 2 : 1 externally

$\therefore$ B(6, 0)

Hence let equation of common tangent is

$y - 0 = m(x - 6)$

$\Rightarrow mx - y - 6m = 0$

length of $\perp^r$ dropped from center (0, 0) = radius

$$\left|\frac{6m}{\sqrt{1+m^2}}\right| = 2$$

$\Rightarrow m = \pm \dfrac{1}{2\sqrt{2}}$

$\therefore$ equation is $x + 2\sqrt{2}\,y = 6$ or $x - 2\sqrt{2}\,y = 6$

So5 Equation of L is

$x - y\sqrt{3} + c = 0$

length of perpendicular dropped from centre = radius of circle

$\therefore \left|\dfrac{3+C}{2}\right| = 1$ $\Rightarrow C = -1, -5$

$\therefore x - \sqrt{3}\,y = 1$ or $x - \sqrt{3}\,y = 5$

Q.6 **(AC)**

Let $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow g^2 - c = 0 \Rightarrow g^2 = c$ ...(i)

$2\sqrt{f^2 - c} = 2\sqrt{7}$ $\Rightarrow f^2 - c = 7$ ...(ii)

$9 + 0 + 6g + 0 + c = 0$ $\Rightarrow 9 + 6g + g^2 = 0$

$\Rightarrow (g + 3)^2 = 0$

$g = -3$ $\therefore c = 9$

$f^2 = 16$ $\Rightarrow f = \pm 4$

$\therefore x^2 + y^2 - 6x \pm 8y + 9 = 0$

Q.7 **(BC)**

Let the cirlce be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)

given circles

$x^2 + y^2 - 2x - 15 = 0$ ...(2)

$x^2 + y^2 - 1 = 0$ ....(3)

(1) & (2) are orthogonal

$\Rightarrow -g + 0 = \dfrac{c-15}{2}$

$0 + 0 = \dfrac{c-1}{2}$

$\Rightarrow$ c = 1 & g = 7

so the cirle is

$x^2 + y^2 + 14x + 2fy + 1 = 0$ it passes thrgouh

(0, 1) $\Rightarrow 0 + 1 + 0 + 2f + 1 = 0$

f = –1

$\Rightarrow x^2 + y^2 + 14x - 2y + 1 = 0$

Centre (–7, 1)

radius = 7

Q.8 **(A,C)**

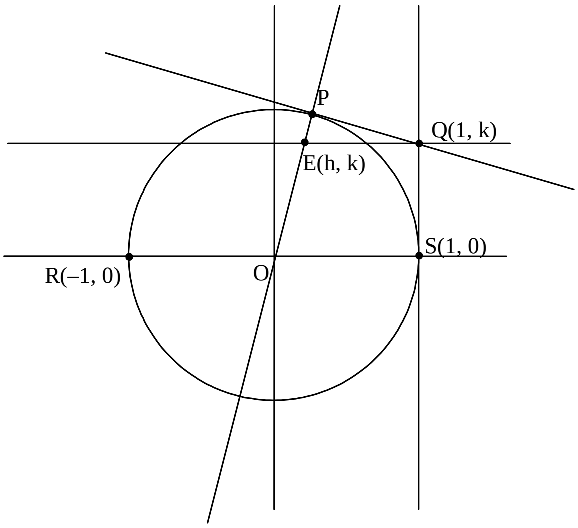
Eq^n of tangent from Q (1, k) is

$y - k = m(x - 1)$

“!$c^2 = a^2 (m^2 + 1)$

$(k - m)^2 = m^2 + 1$

$$m = \frac{k^2 - 1}{2k}$$



So, Eq^n of QP is $\frac{k^2 - 1}{2k} x - y + \frac{k^2 + 1}{2k} = 0$

Hence, P is $\left(\frac{1 - k^2}{1 + k^2}, \frac{2k}{1 + k^2}\right)$

So, Eq^n of OP is $y = \frac{2k}{1 - k^2} x$

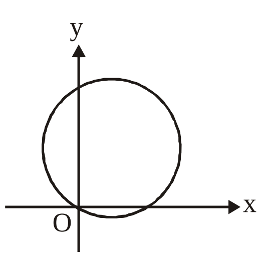
↓E (h, k)

So, locus of E is $1 - y^2 - 2x = 0$

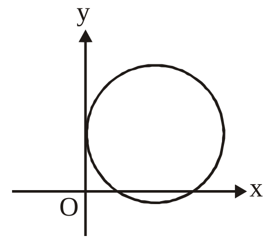
Hence, (a, c)

Q.9 **(2)**

Case-I Passing through origin ⇒ p = 0



Case-II Touches y-axis and cuts x-axis

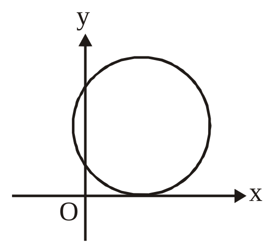


$f^2 - c = 0$ & $g^2 - c > 0$

$4 + p = 0$ $\qquad$ $1 + p > 0$

$p = -4$ Not possible

Case-III Touches x-axis and cuts y-axis



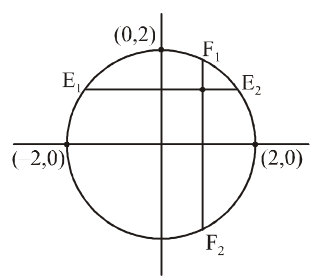
$f^2 - c > 0$ & $g^2 - c = 0$

$4 + p > 0$ $\qquad$ $1 + p = 0$

So two value of p are possible

Comprehenssion # 1 (Q. No. 10 to 14)

Q.10 **(A)**



Co-ordinates of E_1 and E_2 are obtained by solving y = 1 and $x^2 + y^2 = 4$

$\therefore$ $E_1\left(-\sqrt{3}, 1\right)$ and $E_2\left(\sqrt{3}, 1\right)$

Co-ordinates of F_1 and F_2 are obtained by solving x = 1 and $x^2 + y^2 = 4$

$F_1\left(1, \sqrt{3}\right)$ and $F_2\left(1, -\sqrt{3}\right)$

Tangent at E_1 : $-\sqrt{3}x + y = 4$

Tangent at E_2 : $\sqrt{3}x + y = 4$

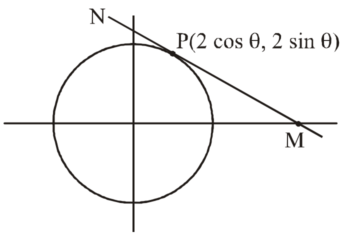
$\therefore$ $E_3(0, 4)$

Tangent at F_1 : $x + \sqrt{3}y = 4$

Tangent at F_2 : $x - \sqrt{3}y = 4$

$\therefore$ $F_3(4, 0)$
and similarly $G_3(2, 2)$
(0, 4), (4, 0) and (2, 2) lies on x + y = 4

Q.11 (D)

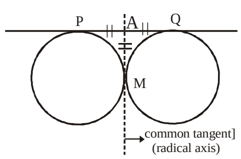


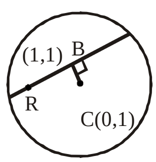
Tangent at P(2 cos θ, 2 sin θ) is x cos θ + y sin θ = 2
M(2 sec θ, 0) and N(0, 2 cosec θ)
Let midpoint be (h, k)
h = secθ, k = cosecθ

$$\frac{1}{h^2}+\frac{1}{k^2}=1$$

$$\frac{1}{x^2}+\frac{1}{y^2}=1$$

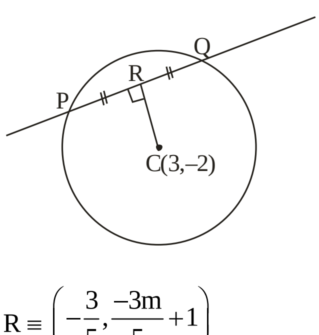
Q.12 (D)





AP = AQ = AM
Locus of M is a cricle having PQ as its diameter
Hence, E_1 : (x – 2) (x + 2) + (y – 7) (y + 5) = 0 and x $\neq$ ±2
Locus of B (midpoint)
is a circle having RC as its diameter
E_2: $x(x-1) + (y-1)^2 = 0$
Now, after checking the options, we get (D)

Q.13 (B)



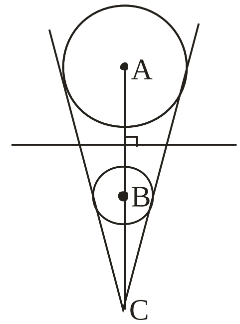
$$R \equiv \left(-\frac{3}{5}, \frac{-3m}{5}+1\right)$$

$$\text{So, } m\left(\frac{-\frac{3m}{5}+3}{-\frac{3}{5}-3}\right)=-1$$

$\Rightarrow m^2 - 5m + 6 = 0 \quad \Rightarrow m = 2, 3$

Q.14 (10.00)

Distance of point A from given line = $\frac{5}{2}$

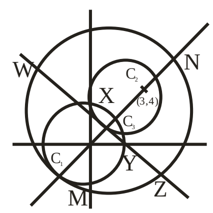


$$\frac{CA}{CB}=\frac{2}{1} \Rightarrow \frac{AC}{AB}=\frac{2}{1} \Rightarrow AC = 2 \times 5 = 10$$

Comprehenssion # 3 (Q. No. 15 to 16)

Q.15 (1)

Q.16 (4)

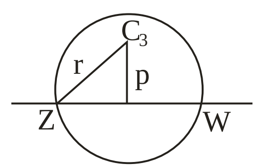


$MC_1 + C_1C_2 + C_2N = 2r$
$\Rightarrow 3 + 5 + 4 = 2r = 6 \Rightarrow$ Radius of $C_3 = 6$
Suppose centre of C_3 be $(0 + r_4 \cos\theta, 0 + r_4 \sin\theta)$,

$$\left\{\begin{matrix} r_4 = C_1C_3 = 3 \\ \tan\theta = \frac{4}{3}\end{matrix}\right\}$$

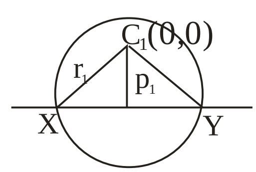
$$C_3 = \left(\frac{9}{5}, \frac{12}{5}\right) = (h,k) \Rightarrow 2h + k = 6$$

Equation of ZW and XY is 3x + 4y – 9 = 0
(common chord of circle $C_1 = 0$ and $C_2 = 0$)



$$ZW = 2\sqrt{r^2 - p^2} = \frac{24\sqrt{6}}{5} \text{ (where r = 6 and p = } \frac{6}{5})$$

$$XY = 2\sqrt{r_1^2 - p_1^2} = \frac{24}{5} \text{ (where } r_1 = 3 \text{ and } p_1 = \frac{9}{5})$$



$$\frac{\text{Length of ZW}}{\text{Length of XY}} = \sqrt{6}$$

Let length of perpendicular from M to ZW be $\lambda, \lambda =$

$$3 + \frac{9}{5} = \frac{24}{5}$$

$$\frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{\frac{1}{2}(MN)\times\frac{1}{2}(ZW)}{\frac{1}{2}\times ZW\times\lambda} = \frac{1}{2}\frac{MN}{\lambda} = \frac{5}{4}$$

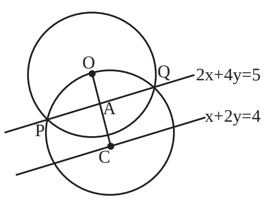
$$C_3 : \left(x-\frac{9}{5}\right)^2 + \left(y-\frac{12}{5}\right)^2 = 6^2$$

$C_1 : x^2 + y^2 - 9 = 0$
common tangent to C_1 and C_3 is common chord of C_1 and C_3 is $3x + 4y + 15 = 0$.
Now $3x + 4y + 15 = 0$ is tangent to parabola $x^2 = 8\alpha y$.

$$x^2 = 8\alpha\left(\frac{-3x-15}{4}\right) \Rightarrow 4x^2 + 24\alpha x + 120\alpha = 0$$

$$D = 0 \Rightarrow \alpha = \frac{10}{3}$$

Q.17 [2]



M-I

$$OA = \frac{\sqrt{5}}{2} \qquad OC = \frac{4}{\sqrt{5}}$$

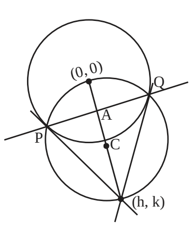
$$CQ = OC = \frac{4}{\sqrt{5}} \text{ and } CA = \frac{3}{2\sqrt{5}}$$

$$\therefore \quad OQ = \sqrt{OA^2 + AQ^2} = \sqrt{OA^2 + \left(CQ^2 - CA^2\right)}$$

$$\Rightarrow \sqrt{\frac{5}{4}+\frac{16}{5}-\frac{9}{20}} = \sqrt{4}$$

$\Rightarrow 2 = r$

M-II



PQ : $hx+ky=r^2$

Given PQ $\quad 2x + 4y = 5$

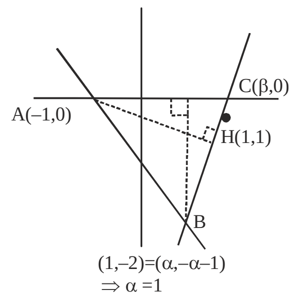
$$\Rightarrow \frac{h}{2} = \frac{k}{4} = \frac{r^2}{5} \Rightarrow h = \frac{2r^2}{5} \quad k = \frac{4r^2}{5}$$

$$\therefore \quad C\left(\frac{r^2}{5}, \frac{2r^2}{5}\right)$$

$$\therefore \quad \text{C lies on } x+2y=4 \quad \Rightarrow \frac{r^2}{5} + 2\left(\frac{2r^2}{5}\right) = 4$$

$\Rightarrow r^2 = 4 \qquad \Rightarrow r = 2$

Q.18 (B)

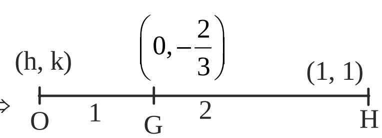


one of the vectex is intersection of x-axis and $x + y + 1 = 0 \Rightarrow A(-1,0)$
Let vertex B be $(\alpha, -\alpha - 1)$
Line $AC \perp BH \Rightarrow \alpha = 1 \Rightarrow B(1, -2)$
Let vertex C be $(\beta, 0)$
Line $AH \perp BC$
$m_{AH}.m_{BC} = -1$

$$\frac{1}{2}.\frac{2}{\beta-1} = -1 \Rightarrow \beta = 0$$

Centroid of ΔABC is $\left(0, -\frac{2}{3}\right)$

Now G (centroid) divides line joining circum centre (O) and ortho centre (H) in the ratio 1 : 2

$\Rightarrow$ 

$2h + 1 = 0 \qquad 2k + 1 = -z$

$$h = -\frac{1}{2} \qquad k = -\frac{3}{2}$$

$$\Rightarrow \text{circum centre is } \left(-\frac{1}{2}, -\frac{3}{2}\right)$$

Equation of circum circle is (passing through C(0,0)) is $x^2 + y^2 + x + 3y = 0$

Parabola

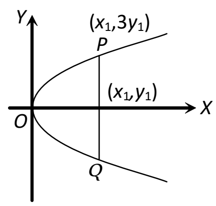
EXERCISES

ELEMENTRY

Q.1 (1)

Required locus is $(3y)^2 = 4ax$

$\Rightarrow 9y^2 = 4ax$



Q.2 (3)

$S \equiv (5,0)$. Therefore, latus rectum $= 4a = 20$.

Q.3 (2)

Distance between focus and directrix is

$$= \left|\frac{3-4-2}{\sqrt{2}}\right| = \frac{\pm 3}{\sqrt{2}}$$

Hence latus rectum $= 3\sqrt{2}$

(Since latus rectum is two times the distance between focus and directrix).

Q.4 (4)

$a = 4,\ = (0,0)$ vertex , focus $= (0,-4)$

Q.5 (3)

Vertex $= (2,0) \Rightarrow$ focus is $(2+2,0) = (4,0)$.

Q.6 (3)

The point $(-3,2)$ will satisfy the equation $y^2 = 4ax$

$\Rightarrow 4 = -12a$, $\Rightarrow 4a = -\frac{4}{3} = \frac{4}{3}$

(Taking positive sign).

Q.7 (3)

$x^2 = -8y \Rightarrow a = -2$ So, focus $= (0,-2)$

Ends of latus rectum $= (4,-2), (-4,-2)$.

Trick : Since the ends of latus rectum lie on parabola, so only points $(-4,-2)$ and $(4,-2)$ satisfy the parabola.

Q.8 (1)

Given equation is $x^2 = -8ay$.

Here $A = 2a$

Focus of parabola $(0,-A)$ i.e. (0, - 2a)

Directrix y = A i.e., y = 2a.

Q.9 (4)

Clearly; $a = \left|\frac{-8}{\sqrt{1+1}}\right| - \left|\frac{-12}{\sqrt{1+1}}\right| = \frac{4}{\sqrt{2}}$

Length of latus rectum = 4a $= 4 \times \frac{4}{\sqrt{2}} = 8\sqrt{2}$.

Q.10 (1)

$(x+1)^2 = 4a(y+2)$

Passes through (3, 6) $\Rightarrow 16 = 4a.8 \Rightarrow a = \frac{1}{2}$

$\Rightarrow (x+1)^2 = 2(y+2) \Rightarrow x^2 + 2x - 2y - 3 = 0$

Q.11 (4)

The parabola is $(x-2)^2 = (3y-6)$. Hence axis is $x - 2 = 0$.

Q.12 (2)

Let any point on it be (x, y), then from definition of parabola, we get

Squaring and after simplification, we get

$$\frac{\sqrt{(x+8)^2 + (y+2)^2}}{\left|\frac{2x-y-9}{\sqrt{5}}\right|} = 1$$

$x^2 + 4y^2 + 4xy + 116x + 2y + 259 = 0$.

Q.13 (3)

Vertex $(0,4)$; focus (0,2) ; $\therefore$ x = 2

Hence parabola is $(x-0)^2 = -4.2(y-4)$

i.e., $x^2 + 8y = 32$.

Q.14 (2)

Parametric equations of $y^2 = 4ax$ are $x = at^2, y = 2at$

Hence if equation is $y^2 = 8x$, then parametric equations are $x = 2t^2, y = 4t$.

Q.15 (3)

Semi latus rectum is harmonic mean between segments of focal chords of a parabola.

$\therefore\ b = \frac{2ac}{a+c} \Rightarrow a, b, c$ are in H.P.

Q.16 (2)

$S_1 \equiv x^2 - 108y = 0$

$T \equiv xx_1 - 2a(y + y_1) = 0 \Rightarrow xx_1 - 54\left(y + \frac{x_1^2}{108}\right) = 0$

$S_2 \equiv y^2 - 32x = 0$

$T \equiv yy_2 - 2a(x + x_2) = 0 \Rightarrow yy_2 - 16\left(x + \frac{y_2^2}{32}\right) = 0$

$\therefore \frac{x_1}{16}=\frac{54}{y_2}=\frac{-x_1^2}{y_2^2}=r \Rightarrow x_1=16r$ and $y_2=\frac{54}{r}$

$\therefore \frac{-(16r)^2}{(54/r)^2}=r \Rightarrow r=-\frac{9}{4}$

$x_1=-36,\ y_2=-24,\ y_1=\frac{(36)^2}{108}=12,\ x_2=18$.

$\therefore$ Equation of common tangent

$(y-12)=\frac{-36}{54}(x+36) \Rightarrow 2x+3y+36=0$

Aliter : Using direct formula of common tangent $yb^{1/3}+xa^{1/3}+(ab)^{2/3}=0$, where $a=8$ and $b=27$.

Hence the required tangent is $3y+2x+36=0$.

Q.17 (3)

$m=\tan\theta$. The tangent to $y^2=4ax$ is $y=x\tan\theta+c$

Hence $c=\frac{a}{\tan\theta}=a\cot\theta$

$\therefore$ The equation of tangent is $y=x\tan\theta+a\cot\theta$.

Q.18 (2)

Equation of parabola is $Y^2=4X$,

where $X=x+\frac{5}{4}$

Tangent parallel to $Y=2X+7$ is $Y=2X+\frac{a}{m}$

$\Rightarrow y=2\left(x+\frac{5}{4}\right)+\frac{1}{2} \Rightarrow y=2x+3$

i.e., $2x-y+3=0$.

Q.19 (1)

$m=\tan\theta=\tan 60°=\sqrt{3}$

The equation of tangent at (h,k) to $y^2=4ax$ is

$yk=2a(x+h)$

Comparing, we get $m=\sqrt{3}=\frac{2a}{k}$ or $k=\frac{2a}{\sqrt{3}}$

and $h=\frac{a}{3}$.

Q.20 (1)

Any point on $y^2=4ax$ is $(at^2,2at)$, then tangent is

$2aty=2a(x+t^2) \Rightarrow yt=x+at^2$

Q.21 (1)

Normal at (h,k) to the parabola $y^2=8x$ is

$y-k=-\frac{k}{4}(x-h)$

Gradient $=\tan 60°=\sqrt{3}=-\frac{k}{4} \Rightarrow k=-4\sqrt{3}$ and

$h=6$

Hence required point is $(6,-4\sqrt{3})$

Q.22 (3)

$y-\frac{2a}{m}=-\frac{2a/m}{2a}\left(x-\frac{a}{m^2}\right)$

$\Rightarrow y-\frac{2a}{m}=\frac{-1}{m}\left(x-\frac{a}{m^2}\right)$

$\Rightarrow m^3y+m^2x-2am^2-a=0$.

Q.23 (4)

Let normal at (h,k) be $y=mx+c$

then, $k=mh+c$ also $k^2=4a(h-a)$

slope of tangent at (h,k) is m_1 then on differentiating equation of parabola.

$2ym_1=4a \Rightarrow m_1=\frac{2a}{k}$ also $mm_1=-1$

$\Rightarrow m=-\frac{k}{2a}$, solving and replacing (h,k) by (x,y)

$\Rightarrow y=m(x-a)-2am-am^3$.

Q.24 (4)

We have $t_2=-t_1-\frac{2}{t_1}$

Since $a=2,\ t_1=1\ \therefore\ t_2=-3$

$\therefore$ The other end will be $(at_2^2, 2at_2)$ *i.e.*, (18, – 12).

Q.25 (4)

The given point (–1, –60) lies on the directrix $x=-1$ of the parabola $y^2=4x$. Thus the tangents are at right angle.

Q.26 (3)

Equation of tangent at (1, 7) to $y=x^2+6$

$\frac{1}{2}(y+7)=x.1+6 \Rightarrow y=2x+5$ .....(i)

This tangent also touches the circle

$x^2+y^2+16x+12y+c=0$ .....(ii)

Now solving (i) and (ii), we get

$\Rightarrow x^2+(2x+5)^2+16x+12(2x+5)+c=0$

$\Rightarrow 5x^2+60x+85+c=0$

Since, roots are equal so

$b^2 - 4ac = 0 \Rightarrow (60)^2 - 4 \times S \times (85 + c) = 0$

$\Rightarrow 85 + c = 180 \Rightarrow 5x^2 + 60x + 180 = 0$

$\Rightarrow x = -\dfrac{60}{10} = -6 \Rightarrow y = -7$

Hence, point of contact is (–6, 7)

Q.27 (3)

Equation of chord of contact of tangent drawn from a point (x_1, y_1) to parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$ so that $5y = 2 \times 2(x+2) \Rightarrow 5y = 4x + 8.$ Point of intersection of chord of contact with parabola $y^2 = 8x$ are $\left(\dfrac{1}{2}, 2\right), (8, 8)$, so that length $= \dfrac{3}{2}\sqrt{41}$.

Q.28 (1)

The combined equation of the lines joining the vertex to the points of intersection of the line $lx + my + n = 0$ and the parabola $y^2 = 4ax$, is

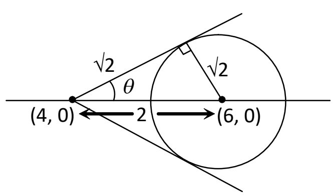
$y^2 = 4ax\left(\dfrac{lx + my}{-n}\right)$ or $4alx^2 + 4amxy + ny^2 = 0$

This represents a pair of perpendicular lines, if $4al + n = 0$.

Q.29 (1)

From diagram, $\theta = 45^o$

$\Rightarrow$ Slope = ± 1.



Q.30 (2)

Any line through origin (0,0) is $y = mx$. It intersects $y^2 = 4ax$ in $\left(\dfrac{4a}{m^2}, \dfrac{4a}{m}\right)$.

Mid point of the chord is $\left(\dfrac{2a}{m^2}, \dfrac{2a}{m}\right)$

$x = \dfrac{2a}{m^2},\ y = \dfrac{2a}{m} \Rightarrow \dfrac{2a}{x} = \dfrac{4a^2}{y^2}$ or $y^2 = 2ax$,

which is a parabola.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)

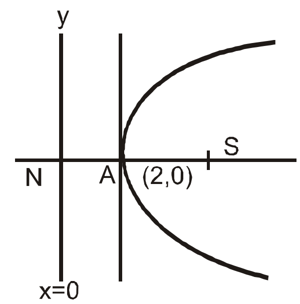
Eq. of the parabola is

$\sqrt{(x+3)^2 + y^2} = |x + 5|$

$x^2 + 6x + 9 + y^2 = x^2 + 25 + 10x$

$y^2 = 4(x + 4)$

Q.2 (3)



A is the mid point of N & S focus is (4, 0)

Q.3 (4)

$(x - 2)^2 + (y - 3)^2 = \left|\dfrac{3x - 4y + 7}{5}\right|^2$

$\therefore$ focus is (2, 3) & directrix is $3x - 4y + 7 = 0$

latus rectum = $2 \times \perp_r$ distance from focus to directrix

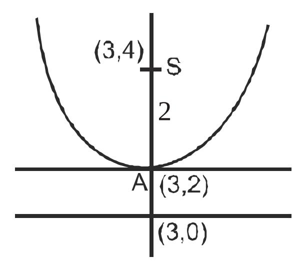
$= 2 \times \dfrac{1}{5} = 2/5$

Q.4 (1)

$y^2 - 12x - 4y + 4 = 0$

$y^2 - 4y = 12x - 4$

$(y - 2)^2 = 12x$



$Y^2 = 12X$

$x^2 = 4ay$

$(X - 3)^2 + 4x^2 (Y - 2)$

$x^2 - 6x + 9 = 8y - 16$

$x^2 - 6x - 8y + 25 = 0$

Q.5 (3)

Directrix : $x + y - 2 = 0$

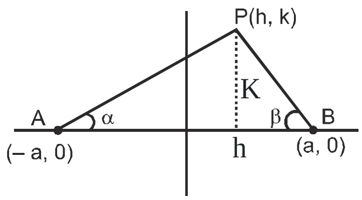
Focus to directrix distance = 2a

$2a = \left|\dfrac{0 + 0 - 2}{\sqrt{2}}\right|$

$2a = \sqrt{2}$

$LR = 4a = 2\sqrt{2}$

Q.6 (2)



$\tan\alpha + \tan\beta = \lambda$(constant)

$$\frac{k}{h+a} + \frac{k}{a-h} = \lambda$$

$$\frac{1}{a+h} + \frac{1}{a-h} = \frac{\lambda}{k}$$

$$\frac{a-h+a+h}{a^2-h^2} = \frac{\lambda}{k}$$

$2ak = (a^2 - h^2)\lambda$

$$\frac{2ay}{\lambda} = (a^2 - x^2) \quad \Rightarrow \quad \boxed{x^2 = -\frac{2ay}{\lambda} + a^2}$$

Q.7 (2)

$x^2 - 2 = -2\cos t$, $y = 4\cos^2\frac{t}{2}$

$\cos t = \dfrac{x^2-2}{-2}$, $y = 4\cos^2\dfrac{t}{2}$

$$y = 2\left(2\cos^2\frac{t}{2}\right)$$

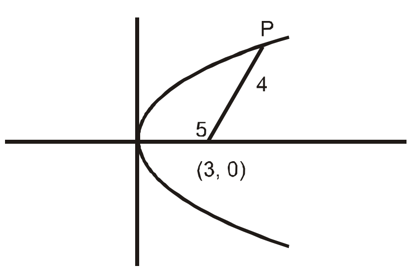
$y = 2(1 + \cos t)$

$$y = 2\left(1 + \frac{x^2-2}{-2}\right)$$

$y = 2 + 2 - x^2$

$y = 4 - x^2$

Q.8 (2)



Let the point P is $(3t^2, 6t)$

and $PS = 3 + 3t^2 = 4$

$t^2 = 1/3$

$$t = \pm\frac{1}{\sqrt{3}}$$

$\therefore$ Points are

$(1, 2\sqrt{3})$ & $(1, -2\sqrt{3})$

Q.9 (2)

$x = t^2 + 1$; $y = 2t \Rightarrow t = \dfrac{y}{2}$

$x = \dfrac{y^2}{4} + 1$ ....(i)

$x = 2s$; $y = \dfrac{2}{s} \Rightarrow s = \dfrac{2}{y}$

$x = \dfrac{4}{y} \Rightarrow \dfrac{4}{y} = \dfrac{y^2}{4} + 1$

$y^3 + 4y - 16 = 0 \Rightarrow \left.\begin{matrix} y = 2 \\ x = 2 \end{matrix}\right\}$ POI

Aliter

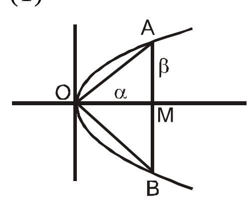
Assume a point on hyperbola $\left(2t, \dfrac{2}{t}\right)$

Put in parabola

$2t = \dfrac{1}{t^2} + 1 \Rightarrow 2t^3 - t^2 - 1 = 0$

t = 1 will satisfy point (2, 2)

Q.10 (1)



$\angle AOM = 30°$ as angle $\angle AOB = 60°$

$$\tan 30° = \frac{\beta}{\alpha}$$

$\alpha = \beta\sqrt{3}$

$\therefore$ A is $(\beta\sqrt{3}, \beta)$

Now A will satisfy equation of parabola $y^2 = 4x$

$\beta^2 = 4\cdot\beta\sqrt{3} \Rightarrow \beta = 4\sqrt{3} \Rightarrow \beta \neq 0$

$\therefore$ AB = $8\sqrt{3}$

<u>Alter</u>

Use parametiric form

at A$(at^2, 2at)$ $\Rightarrow (t^2, 2t)$

$$\tan 30° = \frac{2t}{t^2}$$

$\Rightarrow t = 2\sqrt{3}$; so A$(12, 4\sqrt{3})$

So. ℓ_{OA} = side of $\Delta = 8\sqrt{3}$

Q.11 (1)

Length of chord = $\frac{4}{m^2}\sqrt{a(a-mc)(1+m^2)}$

m = tan 60° = $\sqrt{3}$

Length of chord = $\frac{4}{3}\sqrt{3(3-\sqrt{3}\times 0)(1+3)}$

$= \frac{4}{3}\sqrt{36} = 8$

Q.12 (1)

$y^2 = 4x$ a = 1

$P(t^2, 2t)$ $t_1 t_2 = -1$

For focal chord

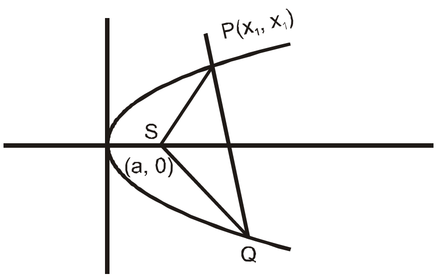
$t_2 = -\frac{1}{t}$

$Q\left(\frac{1}{t^2}, \frac{-2}{t}\right)$

$PQ = \sqrt{\left(t^2-\frac{1}{t^2}\right)^2+\left(2t+\frac{2}{t}\right)^2}$

$= \left(t+\frac{1}{t}\right)\sqrt{\left(t-\frac{1}{t}\right)^2+4} = \left(t+\frac{1}{t}\right)^2$

Q.13 (1)



$y^2 = 4ax$

$x_1^2 = 4ax_1$

$x_1 = 0, 4a$

P(4a, 4a)

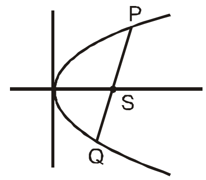
∴ Q is (9a, – 6a) $\left\{\text{using } t_2 = -t_1 - \frac{2}{t_1}\right\}$

$\Rightarrow x^2 - 4mx - \frac{4}{m} = 0$

$D = 0 \Rightarrow 16m^2 + \frac{16}{m} = 0 \Rightarrow m = -1$

slope of PS × slope of QS = – 1

Q.14 (1)



From the property $\frac{1}{PS} + \frac{1}{QS} = \frac{1}{a}$

$\frac{1}{3} + \frac{1}{2} = \frac{1}{a}$

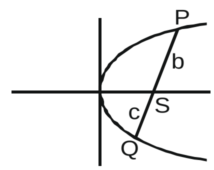
$a = \frac{6}{5}$

∴ Latus rectum = 4a = $\frac{24}{5}$

Q.15 (1)

$y^2 = 8x$

SP = 6



$\frac{1}{b} + \frac{1}{c} = \frac{1}{a}$

$\frac{1}{c} = \frac{1}{a} - \frac{1}{b}$

$c = \frac{ab}{b-a}$

b = 6, a = 2

$= \frac{12}{4} = 3$

Q.16 (4)

$y = 2x - 3,\ y^2 = 4a\left(x - \frac{1}{3}\right)$

$(2x-3)^2 = 4a\left(x-\frac{1}{3}\right)$

$\Rightarrow 4x^2 + 9 - 12x = 4ax - \frac{4}{3}a$

$\Rightarrow 4x^2 - 4(3+a)x + 9 + \frac{4a}{3} = 0$

equal roots D = 0

$16(3+a)^2 - 4\times 4\times\left(9+\frac{4a}{3}\right) = 0$

$\Rightarrow 9 + a^2 + 6a - 9 - \frac{4a}{3} = 0$

$\Rightarrow a^2 + 6a - \frac{4a}{3} = 0 \quad \Rightarrow 3a^2 + 14a = 0$

$a = 0,\ a = -\frac{14}{3}$

Q.17 (4)

Slope of tangent $= \frac{1-0}{4-3} = 1$

also $\frac{dy}{dx} = 2(x-3)$

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2(x_1 - 3) = 1 \Rightarrow x_1 - 3 = \frac{1}{2}$

$x_1 = \frac{7}{2}$

$\therefore\ \ y_1 = \left(\frac{7}{2} - 3\right)^2 = \frac{1}{4}$

Equation of tangent is

$y - \frac{1}{4} = 1\left(x - \frac{7}{2}\right)$

$4y - 1 = 2(2x - 7)$

$4x - 4y = 13$

Q.18 (3)

Let the equation of tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(1)

solving equation (1) with parabola $x^2 = 4y$

$\Rightarrow x^2 = 4\left(mx + \frac{1}{m}\right)$

Now put D = 0 & find the value of m

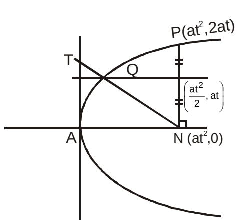
Q.19 (2)

$N(at^2, 0)$

solve $y = at$ with

curve $y^2 = 4ax$

$x = \frac{at^2}{4}$



$Q\left(\frac{at^2}{4}, at\right)$

Equation of QN $y = \frac{dt}{\left(\frac{at^2}{4} - at^2\right)}(x - at^2)$

put x = 0 $y = \frac{4}{3}at$

$T\left(0, \frac{4}{3}at\right)$ $AT = \frac{4}{3}at$

PN = 2at

$\frac{AT}{PN} = \frac{4/3\ at}{2at} = \frac{2}{3}$ so $k = \frac{2}{3}$

Q.20 (1)

Equation of normal to the parabola $y^2 = 4ax$ at points $(am^2, 2am)$ is

$y = -mx + 2am + am^3$

Q.21 (4)

Point $(am^2, -2am)$, where $m = \pm 1$

$\therefore$ point is (1, 2)

Q.22 (3)

Line : $y = -2x - \lambda$

Parabola : $y^2 = -8x$

$c = -2am - am^3$

(condition for line to be normal to parabola)

$-\lambda = -2 \times -2 \times -2 - (-2)(-8)$

$-\lambda = -8 - 16$

$\lambda = 24$

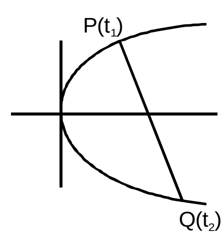
Q.23 (2)

Normal at $P(at_1^2, 2at_1)$

a = 1

$P(t_1^2, 2t_1)$

$y + t_1x = 2t_1 + t_1^3$ .....(1)



slope $= 1 = -t_1$

$t_1 = -1$

P(1, –2) $t_2 = -t_1 - \frac{2}{t_1}$

$Q(t_2^2, 2t_2)$ $t_2 = 1 + 2 = 3$

Q(9, 6)

$PQ = \sqrt{(9-1)^2 + (6+2)^2} = 8\sqrt{2}$

Q.24 (3)

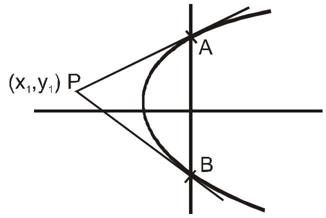
Use $T^2 = SS_1$

$\Rightarrow$ $[y.0 - 4(x+2)]^2 = (y^2 - 8x)(0 - 8(-2))$

$\Rightarrow$ $\cancel{16}(x-2)^2 = \cancel{16}(y^2 - 8x)$

$\Rightarrow$ $y = \pm(x+2)$

Q.25 (3)



Eq. of AB is :

T = 0

$yy_1 = 2(x + x_1)$

$2x - yy_1 + 2x_1 = 0$

...(1)

$4x - 7y + 10 = 0$

.... (2)

equ. (1) & (2) are identical

$\therefore \ \frac{2}{4} = \frac{y_1}{7} = \frac{2x_1}{10}$

$y_1 = \frac{7}{2}$ & $x_1 = \frac{5}{2}$

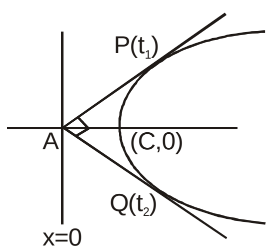
Q.26 (4)

$y^2 = x - c$; $a = 1/4$

Slope of tangent = $\frac{1}{t}$

so $\frac{1}{t_1 t_2} = -1$

$t_1t_2 = -1$ ....(i)



$A(at_1t_2 + C, a(t_1 + t_2))$

$at_1t_2 + C = 0$

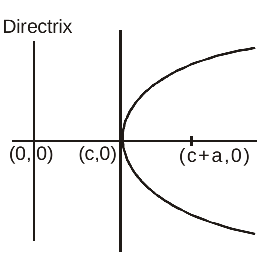
$C = -at_1t_2$

$C = a$

$C = \frac{1}{4}$

Aliter

$\frac{c + a + 0}{2} = c$



$c + a = 2c \Rightarrow c = a$

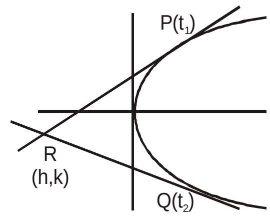
$\Rightarrow c = 1/4$

Q.27 (1)

$y^2 = 4ax$

Slope = $\frac{1}{t}$

$\frac{1}{t_1} = \frac{2}{t_2}$



$\Rightarrow t_2 = 2t_1$ ......(1)

$R[at_1t_2, a(t_1 + t_2)]$

$h = at_1 t_2$, $k = a(t_1 + t_2)$

$k = 3at_1 \Rightarrow t_1 = \frac{k}{3a}$

$h = 2at_1^2$

$h = 2a\frac{k^2}{9a^2}$ $\Rightarrow k^2 = \frac{9}{2}ah$

$\Rightarrow y^2 = \frac{9}{2}ax$

Q.28 (4)

$y^2 + 4y - 6x - 2 = 0$

$y^2 + 4y + 4 - 6x - 6 = 0$; $a = \frac{3}{2}$

$(y+2)^2 = 6(x+1)$

$Y^2 = 6X$ vertex $(-1, -2)$

POI of tangents $t_1 t_2 = -1$

$[at_1t_2, a(t_1 + t_2)]$

$h + 1 = at_1t_2$

$h + 1 = -\frac{3}{2}$

$2h + 2 = -3$

$2h + 5 = 0 \Rightarrow 2x + 5 = 0$

Q.29 (3)

Let point $P(x_1, y_1)$

$x_1 - y_1 + 3 = 0$

C.O.C. w.r.t. (x_1, y_1) of $y^2 = 4ax$

$yy_1 = 4(x + x_1)$

$y(x_1 + 3) = 4x + 4x_1$

$yx_1 + 3y - 4x - 4x_1 = 0$

$(3y - 4x) + x_1(y - 4) = 0$

$L_1 + \lambda L_2 = 0$

$L_1 = 0$ & $L_2 = 0$

$3y = 4x$ $y = 4$

$x = 3$

point (3, 4)

Q.30 (3)

Equation of PQ

$(t_1 + t_2)y = 2x + 2at_1t_2$

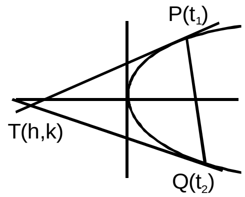
passes through $(-a, b)$

$b(t_1 + t_2) = -2a + 2at_1t_2$ ....(i)

$h = at_1t_2$ & $k = a(t_1 + t_2)$

POI of tangents

$h = at_1t_2$ & $k = a(t_1 + t_2)$



$\frac{bk}{a} = -2a + 2h$

$bk = -2a^2 + 2ah$

$by = -2a^2 + 2ax$

$by = 2a(x - a)$

Q.31 (3)

Tangent at P of $y^2 = 4ax$

$yy_1 = 2a(x + x_1)$

.....(1)

Let Mid point (h, k)

$T = S_1$

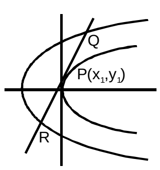
$yk - 2a(x + h) - 4ab = k^2 - 4a(h + b)$

$yk - 2ax - 2ah + 4ah - k^2 = 0$

$yk - 2ax + 2ah - k^2 = 0$ ....(2)

(1) & (2) are same

$$\frac{k}{y_1} = \frac{-2a}{-2a} = \frac{2ah - k^2}{-2ax_1}$$



$k = y_1$; $-2ax_1 = 2ah - k^2$

$-2ax_1 = 2ah - y_1^2$; $y_1^2 = 4ax_1$

Mid point $-2ax_1 = 2ah - 4ax_1$

(x_1, y_1) $2ah = 2ax_1$

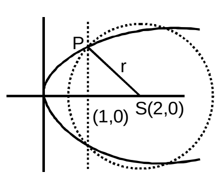
$h = x_1$

Q.32 (2)

$P(1, 2\sqrt{2})$

Intersection point of x = 1 with $y^2 = 8x$

$r^2 = SP^2$



$= (1 - 2)^2 + (2\sqrt{2})^2$

$= 1 + 8 = 9$

equation of circle as centre (2, 0) ; r = 3

$(x - 2) + y^2 = 9$

Q.33 (2)

Eq. of chord is $T = S_1$

$ky - 2(x + h) = k^2 - 4h$

...(1)

$\because$ above eq. passes through focus (1, 0)

$\therefore$ $0.k - 2(1 + h) = k^2 - 4h$

$-2 - 2x = y^2 - 4x$

$y^2 = 2(x - 1)$

Q.34 (1)

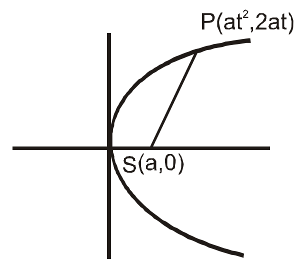
From the property : the feet of the $\perp$r will lie on the tangent at vertex of the parabola.

$y = (x - 1)^2 - 3 - 1$

$(x - 1)^2 = (y + 4)$

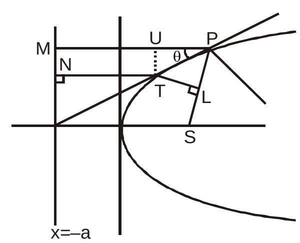
Tangent at vertex of above parabola is y + 4 = 0.

Q.35 (1)



(Note: this is a High light)

Q.36 (3)



Δ PUT $\cong$ Δ PLT

Both Δ are congurrent

Hence PU = PL

PM = SP

PM – PL = SP – PL

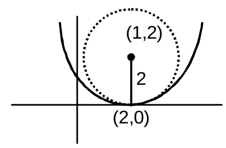
TN = MU = SL

Q.37 (4)

$(x - 1)^2 = 8y$; a = 2 $x - 1 = 0$, $y = 2$

$x^2 = 8y$;   x = 1, y = 2

vertex (1, 0)



Focus (1, 2)

Radius of circle = 2

$(x - 1)^2 + (y - 2)^2 = 4$

$x^2 + y^2 - 2x - 4y + 1 = 0$

Q.38 (3)

$y^2 = 4a(x = \ell_1)$

$x^2 = 4a(y - \ell_2)$

let the POC (h, k)

$2yy' = 4a$

$2x = 4ay'$

$$y' = \left.\frac{2a}{y}\right|_{(h,k)} = \frac{2a}{k} \qquad ...(1)$$

$$y' = \left.\frac{x}{2a}\right|_{(h,k)}$$

(1) and (2) are equal $= \frac{h}{2a}$ ....(2)

$$\frac{2a}{k} = \frac{h}{2a}$$

$hk = 4a^2$

$xy = 4a^2$

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 (C)

$t_1 t_2 = -1$, and the point of intersection tangent in $(a_1 t_1 t_2, a(t_1 + t_2))$

intersection point of Normals is

$(a(t_1^2 + t_2^2 + t_1 t_2 + 2), -at_1t_2(t_1 + t_2))$

using $t_1t_2 = -1$, ordinate of both the section point are equal.

Q.2 (C)

distance of focal chord from (0, 0) is p

equation of chord ;

$2x - (t_1 + t_2) y + 2a t_1 t_2 = 0$

$2x - (t_1 + t_2) y - 2a = 0$ ...... (i)

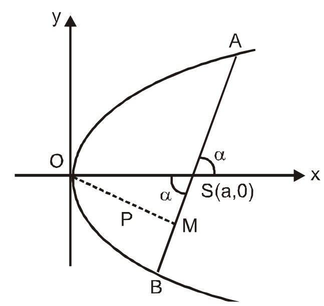
so perpendicular length from (0, 0)

$$\left|\frac{2a}{\sqrt{4 + \left(t_1 - \frac{1}{t_1}\right)^2}}\right| = p \Rightarrow \left|\frac{2a}{\left(t_1 + \frac{1}{t_1}\right)}\right| = p$$

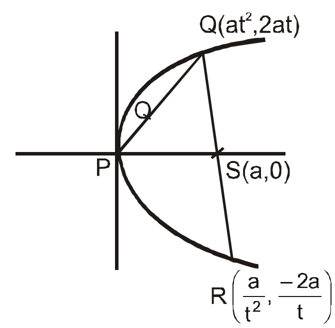
$$\Rightarrow \left(t_1 + \frac{1}{t_1}\right) = \frac{2a}{p}$$

Now length of focal chord is $= a\left(t_1 + \frac{1}{t}\right)^2$

$$= a\,\frac{4a^2}{p^2} = \frac{4a^3}{p^2}$$



Q.3 (C)



Equation of QR is

$2x - (t_1 + t_2) y + 2at_1t_2 = 0$

$2x - \left(t - \frac{1}{t}\right) - 2a = 0$...(1)

⊥r distance from (0, 0) to the line (1) is

$$\left|\frac{2a}{4 + \left(t - \frac{1}{t}\right)^2}\right| = \left|\frac{2a}{\left(t + \frac{1}{t}\right)}\right|$$

Area $= \frac{1}{2} \times$ QR × ⊥r distance from origin

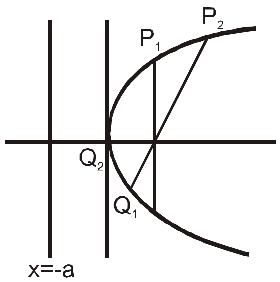
$$= \frac{1}{2} a\left(t + \frac{1}{t}\right)^2 \times \frac{2a}{\left(t + \frac{1}{t}\right)}$$

$$A = a^2\left(t + \frac{1}{t}\right)$$

Now the difference of ordination

$$= \left|2at + \frac{2a}{t}\right| = \left|2a\left(t + \frac{1}{t}\right)\right| = 2a.\frac{A}{a^2} = \frac{2A}{a}$$

Q.4 (A)

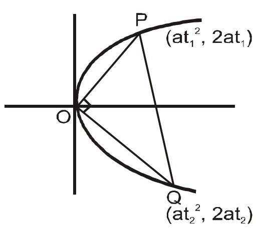


$P_1(at_1^2, 2at_1)$, $Q_1\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$

$P_2(at_2^2, 2at_2)$, $Q_2\left(\frac{a}{t_2^2}, \frac{-2a}{t_2}\right)$

write the equation of P_1P_2 and Q_1Q_2 and then find the x-coordinate of their intersection.

Q.5 (B)



Slope of OP $\propto$ slope of OQ

$t_1 t_2 = -4$ ..(1)

also $t_2 = -t_1 - \frac{2}{t_1}$

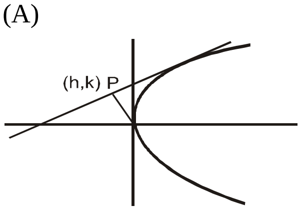
$\frac{-4}{t_1} = \frac{-t_1^2 - 2}{t_1}$

$t_1^2 = 2$

$t_1 = \pm\sqrt{2}$

slope of normal at P $= -t_1 \Rightarrow \tan\theta = \sqrt{2} \Rightarrow \theta = \tan^{-1}(\sqrt{2})$

Q.6 (A)



$y = mx + \frac{a}{m}$ ...(1)

$y = -\frac{1}{m}x$ ...(2)

solving (1) & (2)

$x = \frac{-a}{1+m^2}$

$m^2 = \frac{-a}{x} - 1$

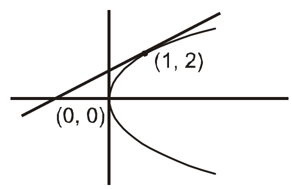
put $m = -\frac{x}{y}$

from equation (2)

$\left(-\frac{x}{y}\right)^2 = -\frac{a}{x} - 1$

$(x^2 + y^2)x + ay^2 = 0$

Q.7 (C)



Equation of tangent at(1, 2) is

$2y = 2(x + 1)$

$x - y + 1 = 0$ .... (i)

image of (0, 0) in the line (i) is (– 1, 1)

$\therefore$ vertex of required parabola will be (– 1, 1)

Q.8 (B)

Equation of tangent is $y = x + A$ ...(1)

and the equation of normal is

$y = mx - 2Am - Am^3$

where $m = -1$

$y = -x + 2A + A$

$x + y - 3A = 0$ ...(2)

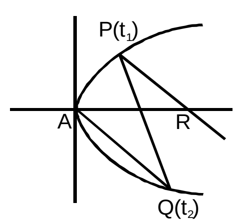
distance b/w (1) & (2) is $\left|\frac{3A + A}{\sqrt{2}}\right| = 2\sqrt{2}$.

Q.9 (C)

Slope of OQ $= \frac{2}{t_2}$

line parallel to AQ

and passing through P



$y - 2at_1 = \frac{2}{t_2}(x - at_1^2)$

For point R put y = 0

$-2at_1 = \frac{2}{t_2}(x - at_1^2)$ $\quad t_2 = -t_1 - \frac{2}{t_1}$

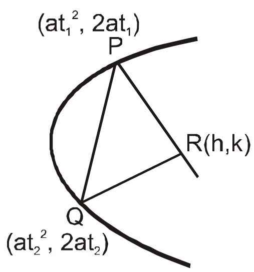
$x = at_1^2 - at_1t_2$

$t_2 + t_1 = -\frac{2}{t_1}$

$= at_1 (t_1 - t_2) = 2at_1 (t_1 + \frac{1}{t_2})$

$x = 2(at_1^2 + a)$ focal distance

Q.10 (A)



Slope of PQ = $\frac{2}{t_1 + t_2} = m$

$\Rightarrow t_1 + t_2 = 2/m$

$h = a(t_1^2 + t_2^2 + t_1t_2 + 2)$

$h = a((t_1 + t_2)^2 - t_1t_2 + 2)$

$h = a\left(\frac{4}{m^2} - t_1 t_2 + 2\right)$ ...(1)

$k = -a\, t_1t_2 (t_1 + t_2) = -at_1t_2\left(\frac{2}{m}\right)$

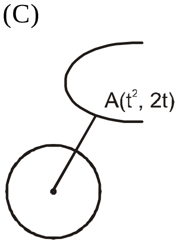
$t_1t_2 = -\frac{mk}{2a}$ ...(2)

using (2) in (1)

$a\left(\frac{4}{m^2} + \frac{mk}{2a} + 2\right) = \frac{8a + m^3k + 4am^2}{2am^2}$

$2xm^2 - m^3y = 4a(2 + m^2)$

Q.11 (C)



shortest distance always lie along the common normal

Equation of normal at $(t^2, 2t)$ to the parabola is

$y + xt = 2t + t^3$ ..... (i)

above equation passes through the center of the circle c(0, 12)

$\therefore \quad 12 = 2t + t^3$

$t^3 + 2t - 12 = 0$

$t = 2$

$\therefore$ point is (4, 4)

Q.12 (B)

Subtangent = $2x_1$

ordinate = y_1 are in G.P.

subnormal = 2a

Q.13 (A)

Equation of Normal In slope form

$y = mx - 2am - am^3$; $a = \frac{1}{4}$

$6 = 3m - \frac{2m}{4} - \frac{m^3}{4}$ (3, 6)

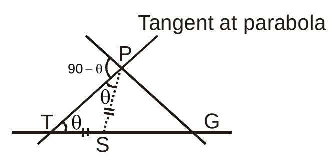
$m^3 - 10m + 24 = 0 \Rightarrow m = -4$

equation of normal

$y - 6 = -4(x - 3) \Rightarrow y + 4x - 18 = 0$

Q.14 (C)

Slope of tangent $\tan\theta = t$

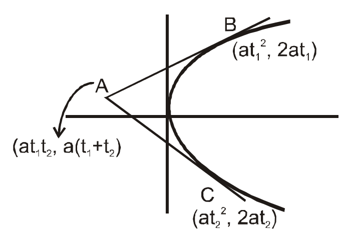


$\tan (90 - \theta) = \cot \theta = \frac{1}{t}$

$\tan \theta = t$

$\theta = \tan^{-1} t$

Q.15 (B)



Let the tangent is x = 0

then, $p_2 = |at_1^2|$

$p_3 = |at_2^2|$

$p_1 = |at_1t_2|$

$\therefore$ p_2, p_1, p_3 are in G.P.

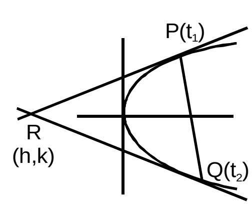
Q.16 (C)

$h = at_1t_2$

$k = a(t_1 + t_2)$

$k = -\frac{2a}{t_2}$

$t_1 = -\frac{2a}{k}$



$t_2 = -t_1 - \frac{-2}{t_1} \Rightarrow t_2 + t_1 = \frac{-2}{t_1}$

$h = at_1t_2 = at_1\left(-t_1 - \frac{2}{t_1}\right)$

$\Rightarrow h = a\left(-\frac{2a}{k}\right)\left(\frac{2a}{k} + \frac{2}{2a/k}\right) = -\frac{2a^2}{k}\left(\frac{2a}{k} + \frac{k}{a}\right)$

$\Rightarrow hk^2 = -4a^3 - 2ak^2 \Rightarrow k^2(h + 2a) + 4a^3 = 0$

$\Rightarrow y^2(x + 2a) + 4a^3 = 0$

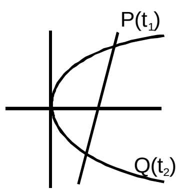
Q.17 (D)

$T = S_1$

$yy_1 - 2a(x + x_1) = y_1^2 - x_1$

$(x_1, y_1) \Rightarrow (2, 1)$

$y - \frac{2}{4}(x + 2) = 1 - 2$



$4y - 2x = 0$

$x = 2y \Rightarrow$ solve with parabola

$y^2 = 2y$

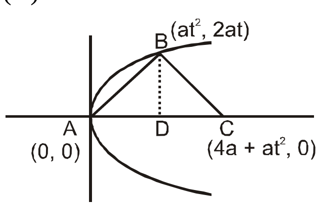
$y = 0, y = 2$

$x = 0, x = 4$

$(0, 0)\ (4, 2)$

$PQ = \sqrt{4 + 16} = 2\sqrt{5}$

Q.18 (C)



Slope of AB $= \frac{2}{t}$

$BC = -\frac{t}{2}$

equation of BC

$y - 2at = -\frac{t}{2}(x - at^2)$

put $y = 0$

$x = 4a + at^2$

in Δ BDC

$DC^2 = BC^2 - BD^2$

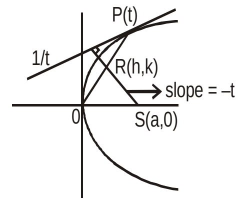
$= 16a^2 + 4a^2t^2 - 4a^2t^2$

$= 16a^2$

$DC = 4a$

Q.19 (B)

Equation of OP



$y = \frac{2}{t}x$

$k = \frac{2}{t}h$ ....(1)

$y - 0 = -t(x - a) \Rightarrow y = -tx + at$

$\Rightarrow k = -th + at \Rightarrow \frac{2}{t}h = -th + at$ from (1)

$(t = \frac{2h}{k})$

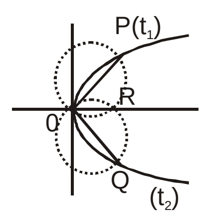
$h = \frac{at^2}{2 + t^2} \Rightarrow h = \frac{a\frac{4h^2}{k^2}}{2 + \frac{4h^2}{k^2}} \Rightarrow h = \frac{2ah^2}{k^2 + 2h^2}$

$\Rightarrow k^2 + 2h^2 = 2ah \Rightarrow 2x^2 + y^2 - 2ax = 0$

Q.20 (A)

$ty = x + at^2$

$\tan\theta_1 = \frac{1}{t_1}$; $\tan\theta_2 = \frac{1}{t_2}$



Circle

$(x - at_1^2)(x - 0) + (y - 0)(y - 2at_1) = 0$

$(x - at_2^2)(x - 0) + (y - 0)(y - 2at_2) = 0$

For Intersection point R

$S_1 - S_2 = 0$

$\Rightarrow (at_2^2 - at_1^2)x + y(2at_2 - 2at_1) = 0$

$\Rightarrow 2y + (t_2 + t_1)x = 0 \Rightarrow y = -\left(\frac{t_1 + t_2}{2}\right)x$

$\tan\theta_1 = \frac{1}{t_1} \Rightarrow \cot\theta_1 = t_1$ & $\cot\theta_2 = t_2$

$\cot\theta_1 + \cot\theta_2 = t_1 + t_2 = -2\tan\phi$

JEE-ADVANCED

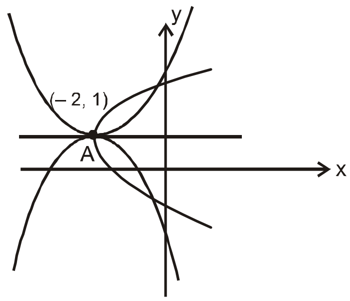
MCQ/COMPREHENSION//COLUMN MATCHING

Q.1 (A,B)

$y^2 - 2y = 4x + 7$

$(y - 1)^2 = 4x + 8$

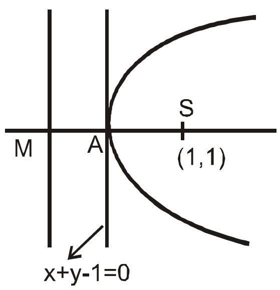
$(y - 1)^2 = 4(x + 2)$



Equation of required parabolas is

$(x + 2)^2 = 8(y - 1)$ & $(x + 2)^2 = -8(y - 1)$

Q.2 (B,C,D)



Point A is $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ M is (0, 0)

$\therefore$ Eq. of Diretrix is $x + y = 0$

$\therefore$ Eq. of parabola is $(x - 1)^2 + (y - 1)^2 = \left(\frac{x+y}{\sqrt{2}}\right)^2$

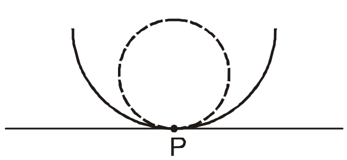
Length of latus vectrum = 2(⊥r distance from focus to the directrix)

$= 2.\left|\frac{1+1}{\sqrt{2}}\right| = 2\sqrt{2}$

Q.3 (A,D)

$at^2 = 2at$

point



t = 0, t = 2

(0, 0), (4, 4)

(I) when P ≡ (0, 0)

$x^2 + y^2 + \lambda (x) = 0$

pass the (1, 0)

$\lambda = -1$

equation tagent al (0, 0)

$y^2 = 4x$

Equ. $x^2 + y^2 - x = 0$

$y. y_1 = 2 (x + x_1)$

$x = 0$

(II) when point (4, 4)

$2x - 2y + 8 = 0$

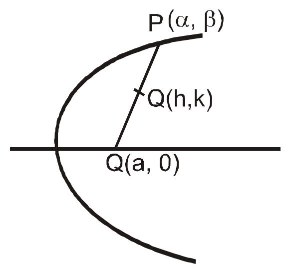
$(x - 4)^2 + (y - 4)^2 + \mu (2x - 2y + 8) = 0$

pass (1, 0)

Equation

$x^2 + y^2 - 13x + 2y + 12 = 0$

Q.4 (A, B, C, D)



$h = \frac{a+\alpha}{2}$, $k = \frac{\beta}{2}$

$\Rightarrow \alpha = 2h - a, \beta = 2k$

α, β satisfies the parabola

$\therefore \beta^2 = 4a\alpha$

$4k^2 = 4a(2n - a)$

$y^2 = a(2x - a)$

$y^2 = 2a\left(x - \frac{a}{2}\right)$

Q.5 (A,B)

$y^2 - 2y - 4x - 7 = 0$

$y^2 - 2y + 1 - 4x - 8 = 0$ LR = 4 = L

$(y - 1)^2 = 4(x + 2)$

vertex (–2, 1) Axis = x-axis

New parabola

$(x + 2)^2 = \pm 8(y - 1)$

+ve $(x + 2)^2 = 8(y - 1)$

$x^2 + 4x - 8y + 12 = 0$

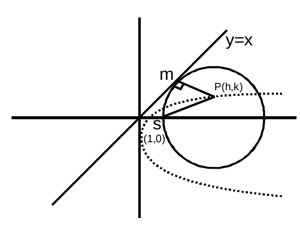
–ve $x^2 + 4x + 4 + 8y - 8 = 0$

$x^2 + 4x + 8y - 4 = 0$

Q.6 (B,C)

$y = \tan (\tan^{-1} x) = x$

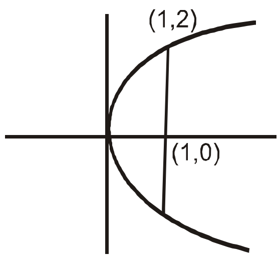
PS = PM $\Rightarrow (h - 1)^2 + (k - 0)^2 = \frac{(h-k)^2}{2}$



$\Rightarrow$ $2(h^2 + 1 - 2h + k^2) = h^2 + k^2 - 2hk$
$\Rightarrow$ $h^2 + k^2 + 2hk + 2 - 4h = 0$
$\Rightarrow$ $x^2 + y^2 + 2xy + 2 - 4x = 0$

Q.7 (A,B)
$y^2 = 4ax$
(A) $(at^2, 2at)$ possible
(B) $(at^2, -2at)$ possible
(C) $(a \sin^2 t, 2a \sin t)$ not possible because sin t will lies only in [–1, 1]
so ans. (A) (B)

Q.8 (A,B,D)



$y^2 = 4x$, the other end of focal chord will be (1, –2) and this satisfy options (A) (B) & (D)

Q.9 (A,C)
Option (A) & (C) are used as a property.

Q.10 (B,C)

Let the equation of tangent is $y = mx + \frac{a}{m}$

$y = mx + \frac{3}{m}$ ...(1)

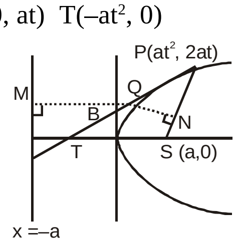
$\tan 45° = \left|\frac{m-3}{1+3m}\right|$

$\Rightarrow \frac{m-3}{1+3m} = \pm 1$ $\Rightarrow$ $m - 3 = \pm(1 + 3m)$

$\Rightarrow m = -2, 1/2$
Put in equation (1)

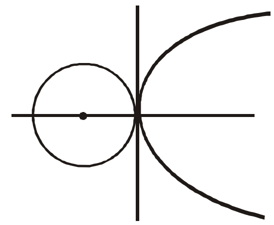
$y = -2x - \frac{3}{2}$ and $y = \frac{1}{2}x + 6$

Q.11 (A,C)
Tangent at P
$ty = x + at^2$
B(0, at) T($-at^2$, 0)



clearly B is the mid point of TP

Q.12 (A, D)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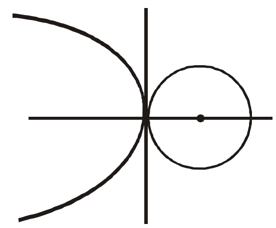


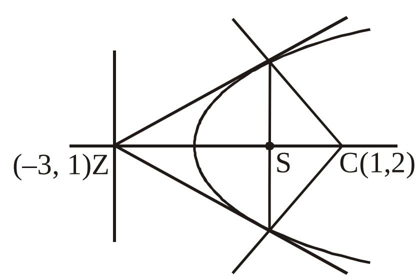
a > 0, b > 0 a < 0 , b < 0

Q.13 Let the normal be $y = mx - 4m - 2m^3$
$\Rightarrow$ $0 = 6m - 4m - 2m^3$ $\Rightarrow$ m = 0, 1, –1
A(0, 0); B(2, 4); C(2, –4)
Area = 8

Centroid $\equiv \left(\frac{4}{3}, 0\right)$, circumcentre ≡ (5, 0).]

Q.14 (A,B)
Tangents are perpendicular $\Rightarrow$ AB is focal chord and Normals meet on axis of parabola $\Rightarrow$ AB is double ordinate $\Rightarrow$ AB is latus rectum.
$\Rightarrow$ Z(–3, 1)
$\therefore$ equation of axis

$y - 1 = \frac{1}{4}(x + 3)$



$4y - 4 = x + 3$

$x - 4y + 7 = 0$

$CZ = 4a = \sqrt{4^2 + 1^2} = \sqrt{17}$ **Ans.**

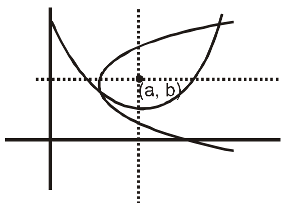
Q.15 (A,B,C,D)
$h = t_1 t_2$ …….(1)
$k = t_1 + t_2$ …….(2)
$t_1^2 = 16t_2^2$ …….(3)
From (1) , (2) and (3)

$k^2 = t_1^2 + t_2^2 + 2h = 17t_2^2 + 2h = \frac{17h}{4} + 2h = \frac{25h}{4}$

$\therefore$ Locus is $y^2 = \left(\frac{25}{4}\right)x$.

Now verify all the options.

Q.16 (A,B)



Equation of both the parabola is given by the equation

$(x – a)^2 + (y – b)^2 = x^2$ .... (i)

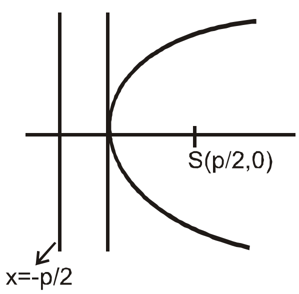
& $(x – a)^2 + (y – b)^2 = y^2$ .... (ii)

(i) - (ii)

$\Rightarrow (x + y)(x – y) = 0$

slope of common chord = 1 & – 1

Q.17 (A, B)



Eq. of circle is given by

$\left(x - \frac{p}{2}\right)^2 + y^2 = r^2$ ...(1)

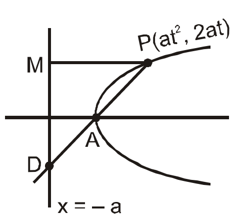
Directrix : $x = -\frac{p}{2}$ in tangent to the circle ...(1)

$\therefore$ r = p

$\therefore$ Eq. of circle is $\left(x - \frac{p}{2}\right)^2 + y^2 = p^2$ ...(2)

solve circle & parabola for point of intersection.

Q.18 (A, D)



Equation of PA is

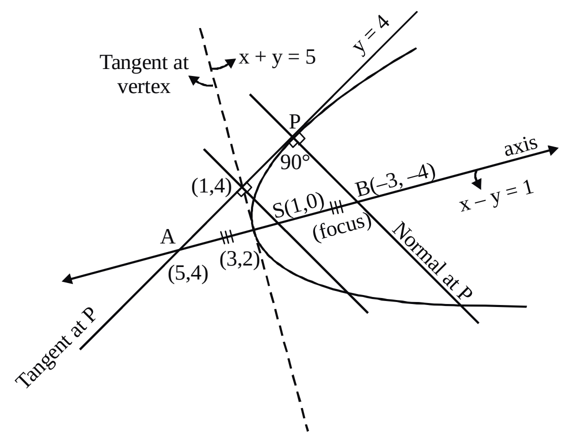
$y = \frac{2}{t}x$ .....(i)

$D\left(-a, \frac{-2a}{t}\right)$ M(– a, 2at)

write the equation of circle with MD as diameter and then solve with x - axis

Comprehenssion # 1 (Q. No. 19 to 21)

Q.19 (A)
Q.20 (D)
Q.21 (D)



(i) Tangent and normal are angle bisectors of focal radius and perpendicular to directrix.

$\therefore$ The equation of circle circumscribing ΔAPB, is $(x – 5)(x + 3) + (y – 4)(y – 4) = 0 \Rightarrow x^2 + y^2 – 2x = 31$

(ii) Two parabolas are called equal when their length of latus rectum is same.

Also, l(L · R) = 4 (Distance of focus from vertex)

$= 4\sqrt{(3-1)^2 + (2-0)^2} = 4\sqrt{8} = 8\sqrt{2}$

(iii) The area of quadrilateral formed by tangent and normals at ends of latus-rectum = $8(VS)^2$

= 8(4 + 4) = 8(8) = 64

Comprehenssion # 2 (Q. No. 22 to 24)

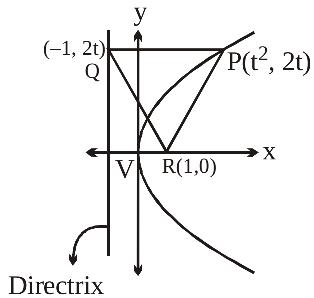
Q.22 (A,B,C,D)
Q.23 (B,C,D)
Q.24 (B,C)[

We have $PM = 1 + t^2$

$PS = \sqrt{(t^2-1)^2 + 4t^2} = (t^2+1)$

$MS = \sqrt{4+4t^2} = 2\sqrt{1+t^2}$

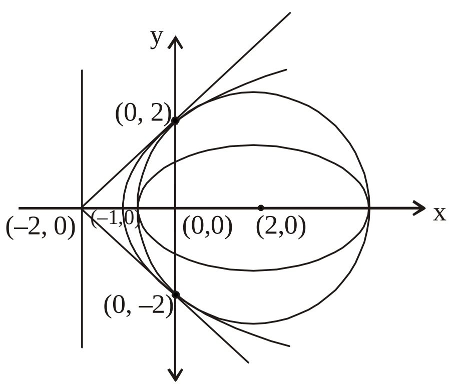
$\Rightarrow 2\sqrt{1+t^2} = 1 + t^2$



$\therefore$ PM = $1 + t^2 = 4 = a = k$ (Given)
Hence $C_1 : y^2 = 4(x + 1)$
Equation of tangent to C_1 at (0, 2) is

$$2y = 4\left(\frac{x+0}{2}+1\right) \Rightarrow y = x + 2.$$

Now circle which touches above line at (0, 2), is
$x^2 + (y – 2)^2 + \lambda(x – y + 2) = 0$.
As above circle is passing through the point (0, –2), so



$0 + 16 + \lambda(4) = 0 \Rightarrow \lambda = – 4$
$\therefore$ $C_2 : x^2 + (y – 2)^2 – 4(x – y + 2) = 0$
or $C_2 : x^2 + y^2 – 4x – 4 = 0$.

Now $C_3 : \dfrac{(x-2)^2}{a^2}+\dfrac{y^2}{b^2} = 1$, $a = 2\sqrt{2}$ and $b = 2$

So $C_3 : \dfrac{(x-2)^2}{8}+\dfrac{y^2}{4} = 1$.

(i) Given $C_1 : y^2 = 4(x + 1)$
(A) Minimum length of focal chord = Latus rectum = 4.
(B) Locus of point of intersection of perpendicular tangents = Director circle which is $x + 2 = 0$.
(C) Clearly distance between focus and tangent at vertex is 1.
(D) Foot of the directrix is clearly (–2, 0).

(ii) We have $C_3 : \dfrac{(x-2)^2}{8}+\dfrac{y^2}{4} = 1$

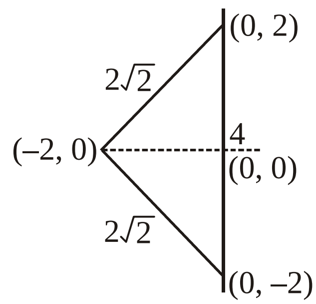
(A) $e = \sqrt{1-\dfrac{4}{8}} = \dfrac{1}{\sqrt{2}}$

(B) Focal length = 2 ae $= 2 \times 2\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right) = 4$

(C) Latus-rectum $= \dfrac{2b^2}{a} = 2\left(\dfrac{4}{2\sqrt{2}}\right) = 2\sqrt{2}$

(D) Director circle is $(x – 2)^2 + y^2 = 12 \Rightarrow x^2 + y^2 – 4x – 8 = 0$

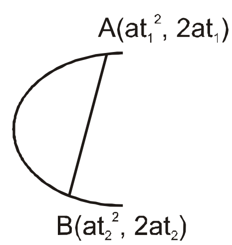
(iii)



Triangle formed by common tangents to the curves C_1 and C_2 and latus-rectum of C_1, is isosceles triangle.

Required area $= \dfrac{1}{2}\times 4\times 2$ = 4 square units.

Q.25 (A) $\to$ (s), (B) $\to$ (r),(C) $\to$ (q), (D) $\to$ (p)
Equation AB



$$y – 2at_1 = \frac{2}{t_1+t_2}(x - at_1^2)$$

(A) AB is a normal chord $t_2 = – t_1 – \dfrac{2}{t_1}$

(B) AB is a focal chord $t_1t_2 = – 1$
(C) AB subtends 90º at the origin then

$$\frac{2at_1-0}{at_1^2-0}\times\frac{2at_2-0}{at_2^2-0} = – 1$$

$$t_1t_2 = – 4 \Rightarrow t_2 = -\frac{4}{t_1}$$

(D) AB is inclinded at 4s° to the axis then slope

$$\frac{2}{t_1 + t_2} = 1$$

$t_1 + t_2 = 2$
$t_2 = -t_1 + 2$

Q.26 A → P, Q, R, S, T; **B** → S, T; **C** → Q, R, S, T

If three normals drawn to any parabola $y^2 = 4ax$ from a given point (h, k) be real, then h > 2a.

(A) ∵ $y^2 - 4x - 2y + 5 = 0$

⇒ $(y-1)^2 = 4(x-1)$

Let y – 1 = Y and x – 1 = x

∴ $y^2 = 4x$

On comparing with $y^2 = 4ax$

∴ a = 1

According to question x > 2a

⇒ x – 1 > 2 or x > 3

∴ x = 4, 5, 6, 7, 8 (P, Q, R, S, T)

(B) ∵ $4y^2 - 32x + 4y + 65 = 0$

⇒ $4(y^2 + y) = 32x - 65$

$$\Rightarrow 4\left(\left(y+\frac{1}{2}\right)^2 - \frac{1}{4}\right) = 32x - 65$$

$$\Rightarrow 4\left(y+\frac{1}{2}\right)^2 = 32x - 64$$

$$\text{or } \left(y+\frac{1}{2}\right)^2 = 8(x-2)$$

Let $y + \frac{1}{2}$ = y and x – 2 = x

∴ $y^2 = 8x$

on comparing with $y^2 = 4ax$

∴ a = 2

According to question x > 2a

⇒ x – 2 > 4 ∴ x > 6

∴ x = 7, 8 (S, T)

(C) ∵ $4y^2 - 16x - 4y + 41 = 0$

⇒ $4(y^2 - y) = 16x - 41$

$$\Rightarrow 4\left\{\left(y-\frac{1}{2}\right)^2 - \frac{1}{4}\right\} = 16x - 41$$

$$\Rightarrow 4\left(y-\frac{1}{2}\right)^2 = 16x - 40$$

$$\text{or } \left(y-\frac{1}{2}\right)^2 = 4\left(x-\frac{5}{2}\right)$$

Let $y - \frac{1}{2}$ = y and $x - \frac{5}{2}$ = x

∴ $y^2 = 4x$

On comparing with $y^2 = 4ax$ ∴ a = 1

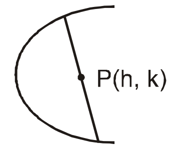
According to question

$x > 2a \Rightarrow x - \frac{5}{2} > 2$ or $x > \frac{9}{2}$

∴ x = 5, 6, 7, 8 (Q, R, S, T)

Q.27 (A) → (r), (B) → (s),(C) → (p), (D) → (q)

$y^2 = 4ax$



$T = S_1$

$T \equiv ky - 2a(x + h)$

$S_1 = k^2 - 2ah$

(A) Equation

$ky - 2a(x + h) = k^2 - 4ah$

This line pass thoh (a, 0)

$0 - 2a(a + h) = k^2 - 4ah$

$-2a^2 - 2ah = k^2 - 4ah$

$k^2 + 2ah - 2a^2 = 0$

Locus $y^2 + 2ax - 2a^2 = 0$ A → r

(B) We know that equation of normal

$y = mx - am^3 - 2am$ ....(i)

$ky - 2ax = k^2 - 2ah$

$$y = \frac{2a}{k}x + \frac{k^2 - 2ah}{k} \quad ....(ii)$$

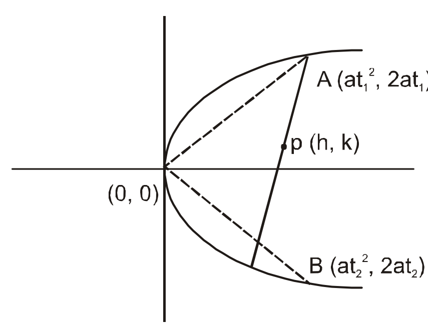
comparing equation (i) and (ii) $m = \frac{2a}{k}$ $am^3 - 2am$

$$= \frac{k^2 - 2ah}{k} \quad \ldots (2)$$

put $m = \frac{2a}{k}$ in equation (2)

we get the locus $y^4 + 2a(2a - x)y^2 + 8a^4 = 0$ B → s

(C) $h = \frac{a(t_1^2 + t_2^2)}{2}$



$k = a(t_1 + t_2)$

$$\text{or } \frac{2}{t_1} = -\frac{2}{t_2}$$

$t_1 + t_2 = 0$ k = 0 ⇒ y = 0

(D) Length of chord = ℓ

$$= -\frac{4}{m^2}\sqrt{a(1+m^2)(a-mc)} = \ell$$

where $m = \frac{2a}{k}$

$$c = \frac{k^2 - 2ah}{k}$$

Let PQ be a variable focal chord of the parabola $y^2 = 4ax$ where vertex is A. Locus of, centroid of triangle APQ is a parabola 'P_1'

NUMERICAL VALUE BASED

Q.1 (4)

$h^2 = ab$

$\Rightarrow 4 = \lambda.1 \Rightarrow \lambda = 4$

Q.2 (**20**)

a = $\perp^r$ distance from (3, 4) to the tangent at vertex

$$= \left|\frac{3+4-7-5\sqrt{2}}{\sqrt{2}}\right|$$

a = 5

LR = 4a = 20

Q.3 (**2**)

$y^2 = 8x$; a = 2

$$\text{Area} = \frac{(y_1^2 - 8x_1)^{3/2}}{4}; \quad (4, 6);$$

$$= \frac{(36-32)^{3/2}}{4} = \frac{8}{4} = 2 \text{ sq. units}$$

Q.4 (3)

$y = mx - 2am - am^3$

Here a = 1

$0 = cm - 2m - m^3$

$m^3 + (2 - c)m = 0$

m = 0

$m^2 = c - 2 \Rightarrow c > 2$

sum $m_1 + m_2 + m_3 = 0$

$$\Sigma m_1m_2 = \frac{2a-h}{a}$$

$$m_1m_2m_3 = \frac{-k}{a}$$

$m_1m_2 = 2 - c$

$-1 = 2 - c$

$\Rightarrow c = 3$

Q.5 (1)

I.F. $(a^2, a - 2)$

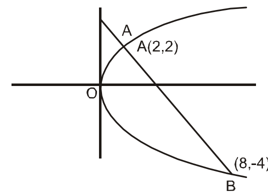
$S \equiv y^2 - 2x$ $\qquad$ $S \equiv y^2 - 2x$

Equation of line AB

$$y - 2 = \frac{-6}{6}(x - 2)$$

$y - 2 = -x + 2$

$L \equiv x + y - 4 = 0$



$S_1 \equiv (a-2)^2 - 2a^2 < 0$ $\qquad$ $\Rightarrow a^2 + 4a - 4 > 0$

$a^2 + 4 - 4a - 2a^2 <$ $\qquad$ $a^2 + 4a + 4 > 8$

$-4a - a^2 + 4 < 0$ $\qquad$ $(a+2)^2 > 8$

$L_1 < 0$ $\qquad$ $a + 2 > 2\sqrt{2}$

$a^2 + a - 6 < 0$

$a + 2 < -2\sqrt{2}$

$-3 < a < 2$

$a > -2 + 2\sqrt{2}$ $\quad$ $a < -2\sqrt{2} - 2$

$\Rightarrow \quad -2 + 2\sqrt{2} < a < 2$

so integral value of a is equal to 1 only.

Q.6 (3)

Here $h^2 - ab = (-12)^2 - 9 \cdot 16 = 144 - 144 = 0$ Also $\Delta \neq 0$

∴ the equation represents a parabola

Now, the equation is $(3x - 4y)^2 = 5(4x + 3y + 12)$

Clearly, the lines $3x - 4y = 0$ and $4x + 3y + 12 = 0$ are perpendicular to each other. So let

$$\frac{3x-4y}{\sqrt{3^2+(-4)^2}} = Y, \quad \frac{4x+3y+12}{\sqrt{4^2+3^2}} = X \qquad ...(i)$$

The equation of the parabola becomes $Y^2 = X = 4 \cdot \frac{1}{4} X$

∴ Here a = 1/4 in the standard equation as $\ell = 2a = 1/2$

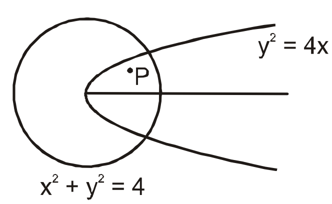
$\Rightarrow \quad 6\ell = 3$

Q.7 (0)

The point P(–2a, a + 1) will be an interior point of both the circle $x^2 + y^2 - 4 = 0$ and the parabola $y^2 - 4x = 0$.

∴ $(-2a)^2 + (a+1)^2 - 4 < 0$

i.e. $5a^2 + 2a - 3 < 0$ $\qquad$ ...(i)



and $(a + 1)^2 - 4(-2a) < 0$

i.e. $a^2 + 10a + 1 < 0$ ...(ii)

The required values of a will satisfy both (i) and (ii)

From (i), $(5a - 3)(a + 1) < 0$

$\therefore$ by sign scheme we get $-1 < a < 3/5$ ...(iii)

Solving (ii), the corresponding equation is

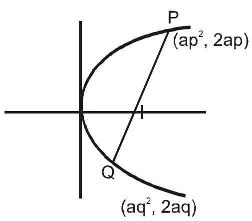
$a^2 + 10a + 1 = 0$ or $a = \dfrac{-10 \pm \sqrt{100-4}}{2} = -5 \pm 2\sqrt{6}$

$\therefore$ by sign scheme for (ii)

$-5 - 2\sqrt{6} < a < -5 + 2\sqrt{6}$ ...(iv)

The set of values of a satisfying (iii) and (iv) is $-1 < a < -5 + 2\sqrt{6}$

Q.8 (2)



slope of PQ $= \dfrac{2a(p-q)}{a(p-q)(p+q)} = 1$

$\therefore$ $p + q = 2$

Q.9 (18)

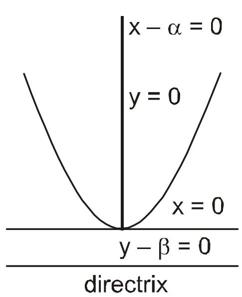
As the axis is parallel to the y-axis, it will be $x - \alpha = 0$ for some α and the tangent to the vertex (which is perpendicular to the axis) will be $y - \beta = 0$ for some β.

Hence the equation of the parabola will be of the form

$(x - \alpha)^2 = 4a(y - \beta)$ ...(i)

when α, β, a are unknown constants, 4a being latus rectum.

(1) passes through (0, 4), (1, 9) and (–2, 6) so



$(0 - \alpha)^2 = 4a(4 - \beta)$,

i.e. $\alpha^2 = 4a(4 - \beta)$ ...(ii)

and $(1 - \alpha)^2 = 4a(9 - \beta)$

i.e. $1 - 2\alpha + \alpha^2 = 4a(9 - \beta)$ ...(iii)

and $(-2 - \alpha)^2 = 4a(6 - \beta)$

i.e. $4 + 4\alpha + \alpha^2 = 4a(6 - \beta)$ ...(iv)

$\therefore$ $\alpha = -\dfrac{3}{4}$

$\therefore$ $a = \dfrac{5}{40} = \dfrac{1}{8}$ or $\beta = \dfrac{23}{8}$

$\therefore$ from (i), the equation of the parabola is

$\left(x + \dfrac{3}{4}\right)^4 = 4 \cdot \dfrac{1}{8} \cdot \left(y - \dfrac{23}{8}\right)$

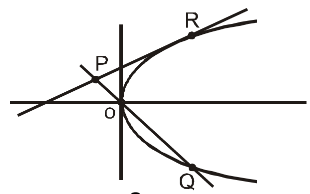
or $x^2 + \dfrac{3}{2}x + \dfrac{9}{16} = \dfrac{1}{2}y - \dfrac{23}{16}$

or $x^2 + \dfrac{3}{2}x - \dfrac{1}{2}y + 2 = 0$

$\therefore$ $2x^2 + 3x - y + 4 = 0 \Rightarrow y = 2x^2 + 3x + 4$

$\Rightarrow$ $\alpha = 2 \times 2^2 + 3 \times 2 + 4 = 18$

Q.10 (16)



$y = mx + \dfrac{a}{m}$ .... (i)

equation of OP is

$y = -\dfrac{1}{m}x$ .... (ii)

$OP = \dfrac{a/m}{\sqrt{1+m^2}}$

equation (ii) meets the parabola at Q

$\dfrac{1}{m^2}x^2 = 4ax \Rightarrow x = 4am^2,\ y = -4am$

$\therefore$ $OQ = 4am\sqrt{1+m^2}$, $OP.\ OQ = 4a^2$

Q.11 (23)

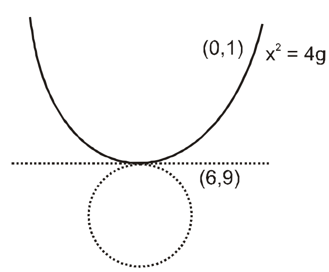
$x x_1 = 2(y + y_1)$

$6x = 2(y + 9)$

$3x = y + 9$ $3x - y - 9 = 0$

from equation of family circle is $S + \lambda L = 0$

$S \equiv (x - 6)^2 + (y - 91^2 + k(3x - y - 9) = 0$



is passes through (0, 1)

$36 + 64 + k(-10) = 0$

$100 - 10k = 0 \qquad k = 10$

$x^2 + 36 - 12x + y^2 + 81 - 18y + 30x - 30y - 90 = 0$

$x^2 + y^2 + 18x - 28y + 27 = 0$

Q.12 (3)

Equation of parabola is $y^2 = 4ax$ ....(1)

Let $A \equiv (at_1^2, 2at_1)$ $B \equiv (at_2^2, 2at_2)$, $C \equiv (at_3^2, 2at_3)$

Equation of the tangents to parabola (1) at A, B, C are

$yt_1 = x + at_1^2$ ....(2)

$yt_2 = x + at_2^2$ ....(3)

and $yt_3 = x + at_3^2$ ....(4)

Let the points of intersection of lines (2), (3) be P; (3), (4) be Q and (2), (4) be R.

Then $P \equiv (at_1t_2, a(t_1 + t_2))$, $Q \equiv (at_2t_3, a(t_2 + t_3))$, $R \equiv (at_1t_3, a(t_1 + t_3))$

Now area of ΔABC,

$$\Delta_1 = \text{modulus of } \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$$

$$= \text{modulus of } \frac{1}{2}\ .\ a.\ 2a \begin{vmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{vmatrix}$$

$= a^2\,|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

Area of ΔPQR

$$\Delta_2 = \text{modulus of } \frac{1}{2}\begin{vmatrix} at_1t_2 & a(t_1+t_2) & 1 \\ at_2t_3 & a(t_2+t_3) & 1 \\ at_3t_1 & a(t_3+t_1) & 1 \end{vmatrix}$$

$$= \text{modulus of } \frac{a^2}{2}\begin{vmatrix} t_1t_2 & t_1+t_2 & 1 \\ t_2t_3 & t_2+t_3 & 1 \\ t_3t_1 & t_3+t_1 & 1 \end{vmatrix}$$

$$= \text{modulus of } \frac{a^2}{2}\begin{vmatrix} t_2(t_1-t_3) & t_1-t_3 & 0 \\ t_3(t_2-t_1) & t_2-t_1 & 0 \\ t_3t_1 & t_3+t_1 & 1 \end{vmatrix}$$

$[R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3]$

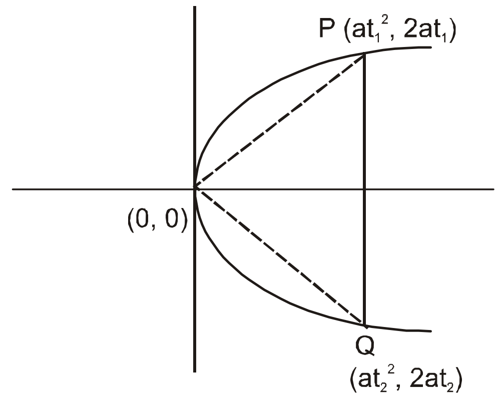
$= \text{modulus of } \frac{a^2}{2}\ (t_1 - t_3)(t_2 - t_1)(t_2 - t_3)$

$= \frac{a^2}{2}\,|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

Clearly $\frac{\Delta_1}{\Delta_2} = \frac{2}{1}$

Q.13 (4)

Equation of parabola



$y^2 = 4ax$

$OQ = \sqrt{a^2t_2^4 + 4a^2t_2^2}$

$= a\,t_2\sqrt{t_2^2 + 4}$

$QQ \geq 2\sqrt{2}a.2\sqrt{3}$

$\geq 4\sqrt{6}\,a \qquad \text{as } t_2 = t_1 - \frac{2}{t_1}$

Q.14 (3)

$y = mx - 2am - am^3$ Here a = 1

$0 = cm - 2m - m^3$

$m^3 + (2 - c)m = 0$

$m = 0 \qquad m^2 = c - 2$

$\Rightarrow c > 2$

sum $m_1 + m_2 + m_3 = 0$

$\Sigma m_1m_2 = \frac{2a - h}{a}$

$m_1m_2m_3 = \frac{-k}{a}$

$m_1m_2 = 2 - c$

$-1 = 2 - c$

$\Rightarrow c = 3$

KVPY

PREVIOUS YEAR'S

Q.1 (B)

Any normal

$y = mx - 2am - am^3$ Here a = 3/2

through $(\lambda, 0)$

$0 = m\lambda - 2am - am^3$

$m = 0,\ \lambda = 2a + am^3$

$m^2 = \frac{\lambda}{a} - 2 > 0$

$\lambda > 2a \Rightarrow \lambda > 3$

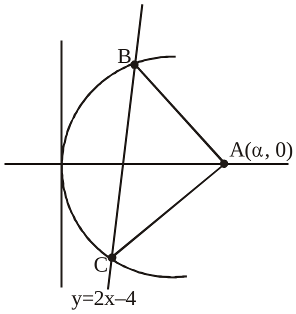
Q.2 (C)

$(2x - 4)^2 = 4x$

$(x - 2)^2 = x$

$x^2 - 5x + 4 = 0$

$x = 1, 4$



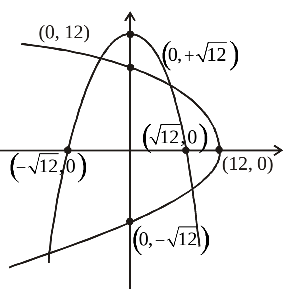
C (1, –2)

B (4, 4) $\because$ AB = AC

$$\sqrt{(\alpha-4)^2+16} = \sqrt{(\alpha-1)^2+4}$$

On solving, we get $\alpha = \frac{9}{2}$

Q.3 (D)



$x + y^2 = x^2 + y = 12$

curve (1) $x + y^2 = 12$

$y^2 = -(x - 12)$

Intersection on x-axis (12,0)

Intersection on y-axis $(0, \pm\sqrt{12})$

curve (2) $x^2 + y = 12$

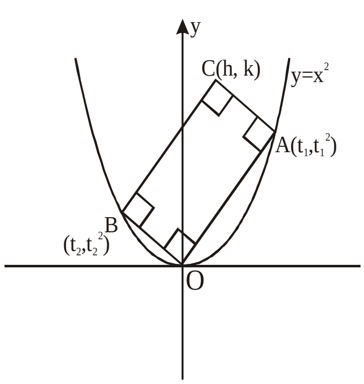
$x^2 = -(y - 12)$

Intersection on x-axis = $\left(\pm\sqrt{12}, 0\right)$

Intersection on y-axis = (0, 12)

four intersection

Q.4 (A)



$\because$ OB $\perp$ OA

So, $t_1 t_2 = -1$

Now $\frac{h}{2} = \frac{t_1 + t_2}{2}$

$t_1 + t_2 = h$ ....(i)

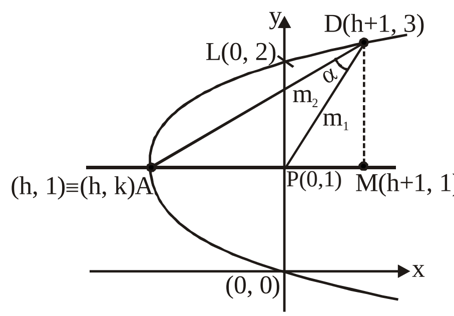
also $t_1^2 + t_2^2 = k$

$(t_1 + t_2)^2 - 2t_1t_2 = k$

$h^2 + 2 = k$

locus is $x^2 + 2 = y$

Q.5 (B)



Curve, S : $(y - k)^2 = 4(x - h)$

LLR = 4 ; Clearly k = 1 ; $\Rightarrow$A (h, 1) & 'M' is focus (h + 1, 1)

So D (h + 1, 3)

$S_{(0,0)} = 0 \Rightarrow k^2 = -4h$

$\Rightarrow h = \frac{-1}{4}$ $\Rightarrow D\left(\frac{3}{4}, 3\right)$

$$\text{Now; } \tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{8}{3} - 2}{1 + \frac{8}{3} \times 2}\right| = \frac{2}{19}$$

$$\text{where, } m_1 = \frac{3-1}{\frac{3}{4} - 0} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$$

$$m_2 = \frac{3-1}{1} = 2$$

JEE MAIN
PREVIOUS YEAR'S

Q.1 (1)

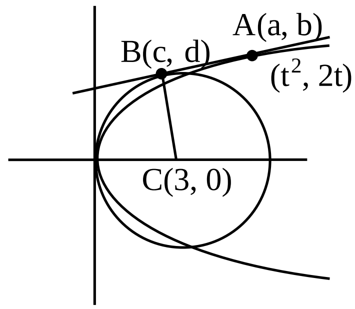
Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line 2x + y = 1)

$\therefore$ tangent is : $y = \frac{x}{y} + 3 \Rightarrow$ $x - 2y + 6 = 0$

Q.2 (9)

Equation of tangent of A



$ty = x + t^2$

$x - yt + t^2 = 0$

$$\left|\frac{3 - 0 + t^2}{\sqrt{1 + t^2}}\right| = 3$$

$(3 + t^2)^2 = 9(1 + t^2)$

$t = 0, \pm\sqrt{3}$

Point A $(3, 2\sqrt{3})$ in first quadrant

For point B foot of perpendicular from c to tangent

$$\frac{x - 3}{1} = \frac{y - 0}{-\sqrt{3}} = -\frac{(3 - 0 + 3)}{4} \Rightarrow x = \frac{3}{2}$$

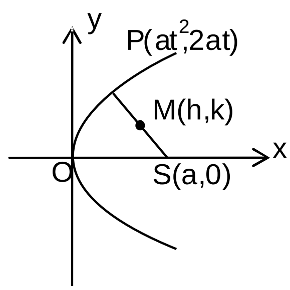
$c = \frac{3}{2}$ and $a = 3$

$2(a + c) = 9$

Q.3 (2)

$$h = \frac{at^2 + a}{2}, k = \frac{2at + 0}{2}$$

$$\Rightarrow t^2 = \frac{2h - a}{a} \text{ and } t = \frac{k}{a}$$



$$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

Q.4 **(4)**

For standard parabola

For more than 3 normals (on axis)

$x > \frac{L}{2}$ (where L is length of L.R.)

For $y^2 = 2x$

L.R. = 2

for (a, 0)

$a > \frac{\text{L.R.}}{2} \Rightarrow a > 1$

Q.5 **(1)**

Given y2 = 4x

Mirror image on y = x $\Rightarrow$ C : x2 = 4y

$$2x = 4\cdot\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2, 1)

$\Rightarrow y - 1 = 1(x - 2)$

$\Rightarrow x - y = 1$

Q.6 **(2)**

Tangent to parabola

$2y = 2(x + 6) - 20$

$\Rightarrow y = x - 4$

Condition of tangency for ellipse.

$16 = 2(1)^2 + b$

$\Rightarrow b = 14$

Option (2)

Q.7 (2)
Q.8 (1)
Q.9 [34]
Q.10 **(2)**
Q.11 **(9)**
Q.12 (2)
Q.13 **(2)**
Q.14 **(3)**
Q.15 (1)
Q.16 (2)
Q.17 **(4)**

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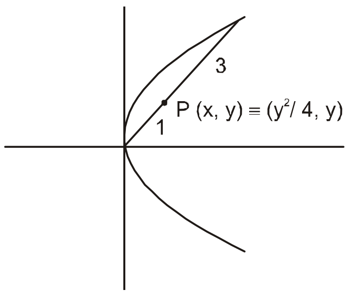
PREVIOUS YEAR'S

Q.1 **(2)**

$\because \quad \Delta_2 = \frac{\Delta_1}{2}$ (by property)

$\because \quad \frac{\Delta_1}{\Delta_2} = 2$

Q.2 **(C)**



$\Rightarrow \quad P\left(\frac{y^2}{16}, \frac{y}{4}\right)$

then locus of P is $x = y^2$

Q.3 **(A, B, D)**

Equation of normal is

$y = mx - 2m - m^3$

(9, 6) satisfies it

$6 = 9m - 2m - m^3$

$m^3 - 7m + 6 = 0 \quad \Rightarrow \quad m = 1, 2, -3$

$m = 1 \quad \Rightarrow \quad y = x - 3$

$m = 2 \quad \Rightarrow \quad y = 2x - 12$

$m = -3 \quad \Rightarrow \quad y = -3x + 33$

Q.4 **(4)**

Focus is S ≡ (2, 0). Points P ≡ (0, 0) and Q = $(2t^2, 4t)$

$$\text{Area of PQS} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 0 & 1 \\ 2t^2 & 4t & 1 \end{vmatrix}$$

$= \frac{1}{2}(8t) = 4t$ ......(i)

Q $(2t^2, 4t)$ satisfies circle

$4t^4 + 16t^2 - 4t^2 - 16t = 0$

$t^3 + 3t - 4 = 0$

$(t - 1)(t^2 + t + 4) = 0$

put t = 1 in Area of PQS.

$\Rightarrow$ Area of PQS is 4

Comprehension # 1 (Q. No. 5 to 6)

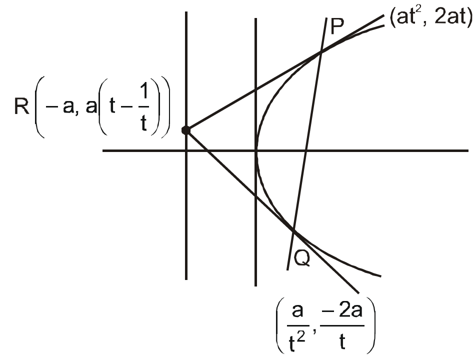
Q.5 (B)

Q.6 (D)

R lies on $y = 2x + a$

$\Rightarrow \quad a\left(t - \frac{1}{t}\right) = -a$

$t - \frac{1}{t} = -1$



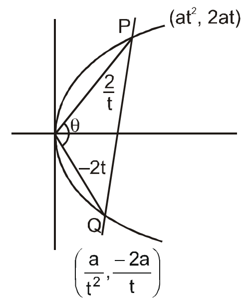
$\Rightarrow \quad \left(t + \frac{1}{t}\right)^2 = 1 + 4 = 5$

$\Rightarrow \quad PQ = a\left(t + \frac{1}{t}\right)^2 = 5a$

Sol. **(D)**

$t - \frac{1}{t} = -1$

$\Rightarrow \quad t + \frac{1}{t} = \sqrt{5}$

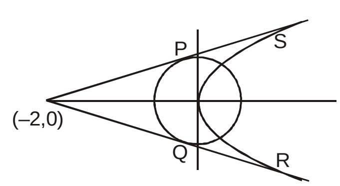


$\tan\theta = \frac{\frac{2}{t} + 2t}{1 - 4}$

$= \frac{2\left(\frac{1}{t} + t\right)}{-3} = \frac{2\sqrt{5}}{-3}$

Q.7 **(D)** $y = mx + \frac{2}{m}$

If it is tangent to $x^2 + y^2 = 2$



Then,

$$\left|\frac{\frac{2}{m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \Rightarrow \frac{4}{m^2(1+m^2)} = 2 \Rightarrow m$$

Hence equation of tangent is y = x + 2 & y = –x – 2.

Chord of contact PQ is –2x = 2 ⇒ x = – 1

Chord of contanct RS is y. 0 = 4 (x – 2) ⇒ x = 2

Hence co-ordinates of P, Q, R, S are (–1, 1) ; (–1, –1) ; (2, – 4) & (2, 4)

Area of trapezium is = $\frac{1}{2}$ (PQ + RS) × Height

= $\frac{1}{2}$ (10) × 3 = 15

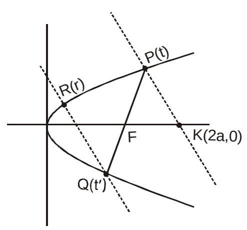
Comprehension # 2 (Q. No. 8 & 9)

Q.8 (D)

Q.9 (B)

$m_{PK} = m_{QR}$

$$\frac{2at-0}{at^2-2a} = \frac{2at'-2ar}{a(t')^2-ar^2}$$



$$\frac{t}{t^2-2} = \frac{t'-r}{(t')^2-r^2}$$

– t′ – tr² = – t – rt² – 2t′ + 2r , tt′ = –1

t′ – tr² = – t + 2r – rt²

–tr² + r(t² – 2) + t′ + t = 0

$$\lambda = \frac{\left(2-t^2\right) \pm \sqrt{\left(t^2-2\right)^2 + 4\left(-1+t^2\right)}}{-2t}$$

$$= \frac{\left(2-t^2\right) \pm \sqrt{t^4}}{-2t} = \frac{2-t^2 \pm t^2}{-2t}$$

$r = -\frac{1}{t}$

It is not possible as the R & Q will be one same.

$r = -\frac{1}{t}$ or $r = \frac{t^2-1}{t}$

(D) Ans.

Sol.9 Tangent at P is ty = x + at²

Normal at S is y + sx = 2as + as²

P ty = x + at²

Sy + sx = 2as + as²

$ty + x = 2a + \frac{a}{t^2}$

$ty = 2a + \frac{a}{t^2} - ty + at^2$

2t³y = at⁴ + 2at² + a

$$y = \frac{a\left(t^2+1\right)^2}{2t^3}$$

Q.10 **(B)**

8x – ky + (k² – 8h) = 0

2x + y – p = 0

Comparing coefficients of x, y and constant term, we get

$$4 = -k = \frac{k^2-8h}{-p}$$

k = – 4

16 – 8h = – 4p

4 – 2h = – p ⇒ p = 2h – 4

Q.11 **(A)**

For a = $\sqrt{2}$, the equation of the circle is : x² + y² = 2

Equation of tangent at (–1, 1) is: – x + y = 2

Point of contact:

$$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right) \Rightarrow \left(\frac{-\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}}\right) \Rightarrow (-1,\ 1)$$

Q.12 **(B)**

(A) $x^2 + y^2 = \frac{13}{4}$

Equation of tangent at $\left(\sqrt{3}, \frac{1}{2}\right)$ is : $x\sqrt{3} + \frac{y}{2} = \frac{13}{4}$.

∴ option (A) incorrect.

(B) Satisfying the point $\left(\sqrt{3}, \frac{1}{2}\right)$ in the curve x² + a²y² = a², we get $3 + \frac{a^2}{4} = a^2$

$$\Rightarrow \frac{3a^2}{4} = 3 \quad \Rightarrow a^2 = 4$$

∴the conic is : x² + 4y² = 4

Equation of tangent at $\left(\sqrt{3},\frac{1}{2}\right)$ is :

$$\sqrt{3}x + 2y = 4$$

Q.13 **(A)**

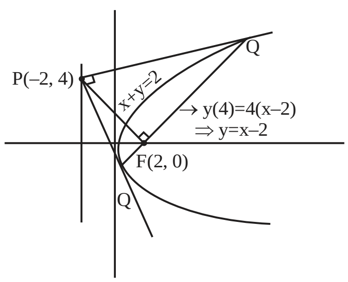
The equation of given tangent is: y = x + 8

Satisfying the point (8, 16) in the curve $y^2 = 4ax$ we get, a = 8.

Now comparing the given tangent with the general tangent to the parabola, $y = mx + \frac{a}{m}$, we get m = 1.

Point of contact is $\left(\frac{a}{m^2},\frac{2a}{m}\right) \Rightarrow (8,16)$

Q.14 (A,B,D)



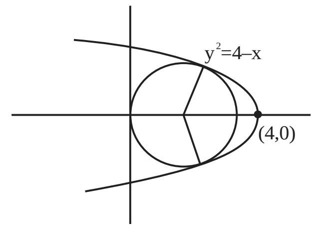
Note that P lies on directrix so triangle PQQ′ is right angled, hence QQ′ passes through focus F.

$PF = 4\sqrt{2}$

Equation of QF is y=x–2 & PFis x+y=2

Hence, A,B,D

Q.15 (1.50)



Let the circle be

$x^2 + y^2 + \lambda x = 0$

For point of intersection of circle & parabola $y^2 = 4 - x$

$x^2 + 4 - x + \lambda x = 0 \Rightarrow x^2 + x(\lambda - 1) + 4 = 0$

For tangency : $\Delta = 0 \Rightarrow (\lambda-1)^2 - 16 = 0 \Rightarrow \lambda = 5$ (rejected)

or $\lambda = -3$

Circle : $x^2 + y^2 - 3x = 0$

Radius $= \frac{3}{2} = 1.5$

Q.16 (2.00)

For point of intersection :

$x^2 - 4x + 4 = 0 \Rightarrow x = 2$ so $\alpha = 2$

Ellipse

EXERCISES

Q.1 (2)

$ae = 2 \Rightarrow a = \frac{2}{e} = \frac{2}{1/2} = 4$

$b^2 = a^2 (1 - e^2) = 16 (1 - 1/4)$

Now equaiton is $\frac{x^2}{16} + \frac{y^2}{16\left(1 - \frac{1}{4}\right)} = 1$

i.e. $\frac{x^2}{16} + \frac{y^2}{12} = 1$

Q.2 (2)

$9x^2 + 5(y^2 - 6y + 9) = 45$

$\Rightarrow \frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$

$a^2(1-e^2) = b^2$

$\Rightarrow 9(1 - e^2) = 5$

$\Rightarrow 1 - e^2 = \frac{5}{9} \Rightarrow e^2 = \frac{4}{9} \Rightarrow e = \frac{2}{3}$

Q.3 (3)

$a = 6, b = 2\sqrt{5}$

$b^2 = a^2(1 - e^2)$ $\frac{20}{36} = (1 - e^2) \Rightarrow e = \sqrt{\frac{16}{36}} = \frac{2}{3}$

But directrices are $x = \pm\frac{a}{e}$

Hence distance between them is $2.\frac{6}{2/3} = 18$.

Q.4 (2)

$\frac{x^2}{(48/3)} + \frac{y^2}{(48/4)} = 1$

$a^2 = 16, b^2 = 12 \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}$

Distance is $2ae = 2 \cdot 4 \cdot \frac{1}{2} = 4$.

Q.5 (2)

Vertex (0,7), directrix $y = 12$, $\therefore$ $b = 7$

Also $\frac{b}{e} = 12 \Rightarrow e = \frac{7}{12}, a = 7\sqrt{\frac{95}{144}}$

Hence equation of ellipse is $144x^2 + 95y^2 = 4655$.

Q.6 (2)

$\frac{x^2}{4} + \frac{y^2}{3} = 1$. Latus rectum $= \frac{2b^2}{a} = 3$

Q.7 (1)

The equation of the ellipse is $16x^2 + 25y^2 = 400$

or $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Here $a^2 = 25, b^2 = 16 \Rightarrow e = \frac{3}{5}$.

Hence the foci are $(\pm 3, 0)$.

Q.8 (1)

Let point P (x_1, y_1)

So, $\sqrt{(x_1 + 2)^2 + y_1^2} = \frac{2}{3}\left(x_1 + \frac{9}{2}\right)$

$\Rightarrow (x_1 + 2)^2 + y_1^2 = \frac{4}{9}\left(x_1 + \frac{9}{2}\right)^2$

$\Rightarrow 9[x_1^2 + y_1^2 + 4x_1 + 4] = 4\left(x_1^2 + \frac{81}{4} + 9x_1\right)$

$\Rightarrow 5x_1^2 + 9y_1^2 = 45 \Rightarrow \frac{x_1^2}{9} + \frac{y_1^2}{5} = 1$,

Locus of (x_1, y_1) is $\frac{x^2}{9} + \frac{y^2}{5} = 1$, which is equation of an ellipse.

Q.9 (3)

In the first case, eccentricity $e = \sqrt{1 - (25/169)}$

In the second case, $e' = \sqrt{1 - (b^2/a^2)}$

According to the given condition,

$\sqrt{1 - b^2/a^2} = \sqrt{1 - (25/169)}$

$\Rightarrow b/a = 5/13$, $(\because a > 0, b > 0)$

$\Rightarrow a/b = 13/5$.

Q.10 (2)

$4(x-2)^2 + 9(y-3)^2 = 36$

Hence the centre is (2, 3).

Q.11 (1)

The ellipse is $4(x-1)^2 + 9(y-2)^2 = 36$

Therefore, latus rectum $= \frac{2b^2}{a} = \frac{2.4}{3} = \frac{8}{3}$

Q.12 (2)

Foci =(3, –3) $\Rightarrow$ ae 3 – 2 = 1

Vertex $= (4,-3) \Rightarrow a = 4-2 = 2 \Rightarrow e = \frac{1}{2}$

$\Rightarrow b = a\sqrt{\left(1-\frac{1}{4}\right)} = \frac{2}{2}\sqrt{3} = \sqrt{3}$

Therefore, equation of ellipse with centre $(2,-3)$ is

$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1.$

Q.13 (2) Check $\Delta \neq 0$ and $h^2 < ab$.

Q.14 (1)

$\frac{(x+1)^2}{\frac{225}{25}} + \frac{(y+2)^2}{\frac{225}{9}} = 1$

$a = \sqrt{\frac{225}{25}} = \frac{15}{5}, b = \sqrt{\frac{225}{9}} = \frac{15}{3} \Rightarrow$

$e = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Focus $= \left(-1, -2 \pm \frac{15}{3}.\frac{4}{5}\right) = (-1, -2 \pm 4)$

$= (-1, 2);\ (-1, -6)$.

Q.15 (3) $3x^2 - 12x + 4y^2 - 8y = -4$

$\Rightarrow 3(x-2)^2 + 4(y-1)^2 = 12$

$\Rightarrow \frac{(x-2)^2}{4} + \frac{(y-1)^2}{3} = 1 \Rightarrow \frac{X^2}{4} + \frac{Y^2}{3} = 1$

$\therefore\ e = \sqrt{1-\frac{3}{4}} = \frac{1}{2}$. $\therefore$ Foci are $\left(X = \pm 2 \times \frac{1}{2}, Y = 0\right)$

i.e., $(x-2 = \pm 1,\ y-1 = 0) = (3,1)$ and $(1,1)$.

Q.16 (3)

Given equation of ellipse is ,

$25x^2 + 9y^2 - 150x - 90y + 225 = 0$

$\Rightarrow 25(x-3)^2 + 9(y-5)^2 = 225$

$\Rightarrow \frac{(x-3)^2}{9} + \frac{(y-5)^2}{25}$

= 1. Here $b > a$

$\therefore$ Eccentricity $e = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Q.17 (3)

Coordinates of any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angle is θ are $(a\cos\theta,\ b\sin\theta)$.

The coordinates of the end points of latus recta are $\left(ae, \pm\frac{b^2}{a}\right)$. $\therefore a\cos\theta = ae$ and $b\sin\theta = \pm\frac{b^2}{a}$

$\Rightarrow \tan\theta = \pm\frac{b}{ae} \Rightarrow \theta = \tan^{-1}\left(\pm\frac{b}{ae}\right)$.

Q.18 (2)

$\because ae = \pm\sqrt{5} \Rightarrow a = \pm\sqrt{5}\left(\frac{3}{\sqrt{5}}\right) = \pm 3 \Rightarrow a^2 = 9$

$\therefore b^2 = a^2(1-e^2) = 9\left(1-\frac{5}{9}\right) = 4$

Hence, equation of ellipse

$\frac{x^2}{9} + \frac{y^2}{4} = 1 \Rightarrow 4x^2 + 9y^2 = 36$

Q.19 (1)

Centre is (3, 0), a = 8, b = $\sqrt{64\left(1-\frac{1}{4}\right)} = 4\sqrt{3}$

Now x = 3 + 8 cosθ

y = $4\sqrt{3}$ sinθ

(3 + 8cosθ , $4\sqrt{3}$ sinθ)

Q.20 (1)

Since $S_1 > 0$. Hence the point is outside the ellipse.

Q.21 (2)

$y = 3x \pm \sqrt{\frac{3.5}{3.4}, 9 + \frac{5}{3} \times \frac{4}{4}}$

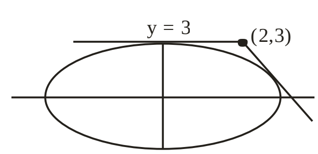
$\Rightarrow y = 3x \pm \sqrt{\frac{155}{12}}$

Q.22 (1)

From the given options it can the easily said

Alternative :

$\frac{x^2}{16} + \frac{x^2}{9} = 1$



As pair of lines of $T^2 = SS_1$

$$\left(\frac{x}{8}+\frac{y}{3}=1\right)^2=\left(\frac{x^2}{16}+\frac{y^2}{9}=1\right)\left(\frac{1}{4}+1-1\right)$$

$$\Rightarrow \frac{x^2}{64}+\frac{y^2}{9}+1-\frac{x}{4}-\frac{2y}{3}+\frac{xy}{12}$$

$$=\frac{x^2}{64}+\frac{y^2}{36}-\frac{1}{4}$$

$$\Rightarrow \frac{y^2}{12}-\frac{2y}{3}-\frac{x}{4}+\frac{xy}{12}+\frac{5}{4}=0$$

$\Rightarrow (y-3)(x+y-5)=0$

Q.23 (4)

By symmetry the quadrilateral is a rhombus. So area is four times the area of the right angled triangle formed by the tangent and axes in the Ist quadrant.

Now, $ae=\sqrt{a^2-b^2}\Rightarrow ae=2$

$\Rightarrow$Tangent (in first quadrant) at end of latus rectum $\left(2,\frac{5}{3}\right)$ is $\frac{2}{9}x+\frac{5}{3}\frac{y}{5}=1$

i.e., $\frac{x}{9/2}+\frac{y}{3}=1$

Area $=4.\frac{1}{2}.\frac{9}{2}.3=27$ sq. unit.

Q.24 (1)

$y=\frac{-l}{m}x+\frac{n}{m}$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if

$\frac{n}{m}=\pm\sqrt{b^2+a^2\left(\frac{l}{m}\right)^2}$ or $n^2=m^2b^2+l^2a^2$.

Q.25 (3)

$SS_1=T^2$

$\tan\theta=2\frac{\sqrt{h^2-ab}}{a+b}$, $a=9$, $b=-4$ and $h=-12$.

Q.26 (3)

The locus of point of intersection of two perpendicular tangents drawn on the ellipse is $x^2+y^2=a^2+b^2$, which is called 'director-circle'.

Given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$, $\therefore$Locus is $x^2+y^2=13$.

Q.27 (3)

Change the equation $9x^2+5y^2-30y=0$ in standard form $9x^2+5(y^2-6y)=0$

$\Rightarrow 9x^2+5(y^2-6y+9)=45\Rightarrow\frac{x^2}{5}+\frac{(y-3)^2}{9}=1$

$\because a^2<b^2$, so axis of ellipse on *y*-axis.

At *y* axis, put x =0, so we can obtained vertex.

Then $0+5y^2-30y=0\Rightarrow y=0,\ y=6$

Therefore, tangents of vertex y =0, y = 6.

Q.28 4)

For $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, equation of normal at point (x_1,y_1),

$$\Rightarrow \frac{(x-x_1)a^2}{x_1}=\frac{(y-y_1)b^2}{y_1}$$

$\therefore (x_1,y_1)\equiv(0,3),\ a^2=5,\ b^2=9$

$\Rightarrow \frac{(x-0)}{0}\ 5=\frac{(y-3).9}{3}$ or x = 0 i.e. , *y*-axis.

Q.29 (1)

Given, equation of ellipse is

$4x^2+9y^2=36$

Tangent at point (3,–2) is $\frac{(3)x}{9}+\frac{(-2)y}{4}=1$ or

$\frac{x}{3}-\frac{y}{2}=1$

$\therefore$ Normal is $\frac{x}{2}+\frac{y}{3}=k$ and it passes through point (3,–2)

$\therefore \frac{3}{2}-\frac{2}{3}=k\Rightarrow k=\frac{5}{6}$

$\therefore$ Normal is, $\frac{x}{2}+\frac{y}{3}=\frac{5}{6}$

Q.30 (1)

We know that the equation of the normal at point (a sinθ, b cos θ) on the curve $x^2+\frac{y^2}{4}=1$ is given by

$$-\frac{ax}{\sin\theta}+\frac{by}{\cos\theta}=-a^2+b^2$$

$$\Rightarrow -\frac{1\cdot x}{\sin\theta}+\frac{2y}{\cos\theta}=3 \qquad ....(i)$$

Comparing equation (i) with $2x-\frac{8}{3}\lambda y=-3$. We get,

$$-\frac{1}{2\sin\theta}=-\frac{2\cdot 3}{8\lambda\cos\theta}=-\frac{3}{3}$$

$\Rightarrow \sin\theta \ \frac{1}{2}$ and $\cos\theta = \frac{3}{4\lambda}$

$\Rightarrow \pm\frac{\sqrt{3}}{2} = \frac{3}{4\lambda}$

$\Rightarrow \lambda = \pm\frac{\sqrt{3}}{2}$

$a\sin\theta = 2$, $b\,\text{cosec}\theta = \frac{8}{3}\lambda$ or $ab = \frac{16}{3}\lambda$ .....(ii)

$\because a = 1, b = 2$; $\therefore 2 = \frac{16}{3}\lambda$ or $\lambda = 3/8$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (1)
PS = ePM

$$\sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2}\left|\frac{x-y-3}{\sqrt{1^2+1^2}}\right|$$

Squaring, we have
$7x^2 + 7y^2 + 7 - 10x + 10y + 2xy = 0$

Q.2 (4)
$4x^2 + 9y^2 + 8x + 36y + 4 = 0$
$4(x^2 + 2x + 1) + 9[y^2 + 4y + 4] = 36$
$4(x+1)^2 + 9(y+2)^2 = 36$

$$\frac{(x+1)^2}{9} + \frac{(y+2)^2}{4} = 1$$

$$\Rightarrow e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

Q.3 (3)

$2 \times \frac{a}{e} = 3 \times 2ae$

$e^2 = \frac{1}{3} \Rightarrow e = \frac{1}{\sqrt{3}}$

Q.4 (2)

$\frac{x^2}{r-2} + \frac{y^2}{5-r} = 1$ For ellipse
$2 < r < 5$

Q.5 (3)
$9x^2 + 4y^2 = 1$

$\frac{x}{1/9} + \frac{y^2}{1/4} = 1 \Rightarrow$ Length of latusrectun $= \frac{2a^2}{b} = \frac{4}{9}$

Q.6 (1)

$e = \frac{5}{8}$; $2ae = 10 \Rightarrow 2a = \frac{10}{e} \Rightarrow 2a = 16$

Latus rectum $= \frac{2b^2}{a} = \frac{2a^2(1-e^2)}{a}$

$= 2a(1 - e^2) = 16\left(1 - \frac{26}{64}\right) = \frac{39}{4}$

Q.7 (1)
$x = 3(\cos t + \sin t)$ $y = 4(\cos t - \sin t)$

$\Rightarrow \frac{x}{3} = \cos t + \sin t$; $\frac{y}{4} = \cos t - \sin t$

square & add $\frac{x^2}{9} + \frac{y^2}{16} = 2$

Ellipse Equation $\frac{x^2}{18} + \frac{y^2}{32} = 1$

Q.8 (3)
$F_1(3, 3)$; $F_2(-4, 4)$
$2ae = F_1F_2$

$2ae = \sqrt{(3+4)^2 + (3-4)^2}$

$2ae = 5\sqrt{2}$ ....(1)
mid point of P_1P_2 will be centre of ellipse

centre $\left(-\frac{1}{2}, \frac{7}{2}\right)$

Ellipse $\frac{\left(x+\frac{1}{2}\right)^2}{a^2} + \frac{\left(y-\frac{7}{2}\right)^2}{b^2} = 1$

Passing through origin $\frac{1}{4a^2} + \frac{49}{4b^2} = 1$ ...(2)

From (1) and (2) $e = \frac{5}{7}$

Q.9 (2)

Max. area $= \frac{1}{2} \times 2ae \times b = \frac{1}{2} \times 2 \times 3 \times 4 = 12$

Q.10 (3)
$4(x^2 - 4x + 4) + 9(y^2 - 64 + 9) = 36$
$4(x-2)^2 + 9(y-3)^2 = 36$

$\frac{(x-2)^2}{9} + \frac{(y-3)^2}{4} = 1.$

Equation of major axis y = 3.
Equation of minor axis x = 2

Q.11 (2)
Let P $(a\cos\theta, b\sin\theta)$
OP = 2
$\Rightarrow OP^2 = 4$
$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta = 4$
$\Rightarrow 6\cos^2\theta + 2\sin^2\theta = 4$

$\cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \pm\frac{\pi}{4}$

Q.12 (4)

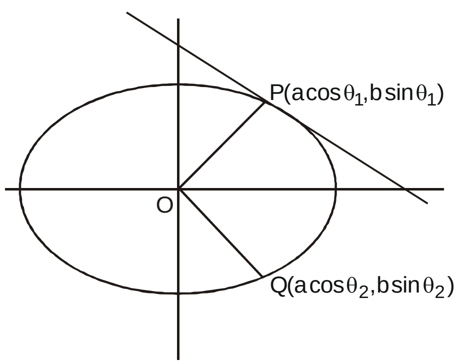
$\frac{de}{dt} = 0.1$

$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{3}{4}$

e = 0.1 t + c ⇒ e = 1/2
when t = 0, e = 1/2
⇒ c = 0.5
e = 0.1 t + 0.5
ellipse become auxiliary circle where e → 1
1 = 0.1 t + 0.5 ⇒ t = 5 sec.

Q.13 (2)

$M_{OP} = \frac{b\sin\theta_1}{a\cos\theta_1} = \frac{b}{a}\tan\theta_1$



$\tan\theta_1 \tan\theta_2 = -\frac{a^2}{b^2}$

$M_{OQ} = \frac{b}{a}\tan\theta_2$

$M_{OP} \times M_{OQ} = \frac{b^2}{a^2}\tan\theta_1 \tan\theta_2$

$= \left(\frac{b^2}{a^2}\right)\left(\frac{-a^2}{b^2}\right) = -1$

So right angle at centre.

Q.14 (2)
Let eccentric angle be θ, then equation of tangent is

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ...(1)

given equation is

$\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ ...(2)

comparing (1) and (2)

$\cos\theta = \sin\theta = \frac{1}{\sqrt{2}}$

⇒ θ = 45°

Q.15 (2)

$C = \pm\sqrt{8\times 4 + 4} = \pm 6$

Q.16 (4)
$3x^2 + 4y^2 = 1$
$3xx_1 + 4yy_1 = 1$
given $3x + 4y = -\sqrt{7}$
comparing

$\because \frac{3x_1}{3} = \frac{4y_1}{4} = \frac{1}{-\sqrt{7}}$

$x_1 = -\frac{1}{\sqrt{7}}$

$y_1 = -\frac{1}{\sqrt{7}}$

Q.17 (4)
Equation of normal
$ax\sec\phi - by\,\mathrm{cosec}\phi = a^2 - b^2$
...(1)
$x\cos\alpha + 4\sin\alpha = p$
...(2)

$\frac{a\sec\varphi}{\cos\alpha} = \frac{-by\,\mathrm{cosec}\,\varphi}{\sin\alpha} = \frac{a^2 - b^2}{p}$

$\Rightarrow \cos\phi = \frac{ap}{(a^2 - b^2)} \times \sec\alpha$ ...(3)

$\Rightarrow \sin\phi = \frac{-bp}{(a^2 - b^2)} \times \mathrm{cosec}\,\alpha$ ...(4)

squaring and adding

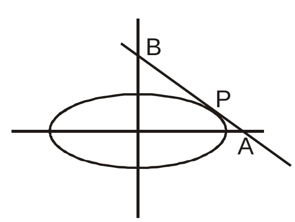
$1 = \frac{p^2}{(a^2 - b^2)^2}[a^2\sec^2\alpha + b^2\mathrm{cosec}^2\alpha]$

Q.18 (1)

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Let the point P(4 cosθ, 3 sinθ)
Tangent at P

$\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$



$A\left(\frac{4}{\cos\theta},0\right)$; $B\left(0,\frac{3}{\sin\theta}\right)$

Let the middle point M(h, k)

$2h = \frac{4}{\cos\theta} \Rightarrow \cos\theta = \frac{2}{h}$

$2k = \frac{3}{\sin\theta} \Rightarrow \sin\theta = \frac{3}{2k}$

square & add

$\frac{4}{h^2} + \frac{9}{4k^2} = 1$

$16k^2 + 9h^2 = 4h^2k^2$

$16y^2 + 9x^2 = 4x^2y^2$

Q.19 (2)

$y = mx \pm \sqrt{(a^2+b^2)m^2+b^2}$

....(1)

$y = mx \pm \sqrt{a^2m^2+(a^2+b^2)}$

.....(2)

Eq^n (1) and (2) are same

$(a^2 + b^2)\, m^2 + b^2 = a^2 m^2 + a^2 + b^2$

$m^2 = a^2/b^2 \Rightarrow m = \pm a/b$

$\Rightarrow by = ax \pm \sqrt{a^4+b^4+a^2b^2}$

Q.20 (2)

Equation of normal $\frac{a^2x}{ae} - \frac{b^2ya}{b^2} = a^2 - b^2$

$\frac{ax}{e} - ay = a^2 - b^2$

$x - ey = ae^3$

Q.21 (3)

$\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$ ......(1)

$x^2 + y^2 = a^2$

$ax\cos\phi + ay\sin\phi = a^2$

$x\cos\phi + y\sin\phi = a$

$\frac{x}{a}\cos\phi + \frac{y}{a}\sin\phi = 1$ ......(2)

Solving (1) and (2) y = 0

Q.22 (4)

$3x^2 + 5x^2 = 15$

$\frac{x^2}{5} + \frac{y^2}{3} = 1$

Equation of director circle.

$x^2 + y^2 = 5+3 = 8$

clearly (2, 2) lies on it

here $\angle\theta = \frac{\pi}{2}$

Q.23 (2)

$ax\sec\theta - by\,\text{cosec}\theta = a^2 - b^2$

slope = $\frac{a\sec\theta}{b\,\text{cosec}\theta} = \frac{5}{3}$

$\frac{\sec\theta}{\text{cosec}\theta} = 1$

$\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

Q.24 (3)

$P(a\cos\theta, b\sin\theta)$

Normal at P ; $ax\sec\theta - by\,\text{cosec}\theta = a^2 - b^2$

$R\left(\frac{a^2-b^2}{a\sec\theta},0\right)$

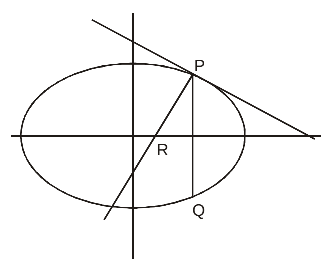
Let mid point of PR is M(h, k)

$2h = \frac{a^2-b^2}{a\sec\theta} + a\cos\theta$

$\cos\theta = \frac{2ha}{2a^2-b^2}$ ....(1)

$2k = b\sin\theta$

$\Rightarrow \sin\theta = \frac{2k}{b}$ ....(2)



Square & odd

$\frac{4h^2a^2}{(2a^2-b^2)^2} + \frac{4k^2}{b^2} = 1$

$\frac{4a^2x^2}{(2a^2-b^2)^2} + \frac{4y^2}{b^2} = 1$ Ellipse

Q.25 (2)

Ellipse $-2x^2 + 5y^2 = 20$, mid point (2, 1)

using $T = S_1$

$2x(2) + 5(y \times 1) - 20 = 2(2)^2 + 5(1)^2 - 20$

$4x + 5y = 13$

Q.26 (1)

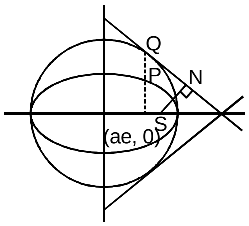
$P(a\cos\alpha, b\sin\alpha)$

$Q(a\cos\alpha, a\sin\alpha)$

Tangent at Q point

$x\cos\alpha + y\sin\alpha = a$

SN $= |ae(\cos\alpha - a)|$



$$SP = \sqrt{(ae - a\cos\alpha)^2 + b^2 \sin^2\alpha}$$

$$= \sqrt{a^2e^2 + a^2\cos^2\alpha - 2a^2e\cos\alpha + b^2 - b^2\cos^2\alpha}$$

$$= \sqrt{a^2 + \cos^2\alpha(a^2 - b^2) - 2a^2e\cos\alpha}$$

$= |ae\cos\alpha - a|$

$\Rightarrow$ SP = SN

Q.27 (**1**)

Same as Previous Question.

Ans.(A) Isosceles triangle

Q.28 (2)

$(S_1 F_1) \cdot (S_2 F_2) = b^2 = 3$

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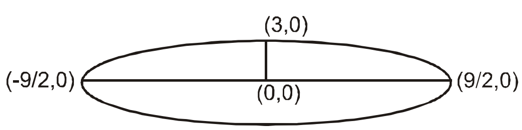
OBJECTIVE QUESTIONS

Q.1 (B)

Equation of ellipse correspoinding to given bridge is

$$\frac{x^2}{\left(\frac{9}{2}\right)^2} + \frac{y^2}{(3)^2} = 1$$



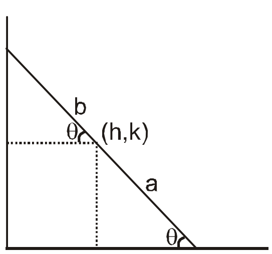
Height of pillar will be y co-ordinat of point on ellipes having x = 2

$$\therefore \frac{(2)^2}{(9/2)^2} + \frac{y^2}{9} = 1 \quad \Rightarrow y = \frac{\sqrt{65}}{3} \simeq \frac{8}{3}$$

Q.2 (A)

Let the fixed lines are co-ordinate axes

from diagram $h = b\cos\theta$



$k = a\sin\theta$

$$\Rightarrow \frac{h^2}{b^2} + \frac{k^2}{a^2} = 1 \rightarrow \text{which is ellipse}$$

Q.3 (A)

$$4\tan\frac{B}{2}\tan\frac{C}{2} =$$

$$4\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = 1$$

$$\Rightarrow 4\frac{(s-a)}{s} = 1$$

$$\Rightarrow s = \frac{4a}{3} = 4 \times \frac{6}{3} = 8$$

but $2s = a + b + c = 16$

$b + c = 10$

Hence locas is an ellipse having center $\equiv (5, 0)$

$2ae = 6$ and $2a = 10$

$b^2 = a^2 - a^2e^2 = 25 - 9 = 16$

$\therefore$ Equation of ellipse

$$\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$$

Q.4 (A)

Given that:

$$\frac{2b^2}{a} = a + b$$

$2b^2 = a^2 + ab$

$b^2 - a^2 = ab - b^2$

$\Rightarrow (b - a)(b + a + b) = 0$

$b = a$

$\Rightarrow$ ellipse becomes a circle

Q.5 (C)

$lx + my + n = 0$ ...(1)

$$|\alpha - \beta| = \frac{\pi}{2}$$

$$\frac{x}{2}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right) \quad ...(2)$$

Equation (1) and (2) are same line of chord

$$\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{a\ell} = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{bm} = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{-n} = \frac{-1}{\sqrt{2}n}$$

$$\cos\left(\frac{\alpha+\beta}{2}\right) = -\frac{a\ell}{\sqrt{2}n} \; ; \; \sin\left(\frac{\alpha+\beta}{2}\right) = \frac{-bm}{\sqrt{2}n}$$

Square and add $\dfrac{a^2\ell^2}{2n^2} + \dfrac{b^2m^2}{2n^2} = 1$

$a^2\ell^2 + b^2m^2 = 2n^2$

Q.6 (A)

$2y = x + 4$

$$y = \frac{x}{2} + 2 \Rightarrow M = \frac{1}{2}$$

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$2 = \pm \sqrt{4m^2 + b^2}$

$\Rightarrow b^2 = 3 \Rightarrow b = \pm\sqrt{3}$

$\Rightarrow \frac{1}{m} = \pm\sqrt{4m^2 + 3}$

$\Rightarrow \frac{1}{m^2} = 4m^2 + 3$

$\Rightarrow 4m^4 + 3m^2 - 1 = 0$

$\Rightarrow m = \pm\frac{1}{2}$

Hence $y = -\frac{1}{2}x - 2,\ 2y + x + y = 1$

Q.7 (A)

tangent

$\frac{x}{a}\cos\frac{\pi}{4} + \frac{y}{b}\sin\frac{\pi}{4} = 1$

$P_1 = \frac{1}{\sqrt{\frac{1}{2a^2} + \frac{1}{2b^2}}} = \frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

Normal

$ax\sec\frac{\pi}{4} - by\cos\frac{\pi}{4} = a^2 - b^2$

$P_2 = \frac{a^2 - b^2}{\sqrt{2}\sqrt{(a^2+b^2)}}$

$\Rightarrow$ Area $= P_1P_2 = \frac{(a^2-b^2)ab}{a^2+b^2}$

Q.8 (C)

$ax\sec\theta - by\,\text{cosec}\,\theta = a^2 - b^2$

$Q \equiv \left(\frac{a^2-b^2}{a}\cos\theta, 0\right) \quad R \equiv \left(0, \frac{-a^2-b^2}{b}\sin\theta\right)$

mid Pt. is (h, k)

$h = \frac{a^2-b^2}{2a}\cos\theta,\ k = \frac{-(a^2-b^2)}{2b}\sin\theta$

$e' = \sqrt{\frac{1-b^2}{a^2}} = e$

Q.9 (C)

Locus of point ‘A’ will be director circle at given ellipse

hence $x^2 + y^2 = a^2 + b^2$

Q.10 (C)

Equation of normal at $P(x_1, y_1)$

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$

$T(x_1e^2, 0)\quad \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$

$y_1^2 = \frac{b^2}{a^2}(a^2 - x_1^2)$

$PT = \sqrt{(x_1 - x_1e^2)^2 + {y_1}^2} = (1-e^2)(a^2 - x_1^2)$

$= \sqrt{x_1^2(1-e^2)^2 + y_1^2}$

$= \frac{b}{a}\sqrt{a^2 - x_1^2e^2}$

$= \frac{b}{a}\sqrt{rr_1}$

$r = a + ex_1$; $r_1 = a - ex_1$

Q.11 (D)

Point of intersection at tangent at point having eccentric angle ‘α’ & ‘β’ is

$h = \frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$k = \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$\because \alpha + \beta$ = constant (let k)

hence $\frac{h}{k} = \frac{a}{b\tan k}$

hence locus is straight line.

Q.12 (B)

Equation of chord of contact at A(4, 3)

$\frac{x}{4} + \frac{y}{3} = 1$

Slope of line EF is $\frac{-3}{4}$

Equation of EF, (EF is tangent of ellipse)

$y = mx + \sqrt{a^2m^2 + b^2}$

$y = \frac{-3}{4}x + \sqrt{16\cdot\frac{9}{16} + 9}$

$y = \frac{-3}{4}x + \sqrt{18}$

EF, $3x + 4y - 4\sqrt{18} = 0$

$$d = \left|\frac{12+12-4\sqrt{18}}{5}\right| = \left|\frac{24-4\sqrt{18}}{5}\right|$$

Q.13 (B)
Point P lies on the director circle
$\Rightarrow$ P, Q and the centre of the ellipse are collinear.
$\Rightarrow$ equation of PQ is $2x - y = 0$]

Q.12 (B)

$$h = \frac{2+2+3\sqrt{2}\cos\theta}{2} \text{ and}$$

$$k = \frac{3+3+3\sqrt{2}\cos\theta}{2}$$

$\therefore$ $(2h-4)^2 + (2k-6)^2 = 18.$

Q.13 (C)
Standard result

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MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A,C,D)
By Definition

Q.2 (A,B,C,D)
$3(x-3)^2 + 4(y+2)^2 = C$
if $C = 0$ a point
if $C > 0$ ellipse
if $C < 0$ no locus.

Q.3 (B,D)

$$2ae = \frac{2b^2}{a}$$

$a^2e = b^2$

$$e = \frac{b^2}{a^2} = 1 - e^2$$

$e^2 + e - 1 = 0$

$$e = \frac{-1\pm\sqrt{5}}{2} \qquad (\because\ 0 < e < 1)$$

$$e = \frac{\sqrt{5}-1}{2}$$

Q.4 (A,B,C)
(A) Direction circle $x^2 + y^2 = a^2 + b^2 = 9 + 5 = 14$
(B) By definition $2.b = 12$
(C)

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \sqrt{\frac{(s-2ae)(s-b)}{s(s-a)}}\sqrt{\frac{(s-2ae)(s-a)}{s(s-b)}}$$

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

$$= \frac{s-2ae}{s} = \frac{a+ae-2ae}{a+ae} = \frac{a-ae}{a+ae} = \frac{1-e}{1+e}$$

Q.5 (A,B)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

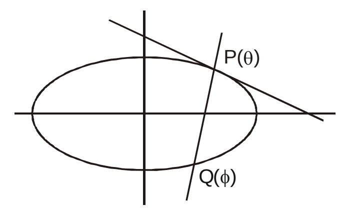
chord PQ :

$$\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

If it is passes through point (d, 0) on axis

$$\frac{d}{a}\cos\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

$$\frac{d}{a} = \frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta+\phi}{2}\right)}$$



C & D

$$\frac{d-a}{d+a} = \frac{\cos\left(\frac{\theta-\phi}{2}\right)-\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)+\cos\left(\frac{\theta+\phi}{2}\right)}$$

$$= \frac{2\sin\frac{\theta}{2}\sin\frac{\phi}{2}}{2\cos\frac{\theta}{2}\cos\frac{\phi}{2}}$$

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{d-a}{d+a}$$

$$d = ae \Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{ae-a}{ae+a} = \frac{e-1}{e+1}$$

$$d = -ae \Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{-ae-a}{-ae+a} = \frac{e+1}{e-1}$$

Q.6 (A,C)

$$2x - \frac{8}{3}\lambda y = -3$$

$$\frac{8}{3}\lambda y = 2x + 3$$

$$y = \left(\frac{3}{4\lambda}\right)x + \left(\frac{9}{8\lambda}\right)$$

$$m = \frac{3}{4\lambda},\ c = \frac{9}{8\lambda}$$

condition of normal

$$c = \frac{-(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$$

$$\frac{9}{8\lambda} = -\frac{[-3].m}{\sqrt{1+4m^2}} \text{ but } \qquad m = \frac{3}{4}\lambda$$

solving

$$\lambda = \pm\frac{\sqrt{3}}{2}$$

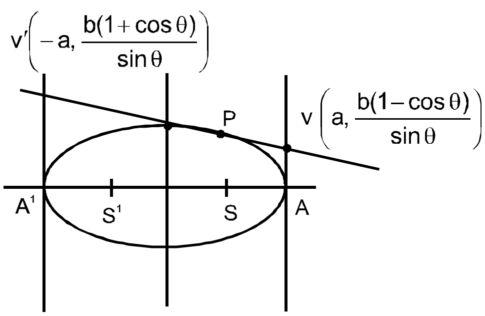
Q.7 (A,B,C,D)

Tangent drawn from points lying on director circle are mutually perpendicular

Equation of director circle given ellipse $\frac{x^2}{4}+\frac{y^2}{5} = 1$ is $x^2 + y^2 = 9$

All points $(1, 2\sqrt{2})$, $(2\sqrt{2}, 1)$, $(2, \sqrt{5})$, $(\sqrt{5}, 2)$ lies on it.

Q.8 (A,C,D)



$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

$$\frac{y}{b}\sin\theta = 1 - \cos\theta \Rightarrow y = \frac{b(1-\cos\theta)}{\sin\theta}$$

$$AV \,.\, A'V' = \frac{b(1-\cos\theta)}{\sin\theta} \times \frac{b(1+\cos\theta)}{\sin\theta} = b^2$$

$\angle V'SV = 90°$ so $V'S'SV$ is a cyclic quadrilaterel

Q.9 (A,C,D)

$(3x^2 + 2y^2 - 5)(3 + 8 - 5) = (3x + 2.y.2 - 5)^2$

$6(3x^2 + 21y^2 - 5) = (3x+4y-5)^2$

$$\tan\theta = 2\frac{\sqrt{h^2-ab}}{a+b} = 2\frac{\sqrt{(24)^2+36}}{9-4} = \frac{12}{\sqrt{5}}$$

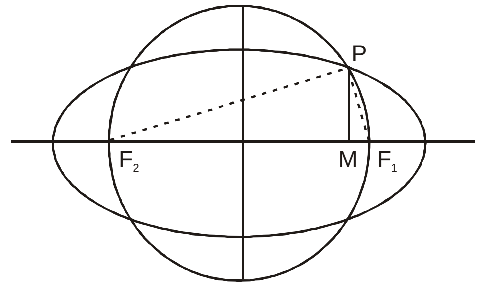
$$\theta = \tan^{-1}\frac{12}{\sqrt{5}}$$

Q.10 (C)

Equation of circle will be

$x^2 + y^2 = (ae)^2$ ....(1)

$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ ....(2)



$$\frac{(ae)^2 - y^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \frac{b}{e}\sqrt{1-e^2} = \frac{a}{e}(1-e^2) \quad 2a = 17$$

$$PM = \frac{a}{e}(1-e^2)$$

Area of $\Delta PF_1F_2 = 30$

$\frac{1}{2}(F_1F_2) \times PM = 30 \quad F_1F_2 = 2ae$

$$\frac{1}{2}(2ae) \times \frac{a}{e}\sqrt{1-e^2} = 30 = 17 \times \frac{13}{17}$$

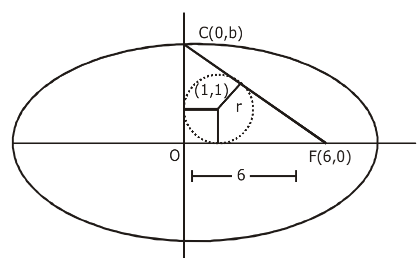
$e = \frac{13}{17} \quad F_1F_2 = 13$

Q.11 **(A)**

Eq^n of CF:

$$\frac{x}{6}+\frac{y}{b} = 1$$

p = r



$$\left|\frac{\frac{1}{6}+\frac{1}{b}-1}{\sqrt{\frac{1}{36}+\frac{1}{b^2}}}\right| = 1$$

$\Rightarrow b = 5/2 \Rightarrow 2b = 5$

$ae = 6$

$e^2 = 1 - b^2/a^2$

$\Rightarrow a^2e^2 = a^2 - b^2 \Rightarrow 36 = a^2 - 25/4$

$\Rightarrow a^2 = 169/4$

$$\Rightarrow a = \frac{13}{2}$$

$2a = 13 \Rightarrow (AB)(CD) = 5 \times 3 = 65$

Comprehension # 1 (Q. No. 16 to 18)

Q.16 (D)
Q.17 (A)
Q.18 **(B)**

$$\left(\frac{3x-4y+10}{5}\right)^2 \times \frac{25}{2} + \left(\frac{4x+3y-15}{5}\right)^2 \times \frac{25}{3} = 1$$

$a^2 = \frac{2}{25} \Rightarrow a = \frac{\sqrt{2}}{5}$ minor axis = 2a = $\frac{2\sqrt{2}}{5}$

$b^2 = \frac{3}{25} \Rightarrow b = \frac{\sqrt{3}}{5}$ major axis = 2b = $\frac{2\sqrt{3}}{5}$

$$e = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{2}{3}} = \frac{1}{\sqrt{3}}$$

centre is point of intersection of
$3x - 4y + 10 = 0$, $4x + 3y - 15 = 0$

$$\left(\frac{6}{5}, \frac{17}{5}\right)$$

Comprehension # 2 (Q. No. 19 to 21)

Q.19 (C)
Q.20 (B)
Q.21 (A)

Sol.19 $y = mx \pm \sqrt{a^2m^2 + b^2}$

$k = mh \pm \sqrt{a^2m^2 + b^2}$
$(k-mh)^2 = a^2 m^2 + b^2$
$m^2 (h^2 - a^2) - 2mhk + k^2 - b^2 = 0$

$\because$ $(m_1 = \tan\theta_1$, $m_2 = \tan\theta_2)$

$$m_1 m_2 = \frac{k^2-b^2}{h^2-a^2} = \tan\theta_1 \tan\theta_2 = 4$$

$$\Rightarrow \frac{y^2-b^2}{x^2-a^2} = 4 \Rightarrow \left(\frac{y-b}{x-a}\right) = 4\left(\frac{x+a}{y+b}\right)$$

Sol.20 $\because \angle QAP = \angle PBQ = 90°$
hence a circle drawn taking 'PQ' as diameter will pass through B,A,P,Q
$\therefore$ center will be mid point of PQ

Sol.21 $m_1 + m_2 = \frac{2hk}{h^2-a^2}$

and $\cot\theta_1 + \cot\theta_2 = \lambda$

$$\Rightarrow \frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2} = \lambda$$

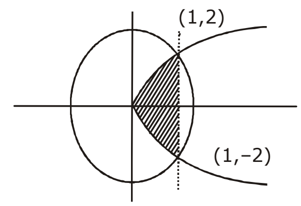
$$\Rightarrow \frac{\tan\theta_1 + \tan\theta_1}{\tan\theta_1 \tan\theta_2} = \lambda$$

$$\Rightarrow \frac{\dfrac{2hk}{h^2-a^2}}{\dfrac{k^2-b^2}{h^2-a^2}} = \lambda$$

$\Rightarrow 2hk = \lambda (k^2 - b^2)$
$2xy = \lambda (y^2 - b^2)$

Comprehension # 3 (Q. No. 22 to 23)

Q.22 (B)
Q.23 (D)



Sol.22 Area = $\int_0^1 \sqrt{4x}\, dx$
= 8/3

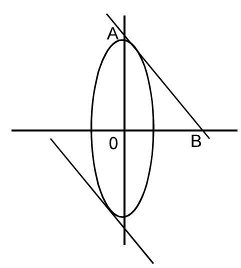
Sol.23 Tangent at P
$y + x = 3$
$\Rightarrow$ T(3, 0)
Normal at P
$x - y = -1$
$\Rightarrow$ G(–1, 0)

Area = $\frac{1}{2} \times 2 \times 4 = 4$

Q.24 (A) → (r), (B) → (p), (C) → (s), (D) → (q)

Sol. (A) $y = mx \pm \sqrt{a^2m^2 + b^2}$

$$y = -\frac{4}{3}x \pm \sqrt{18 \times \frac{16}{9} + 32} \Rightarrow y = -\frac{4}{3}x \pm 8$$



Distance between tangent

$$= \frac{16}{\sqrt{1+\frac{16}{9}}} = \frac{16\times 3}{5} = \frac{48}{5}$$

(B) $y = -\frac{4}{3}x + 8$ A(6, 0) B (0, 8)

Area of $\Delta AOB = \frac{1}{2} \times 6 \times 8 = 24$

(C) point of contact

$$\left(-\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

product of coordinates = $-\dfrac{a^2b^2m}{a^2m^2+b^2} = -\dfrac{18\times32\times\left(-\frac{4}{3}\right)}{64}$

= 12

(D) 4x + 3y = 24 $\ell = \dfrac{4}{24}$ m = $\dfrac{3}{24}$

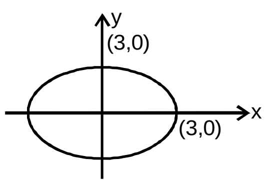
$\dfrac{4}{24}$ x + $\dfrac{3}{24}$ y = 1 $\quad \ell + m = \dfrac{7}{24}$

Q.25 **(A) → (p), (B) → (s), (C) → (p), (D) → (r)**

Point P = $\left(5/\sqrt{2}, 3/\sqrt{2}\right)$

equation of normal at P

5x – 3y = $8\sqrt{2}$ . ....(i)



point A = $\left(\dfrac{8\sqrt{2}}{5}, 0\right)$ & B = $\left(0, \dfrac{-8\sqrt{2}}{3}\right)$.

Tangent at P : 3x + 5y = 15 $\sqrt{2}$ .....(ii)

Point T = $\left(5\sqrt{2}, 0\right)$ check the options.

Q.26 **(A) → (q), (B) → (r), (C) → (s), (D) → (q)**

(A) $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1$

e = $\sqrt{1-\dfrac{16}{25}} = \dfrac{3}{5}$

be = $\dfrac{3}{5}\times5=3$

$\dfrac{2a^2}{b}=\dfrac{2\times16}{5}=\dfrac{32}{5}=\dfrac{4k}{5}$

k = 8

(B) Any pont of ellipse $\dfrac{x^2}{6}+\dfrac{y^2}{2}=1$ is

$\left(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta\right)$

distance from origin $\sqrt{6\cos^2\theta+\sin^2\theta}=2$

⇒ $\cos^2\theta = \dfrac{1}{2}$ ⇒ $\cos\theta = \dfrac{1}{\sqrt{2}}$

(C) ae – $\dfrac{a}{e}$ = 8

$a\left[\dfrac{1}{2}-2\right]=8$

$\dfrac{3}{2}a=8 \Rightarrow a = \dfrac{16}{3}$

∵ $b^2 = a^2(1-e^2)$

∴ $b^2 = \left(\dfrac{16}{3}\right)^2\left(1-\dfrac{1}{4}\right)$

⇒ $b^2 = \dfrac{64}{3}$

⇒ b = $\dfrac{8}{\sqrt{3}}$

⇒ k = 8

(D) By definition of ellipse

NUMERICAL VALUE BASED

Q.1 (13)

$PF_1 + PF_2 = 17$

$\dfrac{1}{2} PF_1 \cdot PF_2 = 30$

$(F_1F_2)^2 = PF_1^2 + PF_2^2 = 289 - 120 = 169$

$F_1F_2 = 13$

Q.2 (85)

Center of ellipse = (29, 75/2)

foot of perpendicular from focii

lie on auxillary circle

equaton of auxillary circle

$(x-29)^2 + (y - 75/2) = a^2$

↓ (9,0) foot of perpendicular

2a = 85.

Q.3 (65)

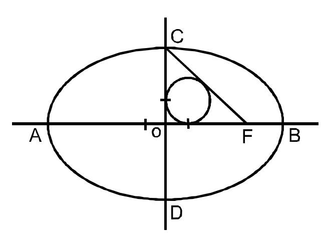
ae = 6

$b^2 + 36 = (b+4)^2$

36 = 16 + 8b

b = $\dfrac{5}{2}$

$a^2 = a^2e^2 + b^2$



= 36 + $\dfrac{25}{4}$ = $\dfrac{169}{4}$

a = $\dfrac{13}{2}$

(2a) (2b) = 65

Q.4 (24)

$\frac{x^2}{18} + \frac{y^2}{32} = 1$ a < b

Tangent Equation slope form

x = my + $\sqrt{a^2m^2 + b^2}$

Slope = $\frac{1}{m} = -\frac{4}{3} \Rightarrow m = -\frac{3}{4}$

$x = -\frac{3}{4}y + \sqrt{32\left(\frac{9}{16}\right) + 18}$

4x + 3y = 24

$\frac{x}{6} + \frac{y}{8} = 1$

Intercept on axis is 6 and 8

So area of Δ CAB = $\frac{1}{2} \times 6 \times 8 = 24$ sq. units.

Q.5 (7)

Property ℓ = a + b = 4 + 3 = 7

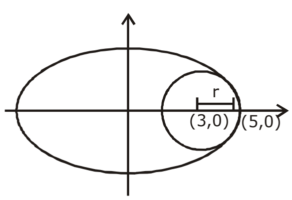
Q.6 (2)

2a = 10 ⇒ a = 5; 2b = 8 ⇒ b = 4

$e = \sqrt{1 - \frac{b^2}{a^2}} = 3/5$

Focus (± ae, 0)

⇒ (±3, 0)



r = 5 – 3 = 2

⇒ r = 2

Q.7 (16)

$x^2 + 9y^2 - 4x + 6y + 4 = 0$

$(x-2)^2 + \frac{(y+1/3)^2}{1/9} = 1$

Let x – 2 = cos θ ⇒ x = 2 + cos θ

$y + \frac{1}{3} = \frac{1}{3}\sin\theta \Rightarrow y = -\frac{1}{3} + \frac{1}{3}\sin\theta$

z = 4x – 9y

$4(2+\cos\theta) - 9\left(-\frac{1}{3} + \frac{1}{3}\sin\theta\right)$

= 11 + 4 cos θ – 3 sin θ

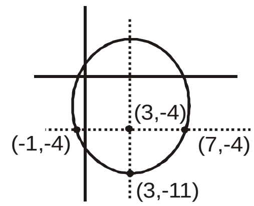
$Z_{max} = 11 + 5 = 16$

Q.8 **(186)**

Equation of parabola,

$(x-3)^2 = k(y+11)$

which is passing through



(7, –4) ⇒ k = 16/7

∴ $16y = 7(x-3)^2 - 176$

⇒ a + h + k = 186

Q.9 (19)

Point P = $\left(\sqrt{2}, 1/\sqrt{2}\right)$

shifting the ellipse by leting the origin at $\left(\sqrt{2}, 1/\sqrt{2}\right)$

$\left(x+\sqrt{2}\right)^2 + 4\left(y + 1/\sqrt{2}\right)^2 = 4$

$\Rightarrow x^2 + 4y^2 + 2\sqrt{2}x + 8\sqrt{2}y = 0$ ....(1)

Let the line AB $\ell x + my = 1$ ....(2)

Homozining (1) with (2) & as the angle between the chords is 90º so coff. of x^2 + coff. of y^2 = 0

$\Rightarrow 2\sqrt{2}\ell + 4\sqrt{2}m = -5$ ....(3)

using (2) & (3) $\left(\frac{-5}{2\sqrt{2}}x - 1\right) + m(y - 2x) = 0$

....(4)

which shows a family of line & passes through a fixed point which is point of intersection of two line A.

$\Rightarrow x = -\frac{2\sqrt{2}}{5}$ & $y = \frac{4\sqrt{2}}{5}$

again $x = -\frac{2\sqrt{2}}{5} - \sqrt{2} = -\frac{3\sqrt{2}}{5}$ & $y = \frac{3\sqrt{2}}{10}$

$a^2 + b^2 = \frac{9}{10} \Rightarrow a + b = 19$

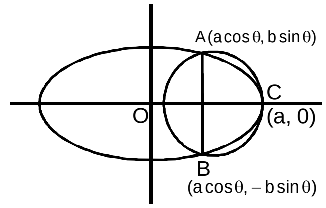
Q.10 (17)

AB = 2 b sin θ

AC = AB/2

$\Rightarrow b^2\sin^2\theta = a^2(1-\cos\theta)^2$

$\Rightarrow \frac{16}{15} = \frac{2\cos\theta}{1+\cos\theta}$



$\Rightarrow \sin\theta = \frac{15}{17}$ & $b = \frac{39}{5}$

so $AB = \frac{180}{17}$

KVPY
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Q.1 (B)

ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Any tangent $\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1$

y intercept = 5 $\Rightarrow \sin\theta = \frac{3}{5}$; $\theta \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \cos\theta = -\frac{4}{5}$

tangent $\Rightarrow -\frac{x}{5} + \frac{y}{5} = 1 \Rightarrow$ slope = 1

Q.2 (C)

$ex^2 + \pi y^2 - 2e^2x - 2\pi^2 y + e^3 + \pi^3 = \pi e$

$e(x^2 - 2ex + e^2) + \pi(y^2 - 2\pi y + \pi^2) = \pi e$

$$\frac{(x-e)^2}{\pi} + \frac{(y-\pi)^2}{e} = 1$$

$a^2 = \pi \Rightarrow a = \sqrt{\pi}$ $\qquad\qquad \pi > e$

$PS_1 + PS_2 = 2a$ Major axis is || ot axis

$PS_1 + PS_2 = 2\sqrt{\pi}$

Q.3 (A)

$4x^2 + 9y^2 - 8x - 36y + 15 = 0$

$4(x^2 - 2x) + 9(y^2 - 4y) = -15$

$4(x^2 - 2x + 1) + 9(y^2 - 4y + 4) = -15 + 4 + 36$

$4(x - 1)^2 + 9(y - 2)^2 = 25$

$$\frac{(x-1)^2}{\left(\frac{5}{2}\right)^2} + \frac{(y-2)^1}{\left(\frac{5}{3}\right)^2} = 1 \text{......................(1)}$$

$x^2 - 2x + y^2 - 4y + 5$

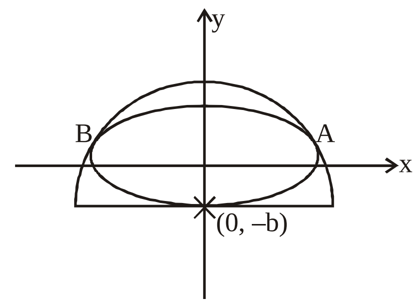
$(x - 1)^2 + (y - 2)^2$

min of $((x - 1)^2 + (y - 2)^2 = \frac{25}{9}$

max of $((x - 1)^2 + (y - 2)^2) = \frac{25}{4}$

$= \frac{25}{9} + \frac{25}{4} = \frac{325}{36}$

Q.4 (D)



Let ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and circle $x^2 + (y + b)^2 = r^2$ $\qquad$ {let radius = r }

put $x^2 = a^2 - \frac{a^2y^2}{b^2}$

in circle $a^2 - \frac{a^2y^2}{b^2} + (y + b)^2 = r^2$

$\Rightarrow \left(1 - \frac{a^2}{b^2}\right)y^2 + 2by + (a^2 + b^2 - r^2) = 0$

$D = 0 \Rightarrow r^2 = \frac{a^4}{a^2 - b^2}$

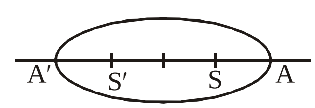
$\Rightarrow b = a\sqrt{1 - \frac{a^2}{r^2}}$

Area $= \Delta = \pi ab = \pi a^2\sqrt{1 - \frac{a^2}{r^2}}$

$\frac{d\Delta}{da} = 0 \Rightarrow a^2 = \frac{2r^2}{3} \Rightarrow a = \sqrt{\frac{2}{3}}\,r$

$\therefore b = a\sqrt{1 - \frac{2}{3}} = \frac{a}{\sqrt{3}} \Rightarrow e = \sqrt{\frac{2}{3}}$

Q.5 (D)



A'S' = SS' = SA

2ae = a – ae

3ae = a

e = 1/3

$1 - \frac{b^2}{a^2} = \frac{1}{9} \Rightarrow \frac{b^2}{a^2} = \frac{8}{9}$

$\Rightarrow \frac{8}{a^2} = \frac{8}{9} \Rightarrow a = 3$

Q.6 (B)

$$\left.\begin{aligned}\frac{x\cos\theta}{3} + \frac{y\sin\theta}{2} = 1\\ \frac{x\cos\theta}{3} - \frac{y\sin\theta}{2} = 1\end{aligned}\right\} x = 3\sec\theta, y = 0$$

$$\frac{-x\cos\theta}{3} + \frac{y\sin\theta}{2} = 1$$

$$\frac{-x\cos\theta}{3} - \frac{y\sin\theta}{2} = 1$$

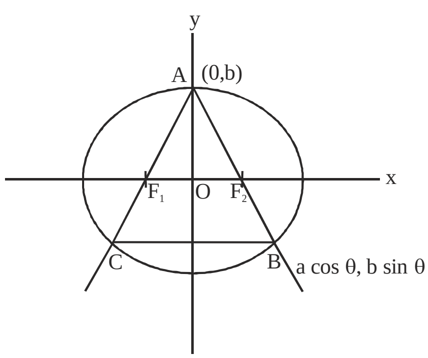
x = 0, y = 2cosθ

area = 4. $\frac{1}{2}3\sin\theta.2\cos\theta = \frac{12}{\sin\theta\cos\theta} = \frac{24}{\sin 2\theta}$

$\therefore$ min. area = 24

Q.7 (D)

$$m_{AB} = \frac{b\sin\theta - b}{a\cos\theta} = -\sqrt{3} \quad \Rightarrow \frac{b}{a}\left(\frac{\sin\theta - 1}{\cos\theta}\right) = -\sqrt{3}$$



y – b = $= -\sqrt{3}$ (x – 0)

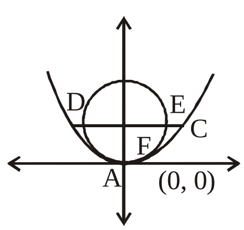
o + b $= +\sqrt{3}$ae

$b^2 = 3a^2e^2 = a^2 (I - e^2)$

$\Rightarrow 4e^2 = 1 \Rightarrow e = \frac{1}{2}$

Q.8 (C)

n for the parabola;
verter A (0,0)
Four F : (o, k)
end point of latus rectum:
Length of BC = 4k;
BD = DE = EC



And BD + DE + EC = $\frac{4k}{3}$ .... (i)

So Major Axis of ellipse = 2AF = 2×

minor Axis of Ellipse = DE = $\frac{4k}{3}$

Eccetricity = C = $\sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{1 - \left(\frac{2k}{3}\right)^2}{k^2}} \quad 2 = \frac{\sqrt{5}}{3}$

Q.9 **(C)**

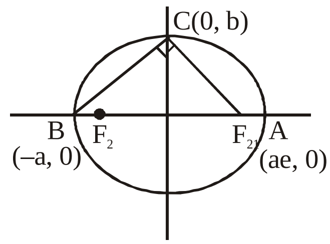
$\frac{b}{-ae} \times \frac{b}{a} = -1$

$\Rightarrow b^2 = a^2e$

$\Rightarrow a^2(1 - e^2) = a^2e$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1 + \sqrt{5}}{2}$



JEE MAIN
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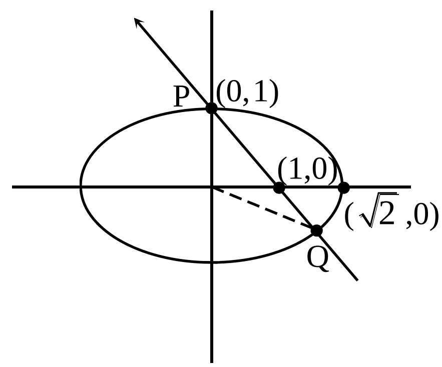
Q.1 (1)



Homogenise Ellipse w.r.t. line, $\frac{x}{2} + \frac{y}{1} = (x+y)^2$

$\therefore \quad x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$

$\Rightarrow \quad x^2 + 4xy = 0$

$\Rightarrow \quad x = 0 , y = \frac{x}{4}$

angle between these line is $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Q.2 (3)

$E : \frac{x^2}{9} + \frac{y^2}{4} = 1$

$C : x^2 + y^2 = \frac{31}{4}$

equation of tangent to ellipse

$y = mx \pm \sqrt{9x^2 + 4}$ .....(i)

equation of tangent to circle

$$y = mx \pm \sqrt{\frac{31}{4}m^2 + \frac{31}{4}} \qquad ......(ii)$$

Comparing equation (i) & (ii)

$$9m^2 + 4 = \frac{31m^2}{4} + \frac{31}{4}$$

$\Rightarrow$ $36m^2+16=31m^2+31$

$\Rightarrow$ $5\ m^2=15$

$\Rightarrow$ $m^2 = 3$

Q.3 **(1)**

$y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so $x^2 = b$

$$\frac{x^2}{16} + \frac{3x^2}{b^2} = 1$$

$$\frac{b}{16} + \frac{3}{b} = 1$$

$b^2 - 16b + 48 = 0$

$(b - 12)(b - 4) = 0$

$b = 12,\ b > 4$

Q.4 **(3)**

Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}} + \frac{y.\sin\theta}{1} = 1, \qquad \theta \in \left(0, \frac{\pi}{2}\right)$$

intercept on x-axis

OA = $3\sqrt{3}\sec\theta$

intercept on y-axis

OB = $\text{cosec}\theta$

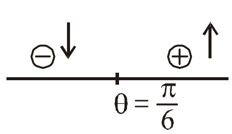
Now, sum of intercept

$= 3\sqrt{3}\ \sec\theta + \text{cosec}\theta = f(\theta)$ let

$f'(\theta) = 3\sqrt{3}\ \sec\theta\tan\theta - \text{cosec}\theta\cot\theta$

$$= 3\sqrt{3}\frac{\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta}$$

$$= \underbrace{\frac{\cos\theta}{\sin^2\theta}.3\sqrt{3}}_{\oplus}\left[\tan^3\theta - \frac{1}{3\sqrt{3}}\right] = 0 \Rightarrow \theta = \frac{\pi}{6}$$



$\Rightarrow$ at $\theta = \frac{\pi}{6}$, $f(\theta)$ is minimum

Q.5 **(1)**

Q.6 (3)

Q.7 (1)

Q.8 (3)

Q.9 (3)

Q.10 (1)

Q.11 (2)

Q.12 (3)

Q.13 **(3)**

Q.14 (2)

Q.15 (1)

Q.16 (1)

Q.17 (15)

JEE-ADVANCED PREVIOUS YEAR'S

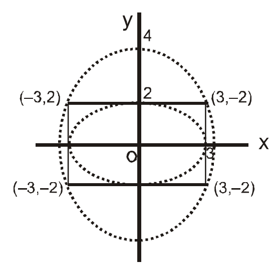
Q.1 (C)

Let required ellipse is

$$E_2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

It passes thorugh (0, 4)

$$0 + \frac{16}{b^2} = 1$$



$\Rightarrow$ $b^2 = 16$

It also passes through (±3, ±2)

$$\frac{9}{a^2} + \frac{4}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{1}{4} = 1$$

$$\frac{9}{a^2} = \frac{3}{4}$$

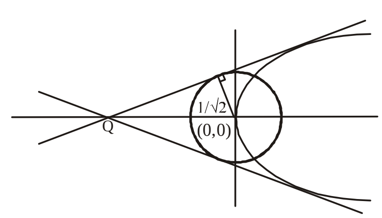
$\Rightarrow$ $a^2 = b^2(1 - e^2)$

$$\frac{12}{16} = 1 - e^2$$

$$e^2 = 1 - \frac{12}{16} = \frac{4}{16} = \frac{1}{4}$$

$$e = \frac{1}{2}$$

Q.2 (A, C)



Let equation of common tangent is y = $mx+\frac{1}{m}$

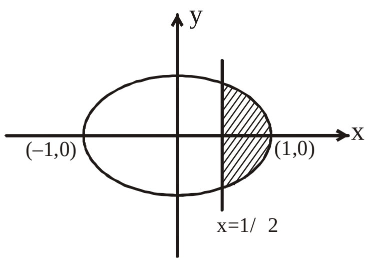
$\therefore \quad \left|\frac{0+0+\frac{1}{m}}{\sqrt{1+m^2}}\right| = \frac{1}{\sqrt{2}} \Rightarrow m^4+m^2-2=0 \Rightarrow m=\pm$

Equation of common tangents are y = x + 1 and y = – x – 1 point Q is (–1, 0)

$\therefore$ Equation of ellipse is $\frac{x^2}{1}+\frac{y^2}{1/2}=1$

(A) $e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$ and LR $\frac{2b^2}{a}=1$

(C)



Area 2.

$\int_{1/\sqrt{2}}^{1}\frac{1}{\sqrt{2}}\cdot\sqrt{1-x^2}dx=\sqrt{2}\left[\frac{x}{2}\sqrt{1-x^2}\ \frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1}$

$=\sqrt{2}\left[\frac{\pi}{4}-\left(\frac{1}{4}+\frac{\pi}{8}\right)\right]=\sqrt{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)=\frac{\pi-2}{4\sqrt{2}}$

correct answer are (A) and (D)

Q.3 (A)

$y^2=4\lambda x$, P(λ, 2λ)

Slope of the tangent to the parabola at point P

$\frac{dy}{dx}=\frac{4\lambda}{2y}=\frac{4\lambda}{2x2\lambda}=1$

Slope of the tangent to the ellipse at P

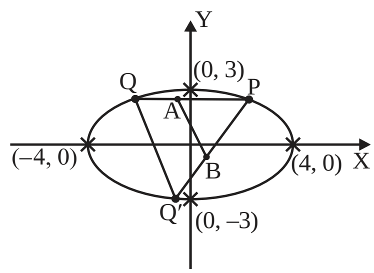
$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0$

As tangents are perpendicular y' = –1

$\Rightarrow \frac{2\lambda}{a^2}-\frac{4\lambda}{b^2}=0 \Rightarrow \frac{a^2}{b^2}=\frac{1}{2}$

$\Rightarrow e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

Q.4 (4)



A and B be midpoints of segment PQ and PQ′ respectively

AB=distance between M(P,Q) and M(P,Q′) $=\frac{1}{2}\cdot QQ'$

Since, Q,Q′ must be on E, so,
maximum of QQ′=8

$\therefore$ Maximum of AB= $\frac{8}{2}=4$

Hyperbola

EXERCISES-I

Q.1 (1)

$e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow e^2=\frac{a^2+b^2}{a^2}$

$e_1=\sqrt{1+\frac{a^2}{b^2}} \Rightarrow e_1^2=\frac{b^2+a^2}{b^2} \Rightarrow \frac{1}{e_1^2}+\frac{1}{e^2}=1$.

Q.2 (1)

Conjugate axis is 5 and distance between foci = 13
$\Rightarrow$ 2b = 5 and 2ae = 13.
Now, also we know for hyperbola

$b^2=a^2(e^2-1) \Rightarrow \frac{25}{4}=\frac{(13)^2}{4e^2}(e^2-1)$

$\Rightarrow \frac{25}{4}=\frac{169}{4}-\frac{169}{4e^2}$ or $e^2=\frac{169}{144} \Rightarrow e=\frac{13}{12}$

or $a=6, b=\frac{5}{2}$ or hyperbola is $\frac{x^2}{36}-\frac{y^2}{25/4}=1$

$\Rightarrow 25x^2-144y^2=900$.

Q.3 (3)

Vertices $(\pm 4,0)\equiv(\pm a,0) \Rightarrow a=4$

Foci $(\pm 6,0)\equiv(\pm ae,0) \Rightarrow e=\frac{6}{4}=\frac{3}{2}$

Q.4 (3)

$(4x+8)^2-(y-2)^2=-44+64-4$

$\Rightarrow \frac{16(x+2)^2}{16}-\frac{(y-2)^2}{16}=1$

Transverse and conjugate axes are y = 2, x= –2

Q.5 (3)

Foci $(0,\pm 4)\equiv(0,\pm be) \Rightarrow be=4$

Vertices $(0,\pm 2)\equiv(0,\pm b) \Rightarrow b=2 \Rightarrow a=2\sqrt{3}$

Hence equation is $\frac{-x^2}{(2\sqrt{3})^2}+\frac{y^2}{(2)^2}=1$ or

$\frac{y^2}{4}-\frac{x^2}{12}=1$.

Q.6 (1)

Directrix of hyperbola $x=\frac{a}{e}$,

where $e=\sqrt{\frac{b^2+a^2}{a^2}}=\frac{\sqrt{b^2+a^2}}{a}$

Directrix is, $x=\frac{a^2}{\sqrt{a^2+b^2}}=\frac{9}{\sqrt{9+4}} \Rightarrow x=\frac{9}{13}$

Q.7 (1)

$(x-2)^2+(y-1)^2=4\left[\frac{(x+2y-1)^2}{5}\right]$

$\Rightarrow 5[x^2+y^2-4x-2y+5]$

$=4[x^2+4y^2+1+4xy-2x-4y]$

$\Rightarrow x^2-11y^2-16xy-12x+6y+21=0$

Q.8 (1)

The equation is $(x-0)^2+(y-0)^2=a^2$.

Q.9 (3)

If $y=2x+\lambda$ is tangent to given hyperabola, then

$\lambda=\pm\sqrt{a^2m^2-b^2}=\pm\sqrt{(100)(4)-144}=\pm\sqrt{256}=\pm 16$

Q.10 (1)

Suppose point of contact be (*h*, *k*), then tangent is $hx-4ky-5=0\equiv 3x-4y-5=0$ or $h=3, k=1$
Hence the point of contact is (3, 1).

Q.11 (1)

Tangent to $\frac{x^2}{1}-\frac{y^2}{3}=1$ and perpendicular to $x+3y-2=0$ is given by

$y=3x\pm\sqrt{9-3}=3x\pm\sqrt{6}$.

Q.12 (2)

$x\cos\alpha+y\sin\alpha=p \Rightarrow y=-\cot\alpha.\, x+p\,\text{cosec}\,\alpha$

It is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Therefore, $p^2\text{cosec}^2\alpha=a^2\cot^2\alpha-b^2$

$\Rightarrow a^2\cos^2\alpha-b^2\sin^2\alpha=p^2$

Q.13 (3)

Equation of normal to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at

$(a\sec\theta, b\tan\theta)$ is $\frac{a^2x}{a\sec\theta}+\frac{b^2y}{b\tan\theta}=a^2+b^2$

Q.14 (1)

Any normal to the hyperbola is

$$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2 \quad .....(i)$$

But it is given by $lx+my-n=0$ .....(ii)

Comparing (i) and (ii), we get

$$\sec\theta=\frac{a}{l}\left(\frac{-n}{a^2+b^2}\right) \text{ and } \tan\theta=\frac{b}{m}\left(\frac{-n}{a^2+b^2}\right)$$

Hence eliminating θ, we get

$$\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$$

Q.15 (4)

Applying the formula, the required normal is

$$\frac{16x}{8}+\frac{9y}{3\sqrt{3}}=16+9 \text{ i.e., } 2x+\sqrt{3}y=25$$

Trick : This is the only equation among the given options at which the point $(8,3\sqrt{3})$ is located.

Q.16 (2)

We know that the equation of the normal of the conic $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at point $(a\sec\theta, b\tan\theta)$ is

$$ax\sec\theta+by\cot\theta=a^2+b^2$$

or $y=\frac{-a}{b}\sin\theta\, x+\frac{a^2+b^2}{b\cot\theta}$ Comparing above equation with equation $y=mx+\frac{25\sqrt{3}}{3}$ and taking $a=4, b=3$.

we get, $\frac{a^2+b^2}{b\cot\theta}=\frac{25\sqrt{3}}{3} \Rightarrow \tan\theta=\sqrt{3}\Rightarrow\theta=60^o$

and $m=-\frac{a}{b}\sin\theta=\frac{-4}{3}\sin 60^o=\frac{-4}{3}\times\frac{\sqrt{3}}{2}=\frac{-2}{\sqrt{3}}$.

Q.17 (2)

The equation of chord of contact at point (h,k) is $xh-yk=9$

Comparing with $x=9$, we have $h=1, k=0$

Hence equation of pair of tangent at point (1,0) is $SS_1=T^2$

$$\Rightarrow (x^2-y^2-9)(1^2-0^2-9)=(x-9)^2$$

$$\Rightarrow -8x^2+8y^2+72=x^2-18x+81$$

$$\Rightarrow 9x^2-8y^2-18x+9=0$$

Q.18 (1)

Tangent to $y^2=8x\Rightarrow y=mx+\frac{2}{m}$

Tangent to $\frac{x^2}{1}-\frac{y^2}{3}=1\Rightarrow y=mx\pm\sqrt{m^2-3}$

On comparing, we get

$m=\pm 2$ or tangent as $2x\pm y+1=0$.

Q.19 (2)

According to question, $S\equiv 25x^2-16y^2-400=0$

Equation of required chord is $S_1=T$ .....(i)

Here, $S_1=25(5)^2-16(3)^2-400$

$=625-144-400=81$

and $T\equiv 25xx_1-16yy_1-400$, where $x_1=5, y_1=3$

$=25(x)(5)-16(y)(3)-400=125x-48y-400$

So from (i), required chord is

$125x-48y-400=81$ or $125x-48y=481$.

Q.20 (4)

Given, equation of hyperbola $2x^2+5xy+2y^2+4x+5y=0$ and equation of asymptotes $2x^2+5xy+2y^2+4x+5y+\lambda=0$ .....(i), which is the equation of a pair of straight lines.

We know that the standard equation of a pair of straight lines is $ax^2+2hxy+by^2+2gx+2fy+c=0$.

Comparing equation (i) with standard equation, we get $a=2, b=2$, $h=\frac{5}{2}, g=2, f=\frac{5}{2}$ and $c=\lambda$.

We also know that the condition for a pair of straight lines is $abc+2fgh-af^2-bg^2-ch^2=0$.

Therefore $4\lambda+25-\frac{25}{2}-8-\frac{25}{4}\lambda=0$

or $-\frac{9\lambda}{4}+\frac{9}{2}=0$ or $\lambda=2$. Substituting value of λ in equation (i), we get

$2x^2+5xy+2y^2+4x+5y+2=0$.

Q.21 (2)

$xy=c^2$ as $c^2=\frac{a^2}{2}$. Here, co-ordinates of focus are

$(ae\cos 45°, ae\sin 45°)\equiv(c\sqrt{2}, c\sqrt{2})$,

$\{\because e=\sqrt{2}, a=c\sqrt{2}\}$

Similarly other focus is $(-c\sqrt{2}, -c\sqrt{2})$

Note : Students should remember this question as a fact.

Q.22 (4)
Since it is a rectangular hyperbola, therefore eccentricity $e=\sqrt{2}$.

Q.23 (3)
Multiplying both, we get $x^2-y^2=a^2$. This is equation of rectangular hyperbola as $a=b$.

Q.24 (2)
Tangent at $(a\sec\theta, b\tan\theta)$ is,

$$\frac{x}{(a/\sec\theta)}-\frac{y}{(b/\tan\theta)}=1 \text{ or}$$

$$\frac{a}{\sec\theta}=1,\ \frac{b}{\tan\theta}=1$$

$\Rightarrow a=\sec\theta \quad b=\tan\theta$ or (a, b)lies on $x^2-y^2=1$

Q.25 (4)
Since eccentricity of rectangular hyperbola is $\sqrt{2}$.

Q.26 (3)
Since the general equation of second degree represents a rectangular hyperbola, if $\Delta\neq 0$, $h^2>ab$ and coefficient of x^2+ coefficient of y^2=0. Therefore the given equation represents a rectangular hyperbola, if $\lambda+5=0$ i.e., $\lambda=-5$

Q.27 (4)

$\because$ Distance between directrices $=\frac{2a}{e}$.

$\because$ Eccentricity of rectangular hyperbola $=\sqrt{2}$

$\therefore$ Distance between directrics $=\frac{2a}{\sqrt{2}}$.

Given that, $\frac{2a}{\sqrt{2}}=10\Rightarrow 2a=10\sqrt{2}$

Now, distance between foci

$=2ae=(10\sqrt{2})(\sqrt{2})=20.$

Q.28 (2)
Eccentricity of rectangular hyperbola is $\sqrt{2}$.

Q.29 (3) It is obvious.

Q.30 (2) Let equation of circle is $x^2+y^2=a^2$

Parametric form of $xy=c^2$ are $x=ct,\ y=\frac{c}{t}$

$$\Rightarrow c^2t^2+\frac{c^2}{t^2}=a^2\Rightarrow c^2t^4-a^2t^2+c^2=0$$

Product of roots will be, $t_1t_2t_3t_4=\frac{c^2}{c^2}=1$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)
Given hyperbola
$(x-2)^2-(y-2)^2=-16$
Rectangular hyperbola
$\therefore\ e=\sqrt{2}$.

Q.2 (3)
If e_1 & e_2 are eccentircities of two conjugate hyperbolas

then $\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$

$\therefore\ e_1=\sec\alpha$ & $e_2=\text{cosec}\alpha$

Q.3 (3)

$\frac{2b^2}{a}=8$ ... (1)

and $2b=\frac{2ae}{2}$ ...(2)

and $e^2=1+\frac{b^2}{a^2}$ ... (3)

by (1), (2), (3) $\quad e=\frac{2}{\sqrt{3}}$ Ans.

Q.4 (4)

$\sqrt{3}x-y-4\sqrt{3}k=0$ ...(1)

$\sqrt{3}kx+ky-4\sqrt{3}=0$ ...(2)

Solve (1) and (2)

$x=2\frac{(1+k^2)}{k}$ and $y=\frac{2\sqrt{3}(1-k^2)}{k}$

$\frac{x^2}{4}-\frac{y^2}{12}=4\Rightarrow\frac{x^2}{16}-\frac{y^2}{48}=1$ Hyperbola

Q.5 (1)

$\frac{2b^2}{a}=8$; $e=\frac{3}{\sqrt{5}}$ $\quad\Rightarrow b^2=4a$; $e^2=\frac{9}{5}$

$1+\frac{b^2}{a^2}=\frac{9}{5}$ $\quad\Rightarrow\frac{b^2}{a^2}=\frac{4}{5}$

$\Rightarrow a=5\Rightarrow b^2=20$

Hyp. $\frac{x^2}{25}-\frac{y^2}{20}=1\Rightarrow 4x^2-5y^2=100$

Q.6 (3)
C(0, 0) $\quad A_1(4, 0)$
F_1 (6, 0)
$CA_1=4$
$CF_1=6$
$\Rightarrow a=4$ $\quad ae=6$

$a^2e^2=36$ $\quad\Rightarrow a^2\left(1+\frac{b^2}{a^2}\right)=36$

$\Rightarrow b^2=36-16$ $\quad\Rightarrow b^2=20$

Hyp. $\frac{x^2}{16} - \frac{y^2}{20} = 1$ or $5x^2 - 4y^2 = 80$

Q.7 (1)

$F_1(6, 5)$ $F_2(-4, 5)$

$e = \frac{5}{4}$

$F_1F_2 = 2ae$ Centre of hyp. is the mid point of $F_1F_2 = (1, 5)$

$2ae = 10$

$\Rightarrow ae = 5 \Rightarrow a^2e^2 = 25 \Rightarrow a^2\left(\frac{25}{16}\right) = 25$

$\Rightarrow a^2 = 16 \Rightarrow b^2 = 9$

Hyp. $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

Q.8 (2)

Centre of hyp. will be

mid point of A_1 & $A_2 = \left(\frac{10+0}{2}, 0\right) = (5, 0)$ & check options

Q.9 (3)

$2a = 7 \Rightarrow a = \frac{7}{2}$

Let the Equation of hyp.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

passes through (5, –2)

$\frac{25}{a^2} - \frac{4}{b^2} = 1$

$\frac{25}{a^2} - 1 = \frac{4}{b^2}$

$b^2 = \frac{4a^2}{25-a^2} = \frac{4 \times \frac{49}{4}}{25 - \frac{49}{4}} = \frac{196}{51}$

Equation $\frac{4x^2}{49} - \frac{51y^2}{196} = 1$

Q.10 (2)

$x^2 - y^2 \sec^2\alpha = 5$

$\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1 \rightarrow e_1$

$e_1 = 1 + \frac{5\cos^2\alpha}{5} = 1 + \cos^2\alpha$ ...(1)

$x^2\sec^2\alpha + y^2 = 25$

$\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1 \rightarrow e_2$

$e_2 = 1 - \frac{25\cos^2\alpha}{25} = 1 - \cos^2\alpha$

$e_1 = \sqrt{3}\, e_2$

$e_1^2 = 3e_2^2$

$1 + \cos^2\alpha = 3 - 3\cos^2\alpha$

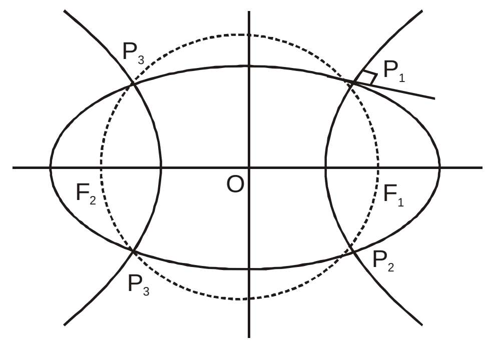
$4\cos^2\alpha = 2$

$\cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}$

Q.11 (1)

If they intersect at right angles then circle will pass through its focus

Circle will be



$x^2 + y^2 = (OF_1)^2$

$x^2 + y^2 = (\sqrt{5})^2$

$x^2 + y^2 = (\sqrt{5})^2$; $F_1(ae, 0)$ $e = \sqrt{5}$

$x^2 + y^2 = 5$; $F_1(\sqrt{5}, 0)$

Q.12 (1)

$\sqrt{2}^2\sec^2\theta + \sqrt{2}^2\tan^2\theta = 6$

$\Rightarrow 1 + 2\tan^2\theta = 3$

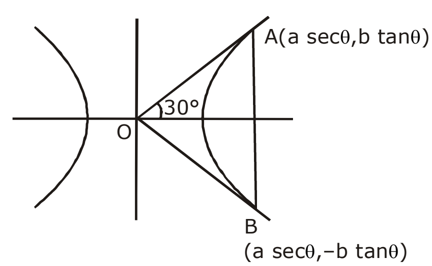
$\therefore \theta = \pi/4$ for first quadrant

Q.13 (4)

$\theta = 30°$

$\frac{b\tan\theta}{a\sec\theta} = \tan 30°$

$\frac{b}{a}\sin\theta = \frac{1}{\sqrt{3}}$



$\frac{b}{a} = \frac{1}{\sqrt{3}\sin\theta}$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{1}{3\sin^2\theta}$

$e^2 > 1 + \frac{1}{3}$

$e > \frac{2}{\sqrt{3}}$

Q.14 (1)

$4x^2 - 9y^2 = 36$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{4} = 1$

$5x + 2y - 10 = 0$

$m = \frac{-5}{2}$

$m' = \frac{2}{5}$

Equation of tangent $y = m'x \pm \sqrt{a^2(m')^2 - b^2}$

$y = \frac{2}{5}x \pm \sqrt{9 \times \frac{4}{25} - 16}$

$y = \frac{2x}{5} \pm \sqrt{-ve}$ so not possible

Q.15 (4)

$(1, 2\sqrt{2})$ lies on director circle

of $\frac{x^2}{25} - \frac{y^2}{16} = 1$ i.e. $x^2 + y^2 = 9$

$\therefore$ Required angle $\pi/2$

Q.16 (3)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Tangent

$y = mx \pm \sqrt{a^2m^2 - b^2}$ ...(1)

$\frac{x^2}{(-b^2)} - \frac{y^2}{(-a^2)} = 1$

$y = mx \pm \sqrt{(-b^2)m^2 + a^2}$

...(2)

(1) and (2) are same

$\frac{1}{1} = \frac{1}{1} = \frac{\sqrt{a^2m^2 - b^2}}{\sqrt{a^2 - b^2m^2}}$

$a^2 - b^2m^2 = a^2m^2 - b^2$

$m^2 = 1 \Rightarrow m = \pm 1$

$y = \pm x \pm \sqrt{a^2 - b^2}$

Q.17 (4)

Locus of the feet of the $\perp^n$ drawn from any focus of the the hyp. upon any tangent is its auxilary circle

Hyp. $\frac{x^2}{\left(\frac{1}{16}\right)} - \frac{y^2}{\left(\frac{1}{9}\right)} = 1$

Auxiliary circle $x^2 + y^2 = \frac{1}{16}$

Q.18 (1)

Tangent to the parabola

$y = mx + \frac{2}{m}$ ...(1)

T angent to the Hyp.

$y = mx \pm \sqrt{m^2 - 3}$ ...(2)

(1) and (2) are same $1 = \frac{2}{m\sqrt{m^2 - 3}}$

$m^2 - 3m^2 - 4 = 0 \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

From (1) $2x \pm y + 1 = 0$

Q.19 (3)

by $T = S_1$ we get $5x + 3y = 16$

Q.20 (1)

by $T = S_1$

$3xh - 2yk + 2(x + h) - 3(y + k)$

$= 3h^2 - 2k^2 + 4h - 6k$

$\Rightarrow x(3h + 2) + y(-2k - 3) = 3h^2 - 2k^2 + 2h - 3k$

If is parallel to $y = 2x$

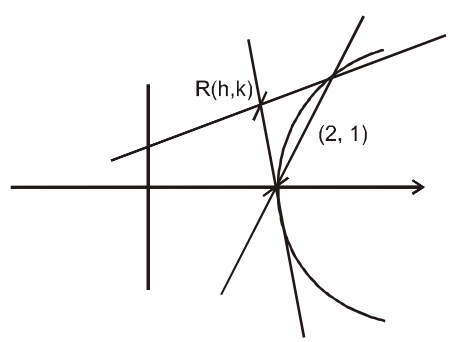
$\therefore \frac{(3h + 2)}{(2k + 3)} = 2 \Rightarrow 3x - 4y = 4$ Ans.

Q.21 (2)

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

Let the point R is (h, k)

So the equation of chord of contact.



$\frac{hx}{16} - \frac{ky}{9} = 1$

It passes through (2, 1) so $\frac{2h}{16}-\frac{k}{9}=1$

$\frac{h}{8}-\frac{k}{9}=1$

so locus of R is 9x – 8y = 72

Q.22 (2)

Slope of the chord = $\frac{25}{16}\times\frac{x_1}{y_1}$

$=\frac{25}{16}\times\frac{6}{2}=\frac{75}{16}$

Equation of chord passing through (6, 2)

$y-2=\frac{75}{16}(x-6)$

16y – 32 = 75x – 450

75x – 16y = 418

Q.23 (1)

Let pair of asymptotes be

$xy - xh - yk + \lambda = 0$

...(1)

where λ : constant

$\therefore$ for (1) represents pair of straight line $\lambda = hk$

$\therefore$ Asymptotes x – k = 0, y – h = 0

Q.24 (1)

$2x^2 + 5xy + 2y^2 + 4x + 5y = 0$

so equation of asymptotes is

$2x^2 + 5xy + 2y^2 + 4x + 5y + c = 0$

it represents a pair of st. line

if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \; \begin{vmatrix} 2 & \frac{5}{2} & 2 \\ \frac{5}{2} & 2 & \frac{5}{2} \\ 2 & \frac{5}{2} & c \end{vmatrix} = 0$

after solving the determinant c = 2

combined equation of asymptotes.

$2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$

Q.25 (1)

Hyp. xy – 3x – 2y = 0

f(x, y) = xy – 3x – 2y

$\frac{\delta f}{\delta x}=0 \Rightarrow y=3$

$\frac{\delta f}{\delta y}=0 \Rightarrow x=2$ Centre (2, 3)

Asy. xy – 3x – 2y + C = 0

will pass through (2, 3)

C = 6

xy – 3x – 2y + 6 =0

(y – 3) (x – 2) = 0

x – 2 = 0, y – 3 = 0

Q.26 (4)

Let the circle on which

P, Q, R, S lie be

$x^2 + y^2 + 2gx + 2fy + C_1 = 0$

How let $\left(ct, \frac{c}{t}\right)$ lie on it

$\Rightarrow c^2t^4 + 2gct^3 + C_1t^2 + 2fct + c^2 = 0$

where $t_1, t_2, t_3\ t_4$ represents the parameters for P, Q, R, S

$\therefore\ t_1t_2t_3t_4 = 1$

also since orthocentre of ΔPQR be

$\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right) \Rightarrow (-x_4, -y_4)$

Q.27 (1)

Let $A\left(ct_1, \frac{c}{t_1}\right)$, $B\left(ct_2, \frac{c}{t_2}\right)$, $C\left(ct_3, \frac{c}{t_3}\right)$

the n orthocentre be

$H\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$ which lies on $xy = c^2$

Q.28 (1)

Curve $xy = c^2$

Point P $\left(ct, \frac{c}{t}\right)$ Point Q $\left(ct', \frac{c}{t'}\right)$

Equation of normal $xt^3 - yt = c(t^4 - 1)$

Point Q satisfy the equation $ct't^3 - \frac{c}{t'}t = c(t^4 - 1)$

$t't^3 - \frac{t}{t'} = t^4 - 1$

$(t')^2 t^3 - t = t'(t^4 - 1)$

$t'^2t^4 + t' - t - t't^4 = 0$

$\Rightarrow t'(t't^3 + 1) - t(1 + t't^3) = 0$

$t' = t$ or $t' = -\frac{1}{t^3}$

so only possibility $t' = -\frac{1}{t^3}$

Q.29 (1)

by $T = S_1$

$\frac{xk+yh}{2} = hk \Rightarrow \frac{x}{h}+\frac{y}{k}=2$

$\therefore\ m = \frac{-1/h}{+1/k}$

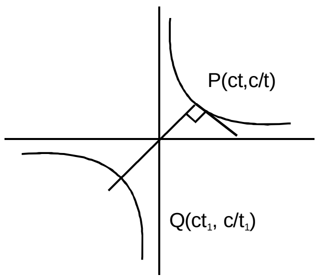
$\Rightarrow k + mh = 0$

$\Rightarrow y + mx = 0$

Q.30 (2)

Slope of tangent at P = $\frac{-1}{t^2}$

So slope of normal = t^2



$$t^2 = \frac{\frac{c}{t_1} - \frac{c}{t}}{(ct_1 - ct)}$$

$$t^2 = \frac{-1}{t_1 t}$$

$t^3 t_1 = -1$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

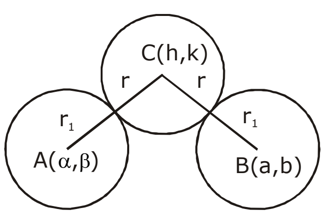
$CA - r_1 = r$

$CB - r_2 = r$

$CA - CB = r_1 - r_2 = k$

$CA - CB = k$

$\Rightarrow$ Locus of C will be hyperbola.



Q.2 (A)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2}$$

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \ (e')^2 = 1 + \frac{a^2}{b^2} = \frac{b^2+a^2}{b^2}$$

$$\frac{1}{e^2} + \frac{1}{(e')^2} = \frac{a^2}{a^2+b^2} + \frac{b^2}{b^2+a^2} = \frac{a^2+b^2}{a^2+b^2} = 1$$

So the point lie on $x^2 + y^2 = 1$

Q.3 (C)

$9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$9(x^2 - 2x) - 16(y^2 - 2y) - 151 = 0$

$9(x^2 - 2x + 1) - 9 - 16(y^2 - 2y + 1) + 16 - 151 = 0$

$9(x-1)^2 - 16(y-1)^2 = 144$

$$\frac{(x-1)^2}{\left(\frac{144}{9}\right)} - \frac{(y-1)^2}{\left(\frac{144}{16}\right)} = 1 \quad \Rightarrow \frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$$

$\ell(TA) = 2a = 8$ $\qquad e^2 = 1 + \frac{b^2}{a^2}$

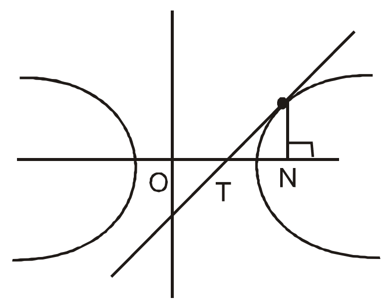
$\Rightarrow e = \frac{5}{4}$

$\ell(LR) = \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$

Directries $x - 1 = \frac{4}{\left(\frac{5}{4}\right)}$ and $x - 1 = -\frac{16}{5}$

$x = \frac{21}{5}$ $\qquad x = -\frac{11}{5}$

Q.4 (B)



Equation of tangent at P (θ)

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

$\therefore$ T (a cosθ, 0), N (a sec θ, 0)

OT. ON = | a cos θ | | a sec θ | = a^2

Q.5 (A)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

tangent at point P (a secθ, b tanθ)

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1 \text{ or } \frac{x}{a\cos\theta} + \frac{y}{(-b\cot\theta)} = 1$$

Point A(acosθ, 0), B(0, – b cotθ)

Cordinate of point P is

(h, k) ≡ (a cosθ, – b cotθ)

$\cos\theta = \frac{h}{a}, \ \cot\theta = -\frac{k}{b}$

$$\cot\theta = \frac{h}{\sqrt{a^2-h^2}} = -\frac{k}{b}$$

$$\frac{h^2}{a^2-h^2} = \frac{k^2}{b^2}$$

$$\frac{a^2}{h^2} - 1 = \frac{b^2}{k^2}$$

So locus is

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

Q.6 (D)

Equation of chord of contact from P (x_1, y_1)

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \quad \text{.......... (1)}$$

similarly from Q $(x_2\ y_2)$, $\frac{xx_2}{a^2} - \frac{yy_2}{b^2} = 1$. .... (2)

$\therefore$ Product of slopes = –1

$$\Rightarrow \frac{x_1x_2}{y_1y_2} = -\frac{a^4}{b^4}$$

Q.7 (C)

Let M (h, k)

Chord with given mid point (h, k)

$$T = S_1 \Rightarrow \frac{hx}{a^2} - \frac{ky}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

$$(\alpha, \beta) \Rightarrow \frac{h\alpha}{a^2} - \frac{k\beta}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

$$\frac{x\alpha}{a^2} - \frac{y\alpha}{b^2} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\frac{x^2}{a^2} - \frac{x\alpha}{a^2} - \left(\frac{y^2}{b^2} - \frac{y\beta}{b^2}\right) = 0$$

$$\frac{x^2}{a^2} - \frac{x\alpha}{a^2} + \frac{\alpha^2}{4a^2} - \frac{\alpha^2}{4a^2} -$$

$$\left(\frac{y^2}{b^2} - \frac{y\beta}{b^2} + \frac{\beta^2}{4b^2} - \frac{\beta^2}{4b^2}\right) = 0$$

$$\left(\frac{x}{a} - \frac{\alpha}{2a}\right)^2 - \left(\frac{y}{b} - \frac{\beta}{2b}\right)^2 = \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2}$$

Centre will be $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ And Hyperbola

Q.8 (D)

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

locus of perpendicular tangents

(Director circle) $x^2 + y^2 = a^2 - b^2$

$x^2 + y^2 = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

But $0 < \alpha < \frac{\pi}{4}$

$\cos\theta < x^2 + y^2 < \cos\frac{\pi}{4}$

$0 < x^2 + y^2 < 1$

So there are infinite points.

Q.9 (A)

Let P$(a\cos\theta, a\sin\theta)$

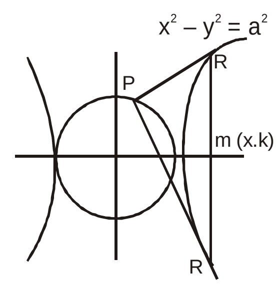
Equation of QR (c.o.c. w.r.t. p) T = 0

$x\cos\theta - y\sin\theta = a$ ...(1)

and $T = S_1$

$hx - ky = h^2 - k^2$ ...(2)

(1) and (2) are same



$$\frac{\cos\theta}{h} = \frac{\sin\theta}{k} = \frac{a}{h^2-k^2}$$

square & add

$(x^2 - y^2)^2 = a^2(x^2 + y^2)$

Q.10 (D)

Let the point $(a\sec\theta, b\tan\theta)$

C.O.C. : $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 2$ ....(1)

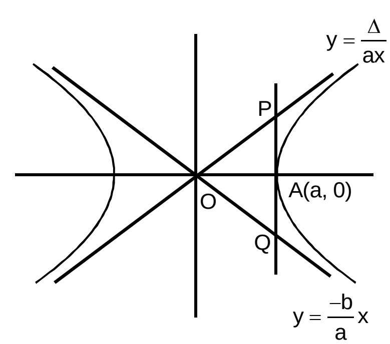
PoI of asymptotes and Eq^n (1)

A[2a (secθ + tanθ), 2b (secθ + tanθ)

B[2a (secθ – tanθ), –2b (secθ – tanθ)

Area of Triangle OAB = $\frac{1}{2}$ (8ab) = 4ab

Q.11 (A)



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Let the any point be (a, 0)

P(a, b), Q (a, –b)

PQ = 2b

OA = a

Area of ΔOPA = $\frac{1}{2} \times a \times 2b = ab$

$\Rightarrow ab = a^2 \tan\lambda$

$\Rightarrow \frac{b}{a} = \tan\lambda$

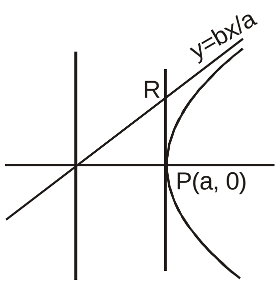
$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\tan^2\lambda} = \sec\lambda$

Q.12 (C)

P(a, 0) ; Q(a, b)

Let M (h, k)

$2h = 2a \Rightarrow h = a$



$k = \frac{b}{2}$

$\left(\frac{h}{a}\right)^2 - \left(\frac{k}{b}\right)^2 = 1 - \frac{1}{4}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{3}{4}$ So $k = \frac{3}{4}$

Q.13 (D)

$xy = c^2$

Let the point is (x_1, y_1) so $x_1y_1 = c^2$

slope of tangent at (x_1, y_1) $y' = -\frac{y_1}{x_1}$

Equation of tangent $(y - y_1) = -\frac{y_1}{x_1}(x - x_1)$

$\frac{x}{2x_1} + \frac{y}{2y_1} = 1$

Foot of perpendicular from origin (0, 0)

$$\frac{x-0}{\frac{1}{2x_1}} = \frac{y-0}{\frac{1}{2y_1}} = -\left(\frac{-1+0+0}{\frac{1}{4x_1^2}+\frac{1}{4y_1^2}}\right)$$

$$x = \frac{\frac{1}{2x_1}}{\frac{1}{4}\left(\frac{1}{x_1^2}+\frac{1}{y_1^2}\right)} = \frac{2x_1y_1^2}{x_1^2+y_1^2}$$

$$y = \frac{2y_1x_1^2}{x_1^2+y_1^2}$$

So $h = \frac{2x_1y_1^2}{x_1^2+y_1^2}$, $k = \frac{2y_1x_1^2}{(x_1^2+y_1^2)}$

$hk = \frac{4x_1^3y_1^3}{(x_1^2+y_1^2)^2} = \frac{4c^6}{(x_1^2+y_1^2)^2}$ .... (i)

$h^2 + k^2 = \frac{4x_1^2y_1^2(x_1^2+y_1^2)}{(x_1^2+y_1^2)^2}$

$(h^2 + k^2) = \frac{4c^4}{(x_1^2+y_1^2)}$

$(x_1^2+y_1^2) = \frac{4c^4}{(h^2+k^2)}$

Put the value in equation (i)

$hk = \frac{4c^6}{16c^8} \times (h^2 + k^2)^2$

$4c^2hk = (h^2 + k^2)^2$

So locus is

$(x^2 + y^2)^2 = 4c^2xy$

Q.14 (B)

Let P $(ct_1, c/t_1)$, Q $(ct_2, c/t_2)$, R $(ct_3, c/t_3)$ and S $(ct_4, c/t_4)$

$\therefore$ by $m_{PQ} \cdot m_{RS} = -1$

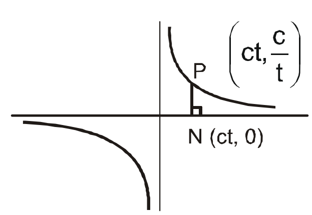
$\Rightarrow t_1 t_2 t_3 t_4 = -1$

and $m_{CP} \times m_{CQ} \times m_{CR} \times m_{CS} = \frac{1}{t_1^2} \times \frac{1}{t_2^2} \times \frac{1}{t_3^2} \times \frac{1}{t_4^2} = 1$

Q.15 (D)

Mid point of PN is $\left(ct, \frac{c}{2t}\right)$

Let it be (h, k)



$\therefore \quad hk = \dfrac{c^2}{2}$

$\Rightarrow$ locus $xy = \dfrac{c^2}{2}$ Hyperbola.

Q.16 (D)

On solving

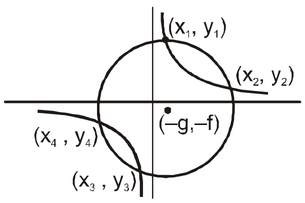
$xy = c^2$ with

circle

$x^2 + y^2 + 2gx + 2fy + \lambda = 0$

$x^2 + \dfrac{c^4}{x^2} + 2gx + \dfrac{2fc^2}{x} + \lambda = 0$

$x^4 + 2gx^3 + \lambda x^2 + 2fc^2x + c^4 d = 0$



$\therefore \quad \sum x_1 = -2g$

$\sum x_1x_2 = \lambda$

and again by eleminating x from equation of circle and hyperbola we have

$\Rightarrow \quad y^4 + 2fy^3 + \lambda y^2 + 2gc^2 y + c^4 = 0$

$\therefore \quad \sum y_1 = -2f$

$\sum y_1y_2 = \lambda$

Now $CP^2 + CQ^2 + CR^2 + CS^2$

$\sum x_1^2 + \sum y_1^2$

$\Rightarrow \quad \left(\sum x_1\right)^2 + \left(\sum y_1\right)^2 - 2\left(\sum x_1x_2 + \sum y_1y_2\right)$

$\Rightarrow \quad 4g^2 + 4f^2 - 4\lambda$

$\Rightarrow \quad 4r^2$

Q.17 (A)

Let $P(ct_1, c/t_1)$ $Q(ct_2, c/t_2)$

$$M_{PQ} = \frac{\dfrac{c}{t_2} - \dfrac{c}{t_1}}{c(t_2 - t_1)} = \frac{-1}{t_1t_2}$$

Equation $y - \dfrac{c}{t_1} = \dfrac{-1}{t_1t_2}(x - ct_1)$

$x + t_1t_2y = c(t_1 + t_2)$

$$\Rightarrow \frac{x}{(ct_1 + ct_2)} + \frac{y}{\left(\dfrac{c}{t_2} + \dfrac{c}{t_1}\right)} = 1$$

$$\frac{x}{x_1 + x_2} + \frac{y}{(y_1 + y_2)} = 1$$

Q.18 (C)

Equation of tangent to $xy = c^2$

at $\left(ct, \dfrac{c}{t}\right)$ is

$\left(y - \dfrac{c}{t}\right) = -\dfrac{1}{t^2}(x - ct)$

$\therefore \quad x_1 = 2ct,\ y_1 = \dfrac{2c}{t}$

and normal $\left(y - \dfrac{c}{t}\right) = t^2(x - ct)$

$\therefore \quad x_2 = ct - \dfrac{c}{t^3},\ y_2 = -ct^3 + \dfrac{c}{t}$

$\therefore \quad x_1x_2 + y_1y_2 = 0$

Q.19 (C)

Tangent at P

$\dfrac{x}{t} + ty = 2c \qquad$ ....(1)

Normal at P

$y - \dfrac{c}{t} = xt^2 - ct^3 \quad$ ....(2)

T (2ct, 0) ; T' (0, 2c/t)

$N\left(ct - \dfrac{c}{t^3}, 0\right)$; $N'\left(0, \dfrac{c}{t} - ct^3\right)$

Δ = Area of $\Delta PNT = \dfrac{1}{2} \times \dfrac{c}{t}\left[2ct - ct + \dfrac{c}{t^3}\right]$

$\Delta = \dfrac{c^2}{2t^4}(t^4 + 1)$

Δ' = Area of $\Delta PN'T'$

$= \dfrac{1}{2} \times ct \times \left[\dfrac{2c}{t} - \dfrac{c}{t} + ct^3\right] = \dfrac{1}{2}c^2(t^4 + 1)$

$$\frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{2}{c^2}$$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (C,D)

Given Hyperbola

$9(x^2 + 2x + 1) - 16(y^2 - 2y + 1)$

$= 151 + 9 - 16$

$\Rightarrow \dfrac{(x+1)^2}{16} - \dfrac{(y-1)^2}{9} = 1$

foci (4, 1), (–6, 1)

Q.2 (B,C)

Asymptotes are $\frac{x}{a} = \pm \frac{y}{b}$

$\tan\theta = \left|\frac{\frac{2b}{a}}{1-\frac{b^2}{a^2}}\right|$ & $e^2 = 1 + \frac{b^2}{a^2}$

$1 - \frac{b^2}{a^2} - \frac{2b}{a}\cot\theta = 0$ ...... (i)

or $\frac{b^2}{a^2} - 1 - \frac{2b}{a}\cot\theta = 0$..... (ii)

by (i) & (ii)

$\left(\frac{b}{a}\right)^2 \pm \frac{2b}{a}\cot\theta - 1 = 0$

$\left(\frac{b}{a}\right) = \frac{\pm 2\cot\theta \pm \sqrt{4\cot^2\theta + 4}}{2}$

$\frac{b}{a} = \pm(\cot\theta \pm \text{cosec}\theta)$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \cot^2\theta + \text{cosec}^2\theta \pm 2\cot\theta\,\text{cosec}\theta$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \cot^2\theta + \text{cosec}^2\theta \pm 2\cot\theta\,\text{cosec}\theta$

$e^2 = 2\,\text{cosec}\theta\,(\cot\theta \pm \text{cosec}\theta)$

$e = \sec\frac{\theta}{2}$ or $e = \text{cosec}\frac{\theta}{2}$

So $\cos\frac{\theta}{2} = \frac{1}{e}$ or $\frac{\sqrt{e^2-1}}{e}$

Q.3 (A,D)

Distance between foci = $\sqrt{19^2+5^2} = \sqrt{386}$

Now by PS + S′P = 2a (for ellipse)

(take point P at origin) we get a = 19

$\therefore$ $2ae = \sqrt{386} \Rightarrow e = \frac{\sqrt{386}}{38}$

If conic is hyperbola

$|PS - PS'| = 2a \Rightarrow a = 6$

by $2ae' = \sqrt{386}$

$e' = \frac{\sqrt{386}}{12}$

Q.4 (A,B,C,D)

$\frac{x^2}{16} + \frac{y^2}{7} = 1$ ....(1)

$\Rightarrow$ $a^2 = 16, b^2 = 7$

i.e. $a = 4, b = \sqrt{7}$

$\therefore$ $e^2 = \frac{a^2-b^2}{a^2}$ $\Rightarrow$ $e = \frac{3}{4}$

$\therefore$ foci ≡ (± ae, 0) = (± 3, 0)

$\frac{x^2}{(144/25)} - \frac{y^2}{(81/25)} = 1$ .....(2)

$\Rightarrow a^2 = \frac{144}{25}, b^2 = \frac{81}{25}$

i.e. $a = \frac{12}{5}, b = \frac{9}{5}$

$\therefore e^2 = \frac{a^2+b^2}{a^2}$ $\Rightarrow$ $e = \frac{5}{4}$

foci ≡ (± ae, 0) = (± 3, 0)

solving (1) and (2) we get $y^2 = \frac{63}{25}$

$\Rightarrow y = \pm\frac{3\sqrt{7}}{5}$ $\Rightarrow$ $x = \pm\frac{16}{5}$

one of the point of intersection is $\left(\frac{16}{5}, \frac{3\sqrt{7}}{5}\right)$

The equation of the asymptote is

$\frac{x^2}{144} - \frac{y^2}{81} = 0$

The abscissa of P is $\frac{16}{5}$

Its ordinate is given by $\frac{y^2}{81} = \frac{16\times16}{25\times144}$

$\therefore$ $y = \pm\frac{12}{5}$

$\therefore$ $P \equiv \left(\frac{16}{5}, \frac{12}{5}\right)$

$\Rightarrow$ $\left(\frac{16}{5}\right)^2 + \left(\frac{12}{5}\right)^2 = 16$

Equation of the auxiliary circle formed on major axis of ellipse $x^2 + y^2 = 16$ P lies on it.

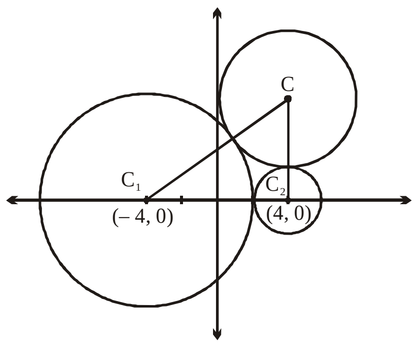
Q.5 (B,C,D)

As, $|cc_1 - cc_2| = |(r + r_1) - (r + r_2)|$ = constant where $|r_1 - r_2| < c_1c_2$

$\Rightarrow$ locus of C is a hyperbola with foci c_1 and c_2 i.e., (–4, 0) and (4, 0).

Also, $2a = |r_1 - r_2| = 2 \Rightarrow a = 1$

Now, $e = \dfrac{2ae}{2a} = \dfrac{8}{2} = 4$



So, $b^2 = 1^2 (4^2 - 1) = 15$

Hence, locus of centre of circle is hyperbola, whose equation

is $\dfrac{x^2}{1} - \dfrac{y^2}{15} = 1$.

Now, verify the options.

Q.6 (B,C)

$H : \sqrt{3}(x-1)^2 - y^2 = -3$

$\Rightarrow H : \dfrac{(x-1)^2}{\sqrt{3}} - \dfrac{y^2}{3} = -1$

auxiliary circle is $(x-1)^2 + y^2 = 3$

$\Rightarrow \quad x^2 + y^2 - 2x - 2 = 0$

$e = \sqrt{1 + \dfrac{\sqrt{3}}{3}} = \sqrt{\dfrac{3+\sqrt{3}}{3}}$

area of $\Delta LOL'$ is $= \dfrac{1}{2}\left(\dfrac{2a^2}{b}\right) \times (be) = a^2e$

$= \sqrt{3}\,e = \sqrt{3+\sqrt{3}}$ sq. units

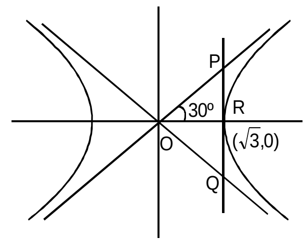
Q.7 (B,C)

$\dfrac{x^2}{3} - \dfrac{y^2}{1} = 1$

Asyp $y = \pm \dfrac{1}{\sqrt{3}} x$

ΔOPQ will be equilateral triangle.

PR = 1



area of $\Delta OPQ = \dfrac{1}{2} \times \sqrt{3} \times (2) = \sqrt{3}$ sq. units

Q.8 (A,B,C)

Normal at P (θ) ≡ P (2secθ , 2 tan θ)

$2x\cos\theta + 2y\cot\theta = 8$

$\Rightarrow \quad x\cos\theta + y\cot\theta = 4$

$\therefore \quad$ G (4secθ, 0), g (0, 4 tan θ) and c (0, 0)

$PG = \sqrt{4\sec^2\theta + 4\tan^2\theta}$ = PC = Pg

Q.9 (A,B,C,D)

Let the point is P(t) so equation of normal at this is

$xt^3 - yt = c(t^4 - 1)$

satisfy by (3, 4)

so $3t^3 - 4t = \sqrt{2}(t^4 - 1)$ [Given xy = 2]

$t^4 - \dfrac{3}{\sqrt{2}} t^3 + 2\sqrt{2}\, t - 1 = 0$

here $t_1t_2t_3t_4 = -1$

& $t_1 + t_2 + t_3 + t_4 = \dfrac{3}{\sqrt{2}}$

But in Cartesian from (x_1, y_1) is

$x_1 = ct_1$ & $y_1 = \dfrac{c}{t_1}$

$\dfrac{x_1x_2x_3x_4}{c^4} = -1$

$x_1x_2x_3x_4 = -c^4 = -4$

similarly $y_1y_2y_3y_4 = \dfrac{c^4}{t_1t_2t_3t_4} = \dfrac{4}{-1} = -4$

$y_1 + y_2 + y_3 + y_4 = c\left(\dfrac{\sum t_1t_2t_3}{t_1t_2t_3t_4}\right) = 4$

$x_1 + x_2 + x_3 + x_4 = c(t_1 + t_2 + t_3 + t_4) = \sqrt{2}\left(\dfrac{3}{\sqrt{2}}\right) = 3$

Q.10 (A,B)

Let the point $P(x_1, y_1)$

tangent at P

$xx_1 - 9yy_1 = 9$

$x\left(\dfrac{x_1}{9}\right) - y(y_1) = 1 \qquad ....(1)$

$\left(\frac{5}{19}\right)x + \left(\frac{12}{19}\right)y = 1$ ....(2)

By comparing (1) & (2)

$x_1 = \frac{45}{19} : y_1 = \frac{-12}{19}$

Q.11 (B,D)

Hyperbola if

$h^2 > ab$

$\Rightarrow \lambda^2 > (2 + \lambda)(\lambda - 1)$

$\Rightarrow \lambda < 2$

and $D \neq 0 \Rightarrow -2[3\lambda - 4] \neq 0 \Rightarrow \lambda \neq 4/3$

Q.12 (A,C)

Let tangent given by

$y = mx + \sqrt{m^2 - 5}$

$\because$ it passes through (2, 8)

$(8 - 2m)^2 = m^2 - 5$

$3m^2 - 32m + 69 = 0 \Rightarrow m = 3$ or $23/3$

$\therefore$ tangent can be

$3x - y + 2 = 0$

or $23x - 3y - 22 = 0$

Q.13 (B,D)

$\frac{x^2}{18} - \frac{y^2}{9} = 1$

given line is

$y = x$

$\therefore$ slope of tangent

$\therefore$ equation is

$y = mx \pm \sqrt{a^2m^2 - b^2} \Rightarrow y = -x \pm 3$

Q.14 (B,D)

$e^2 = 1 + \frac{3}{9} = \frac{4}{3} \Rightarrow e = \frac{2}{\sqrt{3}}$

$\Rightarrow$ (B) is correct

$\Rightarrow \theta = 60^0$

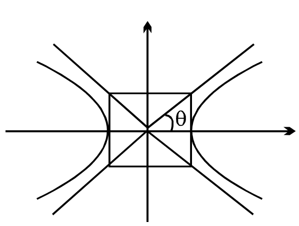
angle between the two asymptotes is 120^0

$\Rightarrow$ acute angle is $60^0 \Rightarrow$ (A) is correct

C : L.L.R. $= \frac{2b^2}{a} = 2.\frac{3}{3} = 2$

$\Rightarrow$ (C) is correct

$p_1p_2 = \frac{ab(\sec\theta + \tan\theta)}{\sqrt{a^2 + b^2}} \frac{ab(\sec\theta - \tan\theta)}{\sqrt{a^2 + b^2}}$



$= \frac{a^2b^2}{a^2 + b^2}(\sec^2\theta - \tan^2\theta) = \frac{9.3}{12} = \frac{9}{4}$

$\Rightarrow$ (D) is incorrect]

Q.15 (A,D)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Asyp. $y = \pm \frac{b}{a}x$

$m_1 = \frac{b}{a}$ and $m_2 = -\frac{b}{a}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\frac{b}{a} + \frac{b}{a}}{1 - \frac{b^2}{a^2}}\right|$

$\tan\theta = \frac{2ab}{a^2 - b^2} \Rightarrow \tan\frac{\theta}{2} = \frac{b}{a}$ and $-\frac{a}{b}$

$\sec\frac{\theta}{2} = \sqrt{1 + \frac{b^2}{a^2}}$ and $\sec\frac{\theta}{2} = \sqrt{1 + \frac{a^2}{b^2}} = e = \frac{1}{e}$

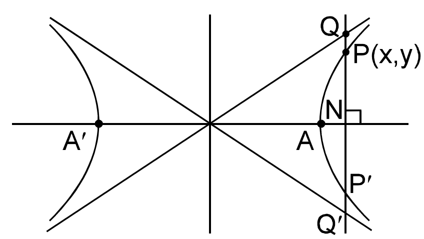
Comprehension # 1 (Q. No. 16 to 18)

Q.16 (B)

Q.17 (D)

Q.18 (B)

Sol.16 $\frac{x^2}{a^2} - 1 = \frac{y^2}{b^2}$



$\frac{(x - a)(x + a)}{y^2} = \frac{a^2}{b^2}$

$\frac{(NA)(NA')}{(PN)^2} = \frac{a^2}{b^2}$

Sol.17 $PQ = NQ - NP$

$= \frac{b}{a}x - \frac{b}{a}\sqrt{x^2 - a^2}$

$PQ' = \frac{b}{a}x + \frac{b}{a}\sqrt{x^2 - a^2}$

$\Rightarrow PQ.PQ' = \frac{b^2}{a^2}x^2 - \frac{b^2}{a^2}(x^2 - a^2) = b^2$

Comprehension # 2 (Q. No. 19 to 21)

Q.19 (D)
Q.20 (C)
Q.21 (B)

Sol.19 Let the asymptotes be $2x + 3y + \lambda = 0$ and $3x + 2y + \mu = 0$

Since, asymptotes passes through (1, 2), then

$\lambda = -8$ and $\mu = -7$

Let the equation of hyperbola be

$(2x + 3y - 8)(3x + 2y - 7) + \gamma = 0$ ...(i)

$\because$ It passes through (5, 3), then

$(10 + 9 - 8)(15 + 6 - 7) + \gamma = 0$

$\Rightarrow \quad 11 \times 14 + \gamma = 0$

$\therefore \quad \gamma = -154$

Putting the value of γ in Eq. (i), then

$(2x + 3y - 8)(3x + 2y - 7) = 154$

Sol.20 The transverse axis is the bisector of the angle between asymptotes containing the origin and the conjugate axis is the other bisector. The bisectors of the angle between asymptotes are

$$\frac{(3x - 4y - 1)}{5} = \pm \frac{(4x - 3y - 6)}{5}$$

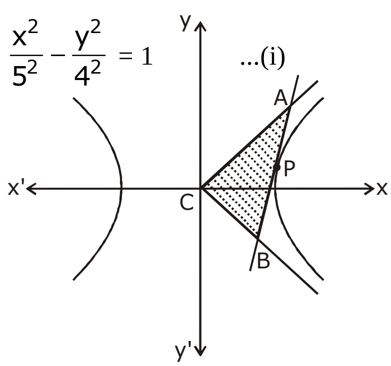
$\Rightarrow \quad (3x - 4y - 1) = \pm (4x - 3y - 6)$

$\Rightarrow \quad x + y - 5 = 0$ and $x - y - 1 = 0$

Hence, transverse axis and conjugate axis are $x + y - 5 = 0$ and $x - y - 1 = 0$

Sol.21 $\because \quad 16x^2 - 25y^2 = 400$

$\Rightarrow \quad \frac{x^2}{5^2} - \frac{y^2}{4^2} = 1$ ...(i)



Let P(5 sec ϕ, 4 tan ϕ) be any point on the hyperbola (i)

Equation of tangent at P is

$$\frac{x}{5}\sec\phi - \frac{y}{4}\tan\phi = 1 \text{...(ii)}$$

And asymptotes of Eq. (i) are

$$y = \pm \frac{4}{5}x \qquad \text{...(iii)}$$

solving Eqs. (ii) and (iii), then

$$\frac{x}{5}\sec\phi \mp \frac{x}{5}\tan\phi = 1$$

or $x = \frac{5}{(\sec\phi \mp \tan\phi)}$

$$= \frac{5(\sec\phi + \tan\phi)(\sec\phi - \tan\phi)}{(\sec\phi \mp \tan\phi)}$$

then we get

A ≡ [5(sec ϕ + tan ϕ), 4(sec ϕ + tan ϕ)]

and B ≡ [(5(sec ϕ – tan ϕ), – 4(sec ϕ – tan ϕ)]

$\therefore$ Area of Δ ABC

$$= \frac{1}{2}\left\| \begin{matrix} 5(\sec\phi + \tan\phi) & 4(\sec\phi + \tan\phi) & 1 \\ 5(\sec\phi - \tan\phi) & -4(\sec\phi - \tan\phi) & 1 \\ 0 & 0 & 1 \end{matrix} \right\|$$

$$= \frac{1}{2}|-20 - 20| = 20 \text{ sq unit}$$

Comprehension # 3 (Q. No. 22 to 24)

Q.22 (C)
Q.23 (B)
Q.24 (A)

Sol.22 PQ – PA = PB – PQ

$\Rightarrow$ QA = BQ

$\therefore$ Q is mid point of AB, Let Q = (h,k)

Equation of chord AB

$T = S_1$

$\frac{1}{2}(xk + yh) = hk$

It passes through P (– 1, 2)

$\therefore$ locus of Q is $2x - y = 2xy$

Sol.23 $\frac{x+1}{\cos\theta} = \frac{y-2}{\sin\theta} = r$

$x = r\cos\theta - 1$, $\qquad y = 2 + r\sin\theta$

Putting it in $xy = c^2$

$r^2 \sin\theta\cos\theta + r(2\cos\theta - \sin\theta) - 2 - c^2 = 0$

$$\text{PA. PB} = \frac{-(2 + c^2)}{\sin\theta\cos\theta} = PQ^2$$

$2 + c^2 + (PQ \sin\theta)(PQ \cos\theta) = 0$

$2 + c^2 + (y - 2)(x + 1)$

$xy + y - 2x + c^2 = 0$

Sol.24 $\frac{2}{PQ} = \frac{PA + PB}{PAPB}$

Gives $2x - y = 2c^2$

Q.25 (A) - (q), (B) - (s), (C) - (s), (D) - (q)

(A) $y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = x \pm \sqrt{5 - b^2}$

$\therefore b = 0, \pm 1, \pm 2$

b can not be zero

$\therefore$ four values are possible

(B) We have, a = 3 and $\frac{b^2}{a}$ = 4 b^2 = 12

Hence, the equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{12} = 1$

$4x^2 - 3y^2 = 36$

(C) The product of the lengths of the perpendiculars from the two focii on any tangent to the hyperbola

$\frac{x^2}{25} - \frac{y^2}{3} = 1$ is 3

$\therefore$ 3 = $\sqrt{k}$, hence k = 9

(D) Equation of the hyperbola can be written as

$\frac{X^2}{5^2} - \frac{Y^2}{4^2} = 1$

where X = x – 3 and Y = y – 2.

$\therefore$ tangent Y = X ± $\sqrt{25-16}$

$\Rightarrow$ y = x + 2 or y = x – 4

Q.26 (A) → (r, t); (B) → (p, s); (C) → (s)

(A) $12x^2 - 4y^2 - 24x + 32y - 127 = 0$

$\Rightarrow 12(x^2 - 2x) - 4(y^2 - 8y) - 127 = 0$

$\Rightarrow 12\{(x-1)^2 - 1\} - 4\{(y-4)^2 - 16\} = 127$

$\Rightarrow 12(x-1)^2 - 4(y-4)^2 = 75$

$\Rightarrow \frac{12(x-1)^2}{75} - \frac{4(y-4)^2}{75} = 1$

$\Rightarrow \frac{75}{4} = \frac{75}{12}(e^2 - 1)$

$\Rightarrow 3 = e^2 - 1$

$\Rightarrow e^2 = 4$

$\therefore$ e = 2

For foci x – 1 = $\pm\left(\frac{5}{2}\times 2\right)$ and y – 4 = 0

$\Rightarrow$ x = 1 ± 5 and y = 4

foci are (– 4, 4) and (6, 4) (r, t)

(B) $8x^2 - y^2 - 64x + 10y + 71 = 0$

$\Rightarrow 8(x^2 - 8x) - (y^2 - 10y) + 71 = 0$

$\Rightarrow 8\{(x-4)^2 - 16\} - \{(y-5)^2 - 25\} + 71 = 0$

$\Rightarrow 8(x-4)^2 - (y-5)^2 = 32$

$\Rightarrow \frac{(x-4)^2}{4} - \frac{(y-5)^2}{32} = 1$

$\Rightarrow 32 = 4(e^2 - 1)$

$\Rightarrow 8 = e^2 - 1$

$\therefore$ e = 3

For foci x – 4 = ± (2 × 3)

and y – 5 = 0

x = 4 ± 6 and y = 5

Foci are (10, 5) and (– 2, 5) (p, s)

(C) $9x^2 - 16y^2 - 36x + 96y + 36 = 0$

$\Rightarrow 9(x^2 - 4x) - 16(y^2 - 6y) + 36 = 0$

$\Rightarrow 9\{(x-2)^2 - 4\} - 16\{(y-3)^2 - 9\} + 36 = 0$

$\Rightarrow 9(x-2)^2 - 16(y-3)^2 = -144$

$\Rightarrow -\frac{(x-2)^2}{16} + \frac{(y-3)^2}{9} = 1$

$\Rightarrow 16 = 9(e^2 - 1)$

$\Rightarrow 25 = 9e^2$

$\therefore$ e = $\frac{5}{3}$

For foci x – 2 = 0

and y – 3 = $\pm\left(3\times\frac{5}{3}\right)$

$\Rightarrow$ x = 2 and y = 3 ± 5

$\therefore$ Foci are (2, – 2) and (2, 8) (s)

Q.27 (A) → (q), (B) → (p), (C) → (q), (D) → (r)

(A) Let P(x_1, y_1)

$\therefore$ normal $y - y_1 = -\frac{y_1}{x_1}(x - x_1)$

$\Rightarrow x_1y + y_1x = 2x_1y_1$

$\therefore$ G($2x_1$, 0) and g(0, $2y_1$)

$\therefore$ PG = PC = Pg = $\sqrt{x_1^2 + y_1^2} = \frac{Gg}{2}$

(B) Since x + y = a touches the hyperbola

$x^2 - 2y^2 = 18$

$\therefore$ $x^2 - 2(a-x)^2 = 18$ has equal roots

i.e. $x^2 - 4ax + 18 + 2a^2 = 0$ has equal roots

$\therefore$ $16a^2 - 4(18 + 2a^2) = 0$

$8a^2 - 72 = 0$

a = ± 3

$\therefore$ | b | = 3

(C) By property, orthocentre always lie on rect. hyperbola

$\therefore$ $\lambda \times 4 = 16$

$\therefore$ $\lambda = 4$

(D) Let P (x, y) and here S(a$\sqrt{2}$,0) and S′ (–a$\sqrt{2}$, 0)

directrices are x = $\frac{a}{\sqrt{2}}$ and x = $-\frac{a}{\sqrt{2}}$

SP . S'P = $\sqrt{2}\left|x - \frac{a}{\sqrt{2}}\right| . \sqrt{2}\left|x + \frac{a}{\sqrt{2}}\right|$

$= 2x^2 - a^2 = x^2 + y^2 = (CP)^2$

NUMERICAL VALUE BASED

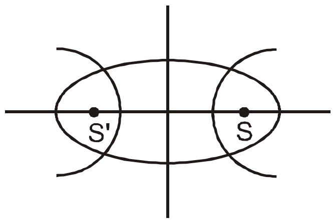
Q.1 (1)

$e = \sqrt{1 - \frac{5}{9}}$, $e' = \sqrt{1 + \frac{45/4}{45/5}}$

$e = \frac{2}{3}$, $e' = \frac{3}{2}$

$\therefore$ e e′ = 1

Q.2 (2)



ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Hyperbola, $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

$\therefore$ $e_1^2 = 1-\frac{b^2}{a^2}$, $e_2^2 = 1+\frac{B^2}{A^2}$

and $2ae_1 = 2Ae_2$
Also, b = B

So, $\frac{b}{ae_1} = \frac{B}{Ae_2}$

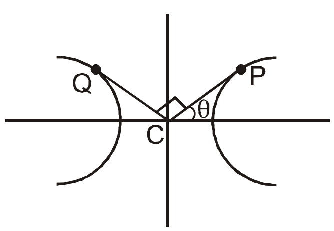
$\therefore$ $e_1^2 = 1 - \frac{B^2}{A^2}\frac{e_1^2}{e_2^2}$

$= 1 - \frac{(e_2^2-1)e_1^2}{e_2^2}$

$e_1^2\ e_2^2 = e_2^2 - e_1^2\ e_2^2 + e_1^2$

$\Rightarrow e_1^{-2} + e_2^{-2} = 2$

Q.3 (1)



CP $\equiv \frac{x-0}{\cos\theta} = \frac{y-0}{\sin\theta} = r_1$ where CP = r_1

$\therefore$ $P(r_1\cos\theta,\ r_1\sin\theta)$

Similarly $Q\left(r_2\cos\left(\frac{\pi}{2}+\theta\right), r_2\sin\left(\frac{\pi}{2}+\theta\right)\right)$

$Q\ (-r_2\sin\theta,\ r_2\cos\theta)$
P & Q lies on Hyperbola

$\therefore$ $r_1^2\left(\frac{\cos^2\theta}{a^2}-\frac{\sin^2\theta}{b^2}\right)=1$

$\therefore$ $r_1^2 = \frac{a^2b^2}{(b^2\cos^2\theta - a^2\sin^2\theta)}$

& $r_2^2 = \frac{a^2b^2}{(b^2\sin^2\theta - a^2\cos^2\theta)}$

$\therefore$ $\frac{1}{r_1^2}+\frac{1}{r_2^2} = \frac{b^2-a^2}{a^2b^2} = \frac{1}{a^2}-\frac{1}{b^2}$ H.P.

Q.4 (6)

Hyp. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ Let the point $P(a\sec\theta,\ b\tan\theta)$

Asy $y = \pm\frac{b}{a}x$

$ay - bx = 0$ and $ay + bx = 0$
$p = p_1 \cdot p_2$

$= \left|\frac{ab\tan\theta - ab\sec\theta}{\sqrt{a^2+b^2}}\right| \left|\frac{ab\tan\theta + ab\sec\theta}{\sqrt{a^2+b^2}}\right|$

$p = \frac{a^2b^2}{a^2+b^2} \Rightarrow \frac{a^2b^2}{a^2+b^2} = 6$ ....(1)

$e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2}$

$\Rightarrow a^2 + b^2 = 3a^2$ ....(2)

(1) and (2) $b^2 = 18$

$\Rightarrow a^2 = 9 \Rightarrow a = 3 = TA = 2a = 6$

Q.5 (0)

by H + H' = 2A we get combined eq^n of Asymptotes as

$A = 0 \Rightarrow x^2 + 3xy + 2y^2 + 2x + 3y + \left(1+\frac{c}{2}\right) = 0$

It represents pair of straight line then c = 0

by $\begin{vmatrix} 1 & 3/2 & 1 \\ 3/2 & 2 & 3/2 \\ 1 & 3/2 & \left(1+\frac{c}{2}\right) \end{vmatrix} = 0$

Q.6 (77)

Let P(x, y) be any point on the hyperbola
Then by focus directrix property

$$\frac{\text{distance of P from the focus}}{\text{distance of P from the directrix}} = e = 3$$

$\therefore$ $\left|\frac{\sqrt{(x+1)^2+(y-1)^2}}{\frac{x-y+3}{\sqrt{1^2+(-1)^2}}}\right| = 3$

or $(x + 1)^2 + (y - 1)^2 = 9 \cdot \left(\frac{x - y + 3}{\sqrt{2}}\right)^2$

or $7x^2 - 18xy + 7y^2 + 50x - 50y + 77 = 0$

Q.7 (32)

Tangent to the hyp. $xy = c^2$

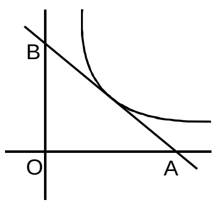
$\frac{x}{x_1} + \frac{y}{y_1} = 2$ (16, 1)

$\frac{x}{16} + \frac{y}{1} = 2$

$x + 16y = 32$

A(32, 0)

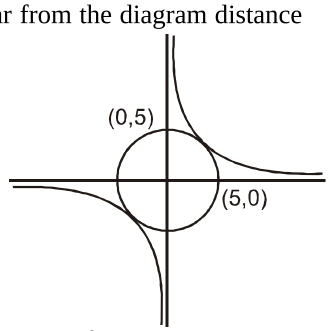
B(0, 2)



Area $= \frac{1}{2} \times 2 \times 32 = 32$ Sq. unit

Q.8 (10)

It is clear from the diagram distance



between point of contacts is 10

Q.9 (22)

Let (x_1, y_1) be the pt, of contact of tangent $3x - 4y = 5$ to $x^2 - 4y^2 = 5$ Solving we have

$\Rightarrow$ $(x_1, y_1) = (3, 1)$

Now any tangent to $\frac{x^2}{25} - \frac{y^2}{16} = 1$ is

$y = mx \pm \sqrt{25m^2 - 16}$

$\Rightarrow$ $y^2 + m^2x^2 - 2mxy = 25m^2 - 16$ ....(i)

$\because$ (1) passes through (3, 1)

$\therefore$ $16m^2 + 6m - 17 = 0$ ....(ii)

Let m_1 & m_2 be the roots of (ii) and $m_1 + m_2 = -\frac{3}{8}$ and $m_1 m_2 = \frac{-17}{16}$

$\therefore$ $32(m_1 + m_2 - m_1 m_2) = 22$

Q.10 (4)

$3x^2 - 2y^2 = 6$

$\frac{x^2}{2} - \frac{y^2}{3} = 1$

Let the equation of tangent

$y = mx + \sqrt{a^2m^2 - b^2}$

passes through (α, β)

$(\beta - m\alpha)^2 = a^2m^2 - b^2$

$m^2\alpha^2 + \beta^2 - 2m\alpha\beta = a^2m^2 - b^2$

$m^2(\alpha^2 - a^2) - 2m\alpha\beta + \beta^2 + b^2 = 0$

$m_1m_2 = \frac{\beta^2 + b^2}{\alpha^2 - a^2} = 2$

$2\alpha^2 - 2a^2 = \beta^2 + b^2$

or $2\alpha^2 - 4 = \beta^2 + 3$

$\beta^2 = 2\alpha^2 - 7$

Q.11 (0030)

Tangent on $(3\sec\phi, 4\tan\phi)$ is

$\frac{\sec\phi}{3}x - \frac{\tan\phi}{4}y = 1$ ......(i)

given that (i) is $\perp$ to $3x + 8y - 12 = 0$

$\Rightarrow$ $\frac{4}{3}\left(\frac{\sec\phi}{\tan\phi}\right)\left(\frac{-3}{8}\right) = -1$

$\Rightarrow$ $\phi = 30^\circ$

Q.12 (0025)

P is $(3\sec\theta, 4\tan\theta)$

Tangent at P is $\frac{x}{3}\sec\theta - \frac{y}{4}\tan\theta = 1$

It meets $4x - 3y = 0$ i.e. $\frac{x}{3} = \frac{y}{4}$ in Q

$\therefore$ Q is $\left(\frac{3}{\sec\theta - \tan\theta}, \frac{4}{\sec\theta - \tan\theta}\right)$

It meets $4x + 3y = 0$

i.e. $\frac{x}{3} = -\frac{y}{4}$ in R

$\therefore$ R is $\left(\frac{3}{\sec\theta + \tan\theta}, \frac{-4}{\sec\theta + \tan\theta}\right)$

$\therefore$ CQ .CR $= \left(\frac{\sqrt{3^2 + 4^2}}{\sec\theta - \tan\theta}\right)\left(\frac{\sqrt{3^2 + 4^2}}{\sec\theta + \tan\theta}\right) = 25$

KVPY PREVIOUS YEAR'S

Q.1 (B)

$x^2 - y^2 = a^2$

A(– a, 0)

B $(a\sec\theta, a\tan\theta)$

B $(a\sec\theta, -a\tan\theta)$

$M_{AB} = \tan 30^\circ = \frac{a\tan\theta}{a\sin\theta + 1} = \frac{1}{\sqrt{3}}$

$\sqrt{3}\tan\theta = 1 + \sin\theta$

$\sqrt{3}\ \tan\theta = 1 + \sec\theta$

$\left(\sqrt{3}\tan\theta - 1\right)^2 = \sec^2\theta$

$3\tan^2\theta - 2\sqrt{3}\ \tan\theta + 1 = 1 + \tan^2\theta$

$3\tan^2\theta - 2\sqrt{3}\ \tan\theta = 0$

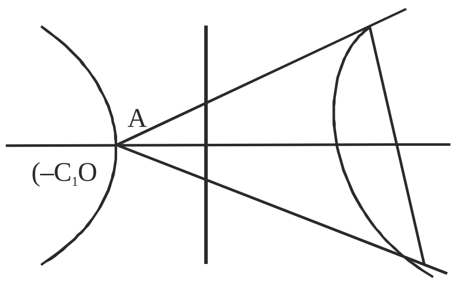
$\tan\theta = \sqrt{3}$

side length = 2a tan θ

$= 2a\sqrt{3}$

$= 2\sqrt{3}\ a$

$K = 2\sqrt{3}$

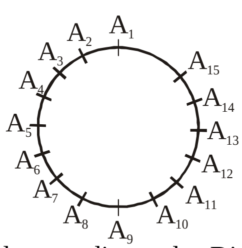


Q.2 **(A)**

Total diagonals = $^{15}C_2 - 15 = 90$

Shortest diagonal = Diagonal connecting (A_1A_3, A_2A_4, ...)

= 15



longest diagonal = Diagonal connecting

(A1A8, A1A9, ...)

= 15

Required probability $= \dfrac{90-15-15}{90}$

$= \dfrac{60}{90} = \dfrac{2}{3}$

JEE MAIN PREVIOUS YEAR'S

Q.1 (2)

$\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

a = 5, b = 4

$e = \sqrt{1 - \dfrac{16}{25}} = \dfrac{3}{5}$

focii : (3, 0), (–3, 0)

let equatio of hyperbola is $\dfrac{x^2}{A^2} - \dfrac{y^2}{B^2} = 1$

satisfy (±3, 0) $\Rightarrow \dfrac{9}{A^2} = 1 \Rightarrow A^2 = 9$

eccentricity of hyperbola

$= \dfrac{1}{\text{eccentricity of ellipse}} = \dfrac{5}{3}$

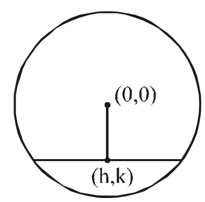
$\Rightarrow \dfrac{5}{3} = \sqrt{1 + \dfrac{B^2}{9}} \Rightarrow 1 + \dfrac{B^2}{9} = \dfrac{25}{9} \Rightarrow B^2 = 16$

equation of hyperbola is

$\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

Q.2 **(4)**

$x^2 + y^2 = 25$



Equation of chord

$y - k = -\dfrac{h}{k}(x - h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y = -\dfrac{hx}{k} \quad \dfrac{h^2 + k^2}{k}$

tangent to $\dfrac{x^2}{9} \quad \dfrac{y^2}{16} = 1$

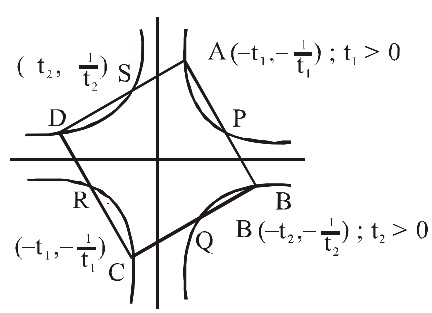
$c^2 = a^2m^2 - b^2$

$\left(\dfrac{h^2 + k^2}{k}\right)^2 = 9\left(-\dfrac{h}{k}\right)^2 - 16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

Q.3 **(80)**

xy = 1, −1



$$\frac{t_1+t_2}{2}\cdot\frac{\frac{1}{t_1}-\frac{1}{t_2}}{2}=1$$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$$\frac{1}{t_1^2}\times\left(-\frac{1}{t_2^2}\right)=-1 \Rightarrow t_1t_2=1$$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_1^2 = 4$

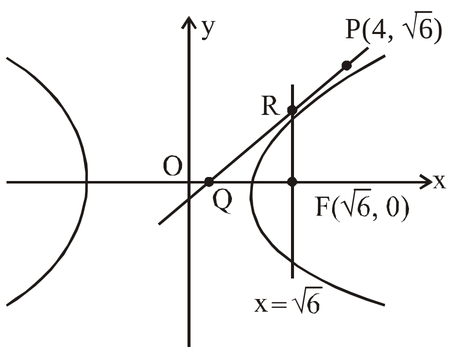
$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2+4} = 2\sqrt{5}$

$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \frac{1}{t_1^2} = \sqrt{5}-2$

$$AB^2 = (t_1-t_2)^2 + \left(\frac{1}{t_1}+\frac{1}{t_2}\right)^2$$

$$= 2\left(t_1^2+\frac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80$$

Q.4 **(3)**



$$\frac{x^2}{4}-\frac{y^2}{2}=1$$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$$

$\therefore$ Focus F(ae, 0) $\Rightarrow F\left(\sqrt{6},0\right)$

equation of tangent at P to the hyperbola is

$2x - y\sqrt{6} = 2$

tangent meet x-axis at Q(1, 0)

& latus rectum $x = \sqrt{6}$ at $R\left(\sqrt{6}, \frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)\right)$

$\therefore$ Area of $\Delta_{QFR} = \frac{1}{2}\left(\sqrt{6}-1\right).\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)$

$= \frac{7}{\sqrt{6}}-2$

Q.5 **(4)**
Q.6 (1)
Q.7 (3)
Q.8 (3)
Q.9 **[5]**

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (B, D)

Eccentricity of ellipse = $\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sqrt{1+\frac{b^2}{a^2}} = \frac{2}{\sqrt{3}}$

$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$

focus of ellipse $\left(\pm\sqrt{3}, 0\right) \Rightarrow \frac{(\sqrt{3})^2}{a^2} = 1$

$\Rightarrow a = \sqrt{3}$

$\Rightarrow$ b = 1 & focus of hyperbola (±2, 0)

Hence equation of hyperbola $\frac{x^2}{3}-\frac{y^2}{1} = 1$

Q.2 (B)

Equation of normal at P(6, 3)

$\frac{a^2x}{6}+\frac{b^2y}{3} = a^2 + b^2$

It passes through (9, 0)

$\frac{3}{2}a^2 = a^2 + b^2$

$\Rightarrow \frac{3}{2} = \frac{a^2+b^2}{a^2} = 1 + \frac{b^2}{a^2}$

$\Rightarrow e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$

Q.3 (AB)

Slope of tangents = 2

Equation of tangents $y = 2x \pm \sqrt{9.4-4}$

$\Rightarrow y = 2x \pm \sqrt{32}$

$\Rightarrow 2x - y \pm 4\sqrt{2} = 0$ ....(i)

Let point of contact be (x_1, y_1)

then equation (i) will be identical to the equation

$\frac{xx_1}{9}-\frac{yy_1}{4} - 1 = 0$

$\therefore \frac{x_1/9}{2} = \frac{y_1/4}{1} = \frac{-1}{\pm 4\sqrt{2}}$

$\Rightarrow (x_1, y_1) \equiv \left(-\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ and $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Q.4 (A,C,D)

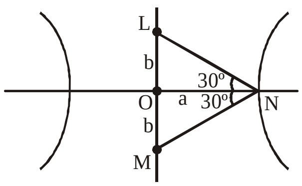
y= 2x + 1 is tangent to $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$

$c^2 = a^2m^2 - b^2$

$1 = 4a^2 - 16 \Rightarrow a^2 = \frac{17}{4}$

[check if $p^2 = q^2 + r^2$]

Q.5 (B)



$\tan 30^\circ = \frac{b}{a}$

$\Rightarrow a = b\sqrt{3}$

Now area of $\Delta LMN = \frac{1}{2}.2b.b\sqrt{3}$

$4\sqrt{3} = \sqrt{3}b^2$

$\Rightarrow b = 2 \;\&\; a = 2\sqrt{3}$

$\Rightarrow e\sqrt{1+\frac{b^2}{a^2}} = \frac{2}{\sqrt{3}}$

P. Length of conjugate axis = 2b=4
So P $\rightarrow$ 4

Q. Eccentricity $e = \frac{2}{\sqrt{3}}$
So Q $\rightarrow$ 3

R. Distance between foci =2ae

$= 2\left(2\sqrt{3}\right)\left(\frac{2}{\sqrt{3}}\right) = 8$

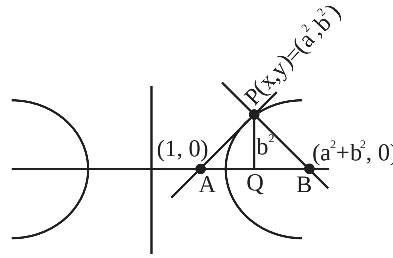
So R $\rightarrow$ 1

S. Length of latus rectum =

$\frac{2b^2}{a} = \frac{2(2)^2}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

S $\rightarrow$ 2

Q.6 (A, D)



Since Normal at point P makes equal intercept on co-ordinate axes, therefore slope of Normal = –1
Hence slope of tangent = 1
Equation of tangent
y – 0 = 1(x–1)
y = x – 1
Equation of tangent at $(x_1 y_1)$

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

x – y = 1 (equation of Tangent)
on comparing $x_1 = a^2$, $y_1 = b^2$
Also $a^2 - b^2 = 1$ ...(1)
Now equation of normal at $(x_1, y_1) = (a^2{}_1, b_1{}^2)$

$y - b^2 = -1(x - a^2)$
$x + y = a^2 + b^2$....(Normal)
point of intersection with x-axis is (a^2+b^2)

Now $e = \sqrt{1+\frac{b^2}{a^2}}$

$e = \sqrt{1+\frac{b^2}{b^2+1}}$ $\left[\text{from}(1)\frac{b^2}{b^2+1} < 1\right]$

$1 < e < \sqrt{2}$ Option (A)

$\Delta = \frac{1}{2}.AB.PQ$

and $\Delta = \frac{1}{2}\left(a^2+b^2-1\right).b^2$

$\Delta = \frac{1}{2}\left(2b^2\right)b^2$ (from (1) $a^2 - 1 = b^2$)

$\Delta = b^4$ so option (D)

Set and Relation

EXERCISES

JEE-MAIN
OBJECTIVE PROBLEMS

Q.1 (2)
A = {2, 3, 4 ....}
B = {0, 1, 2, 3 ....}
$A \cap B = \{2, 3\}$
Then $A \cap B$ is $\{x : x \in R, 2 \le x < 4\}$

Q.2 (2)

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} \ \forall\, a_i \in \{0, 1\}$$

This deter minant will take value O, 1 or –1 only & '1' will be taken same no. of times as –1 ; so n(B) = n(C)

Q.3 (3)
$A = \{\phi, \{\phi\}\}$
P(A) = set containing all subsets
$= \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}$
$= \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

Q.4 (1)
A = {2, 3} ; B = {1, 2}
$A \times B = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

Q.5 (3)
$n(A \cap B) = n(A) + n(B) - n(A' \cap B')$
= 200 + 300 – 100
$n(A \cap B) = 400$
Now $n(A' \cap B') = U - n(A \cup B)$
(De marganistans)
= 700 – 400 = 300

Q.6 (4)
conceptual
2^n

Q.7 (1)

$P : a \rho b$ iff $|a - b| \le \frac{1}{2}$

Reflexive : $a \rho b : |0 - a| \le \frac{1}{2}$ (True)

Symmetric : $a \rho b \Rightarrow b \rho a$

$$|a - b| \le \frac{1}{2} \Rightarrow |b - a| \le \frac{1}{2} \text{ (True)}$$

Transitive : $a \rho b : b \rho a \Rightarrow a \rho c$

$$|a - b| \le \frac{1}{2}\ ;\ |b - c| \le \frac{1}{2}$$

$$\Rightarrow |a - c| \le \frac{1}{2}$$

so not transitive

Q.8 (2)
Reflexive relation : a R a
but identity relation is $y = x : x \in A$ & $y \in A$
so $I \subset R$

Q.9 (2)
R = {(1, 2), (2, 3)}
for Reflexive : a R a
for symmetric : a R b $\Rightarrow$ b R a
for transitive : a R b, b R c $\Rightarrow$ a R c
So elements to be added
{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (1, 3), (3, 1)}

Q.10 (3)
for $x = 2, y = 3 \in N$
$x = 4, y = 2 \in N$
$x = 6, y = 1 \in N$

Q.11 (3)
$(4, 2) \in R$ but $(2, 4) \notin R$ &
$(2, 3) \in R$ but $(3, 2) \notin R$

KVPY
PREVIOUS YEAR'S

Q.1 (A)
for $A \cap B$
cos (sinθ) = 1 or –1 & sin (cosθ) = 0
which is not possible
or cos (sinθ) = 0 & sin (cosθ) = 1 or –1
also not possible
so $A \cap B$ is an empty set

Q.2 (C)
A = {1,2,6,7,11,12,16,17,21,22,26,27,31,32,36,37}
& One of the element which is multiple of 5
B = {3,4,8,9,13,14,18,19,23,24,28,29,33,34,38,39}
& One of the element which is multiple of 5

Q.3 (C)
Good subset is total number of symmetric subset

Q.4 (D)
n+ 1, n + 2, ... n + 18
(A) False, if n = 19
(C) False if n = 15
16 to 33
20, 25, 30 ® only three multiples of 5
(D) no. of odd integers in $S_n = 9$
every third odd integer is multiple of 3
so maximum prime no. = 6

Q.5 (C)

$$100000 \le ababab < 1000000$$

$$\le 10^5 a + 10^4 b + 10^3 a + 100b + 10a + b < 1000000$$

$$\le a\left(10^5 + 10^3 + 10\right) + b\left(10^4 + 10^2 + 1\right) \le 1000000$$

$100000 \le (10^4 + 10^2 + 1)(100a + b) < 100000$

$100000 \le 10101(ab) < 100000$

$9.9 \le ab \le 99$

'ab' number can be obtained as product of ordered pairs (2, 5); (2, 11); (2, 17); (2, 19); (2, 23); (2, 29); (2, 31); (2, 41); (2, 43); (2, 47); (5, 11); (5, 17); (5, 19)

Total number = 13

Q.6 (C)

$$^5C_2 2 + {}^5C_3 \frac{3!}{1!} + {}^5C_4\left[\frac{4!\,2!}{1!\,3!} + \frac{4!}{1!\,2!}\right] + \frac{5!\,2!}{1!\,4!} + \frac{5!\,2!}{2!\,3!}$$

$20 + 10 \times 6 + 5[8 + 6] + 10 + 20 = 180$

Q.7 (C)

As $n \to \infty$

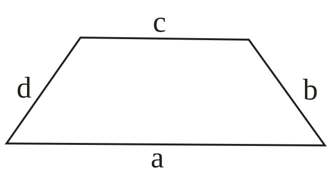
$$\left|\sin\sqrt{x+1} - \sin\sqrt{x}\right| \to 0$$

$\therefore$ There exist infinite natural numbers for which

$$\left|\sin\sqrt{x+1} - \sin\sqrt{x}\right| < \lambda \,\forall\, \lambda > 0$$

Hence $A_{\frac{1}{2}}, A_{\frac{1}{3}}, A_{\frac{2}{5}}$ are all infinite sets

Q.8 **(B)**



$|a - c| < b + d < a + c$

(a, c) (b, d) (1, 3) (5, 6)

(1, 3) (4, 5)

and (1, 3) (4, 6)

Now make different combination. Total of 11 combination are possible.

Q.9 (D)

$\cos x + \cos\sqrt{2}x < 2$

cos x £1 and cos $\sqrt{2}x \le 15$

cos x + cos $\sqrt{2}x \le 15$ at x = 0 cos x + cos $\sqrt{2}x = 2$

Þ x Î R – {0}

Q.10 (C)

$$\frac{2a-1}{b} \ge 1 \Rightarrow a \ge \frac{b+1}{2} \Rightarrow \frac{1}{a} \le \frac{2}{b+1}$$

$$\Rightarrow \frac{2b-1}{a} \le \frac{4b-2}{b+1} = 4 - \frac{6}{b+1} < 4$$

$$\Rightarrow \frac{2b-1}{a} = 1, 2, 3$$

$2b - 1$ is odd $\Rightarrow \frac{2b-1}{a} = 1, 3$

Case (i) Let $\frac{2b-1}{a} = 1$

$$\Rightarrow \frac{2a-1}{b} = \frac{2(2a-1)}{a+1} = 4 - \frac{6}{a+1}$$

for a = 1, $\frac{2a-1}{b} = 4 - 3 = 1 \Rightarrow a = 1, b = 1$

for a = 3, $\frac{2a-1}{b} = 4 - \frac{3}{2} \notin I$

for a = 5, $\frac{2a-1}{b} = 4 - 1 = 3 \Rightarrow a = 5, b = 3$

case (ii) Let $\frac{2b-1}{a} = 3$

$\Rightarrow a = 3, b = 5$ (similar as case (i))

Q.11 (A)

$(1 + a^2)(1 + b^2) = 4ab$

$$\Rightarrow \left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right) = 4$$

$\Rightarrow$ a = 1 and b = 1

but a ≠ 1 so no value of b

Q.12 **(A)**

$f_n = (n + 1)^{1/3} - n^{1/3}$

Rationalising f_n get

$$f_n = \frac{1}{(n+1)^{2/3} + n^{1/3}(n+1)^{1/3} + n^{2/3}} > \frac{1}{3(n+1)^{2/3}}$$

Similarty

$$f_n + 1 = \frac{1}{(n+1)^{2/3} + (n+1)^{1/3} + (n+2)^{1/3} + (n+1)^{2/3}} > \frac{1}{3(n+1)^{2/3}}$$

Hence $f_n + 1 = \frac{1}{3(n+1)^{2/3}} < f_n \forall n \in N$

Hence A = N

Q.13 **(A)**

(I) This relation is reflexive relation because every natural no. divides square of itself a R a $\Leftrightarrow$ a divides a^2

(II) not symmetric eg. 5 R 10 $\Leftrightarrow$ 5 Divide 100

But 10 R 5 $\not\Leftrightarrow$ 10 Divide 25 ?

(III) Not transitivity for example

if 8 R 4 & 4 R 2 $\not\Rightarrow$ 8 R 2

only (I) Option

Q.14 **(D)**

n(A × A) = 100

number of (a,a) type pairs is 10

number of (a,b) and (b,a) type pair of pairs is 45 ($a \neq b$)
so, required number of relations is
$2^{90} - 2^{45}$

JEE MAIN PREVIOUS YEAR'S

Q.1 (5.00)

3 digit number of the form 9K + 2 are {101,109,..........,992}

$\Rightarrow$ Sum equal to $\frac{100}{2}(1093)$

Similarly sum of 3 digit number of the form 9K + 5 is $\frac{100}{2}$ (1099)

$\frac{100}{2}(1093) + \frac{100}{2}(1099) = 100 \times (1096)$

$= 400 \times 274$

$\Rightarrow \ell = 5$

Q.2 **(3)**

$A \cap B \cap C$ is visible in all three venn diagram
Hence, Option (3)

Q.3 (832)

Q.4 **(5143)**

Q.5 (3)

Q.6 (1)

The equivalence class containing (1, –1) for this relation is $x^2 + y^2 = 2$

Q.7 **(4)**

A = {2, 3, 4, 5, ... , 30}
$(a, b) \simeq (c, d) \Rightarrow ad = bc$
$(4, 3) \simeq (c, d) \Rightarrow 4d = 3c$

$\Rightarrow \frac{4}{3} = \frac{c}{d}$

$\frac{c}{d} = \frac{4}{3}$ & $\chi, \delta \in \{2, 3, ..., 30\}$

(c, d) = {(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)}
No. of ordered pair = 7

Q.8 **(3)**

A and B are matrices of $n \times n$ order & ARB iff there exists a non singular matrix P($\det(P) \neq 0$) such that $PAP^{-1} = B$

For reflexive

ARA $\Rightarrow$ PAP–1 = A...(1) must be true
for P = I, Eq.(1) is true so 'R' is reflexive

For symmetric

ARB $\Leftrightarrow PAP^{-1} = B$ ...(1) is true
for BRA iff $PBP^{-1} = A$ ...(2) must be true
Q $PAP^{-1} = B$
$P^{-1}PAP^{-1} = P^{-1}B$
$IAP^{-1}P = P^{-1}BP$
$A = P^{-1}BP$ ...(3)
from (2) & (3) $PBP^{-1} = P^{-1}BP$
can be true some $P = P^{-1} \Rightarrow P^2 = I$ ($\det(P) \neq 0$)
So 'R' is symmetric

For trnasitive

ARB $\Leftrightarrow PAP^{-1} = B$... is true
BRC $\Leftrightarrow PBP^{-1} = C$... is true
now $PPAP^{-1}P^{-1} = C$
$P^2A(P^2)^{-1} = C \Rightarrow$ ARC
So 'R' is transitive relation
$\Rightarrow$ Hence R is equivalence

Q.9 (2)

Q.10 (2)

JEE ADVANCED PREVIOUS YEAR'S

Q.1 **[3748]**

X : 1, 6, 11, ..., 10086
Y : 9, 16, 23, 14128
$X \cap Y$: 16, 51, 86, ...
Let $m = n(X \cap Y)$
$\therefore$ $16 + (m - 1) \times 35 \leq 10086$
$\Rightarrow$ $m \leq 288.71$
$\Rightarrow$ m = 288
$\therefore$ $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$= 2018 + 2018 - 288 = 3748$

Q.2 (A,B,D)

(A) $n_1 = 10 \times 10 \times 10 = 1000$
(B) As per given condition $1 \leq i < j + 2 \leq 10 \Rightarrow j \leq 8$ & $i \geq 1$
for i = 1, 2, j = 1, 2, 3, ..., 8 $\rightarrow$ (8 + 8) possibilities
for i = 3, j = 2, 3, ..., 8 $\rightarrow$ 7 possibilities
i = 4, j = 3, ..., 8 $\rightarrow$ 6 possibilities
i = 9, j = 1 $\rightarrow$ 1 possibility
So $n_2 = (1 + 2 + 3 + ...+ 8) + 8 = 44$
(C) $n_3 = {}^{10}C_4$ (Choose any four)
$= 210$
(D) $n_4 = {}^{10}C_4.4! = (210)(24)$

$\Rightarrow \frac{n_4}{12} = 420$

So correct Ans. (A), (B), (D)

Mathematical Reasoning

EXERCISES

JEE-MAIN
OBJECTIVE PROBLEMS

Q.1 (3)

Here option A, B, & D is mathematical acceptable sentance so these are statement but option C is interogative sentance so it is nto statement.

Q.2 **(3)**

A, B $\rightarrow$ imperative sentence

D $\rightarrow$ exclametry sentence

C $\rightarrow$ Mathematically acceptable statement it is univossal fact

so the sun is a star is a statement.

Q.3 **(3)**

$\sim (p \wedge q) = \sim p \vee \sim q$

$\sim (2 + 3 = 5 \text{ and } 8 < 10) = 2 + 3 \neq 5 \text{ or } 8 \nless 10$

Q.4 **(3)**

$\sim (p \vee q) = \sim p \wedge \sim q$

so monu is not in class X or Anu is not in class XII

Q.5 **(2)**

If p then q is false

p	q	$p \rightarrow q$
T	T	T
T	P	F
F	T	T
F	F	T

$p \rightarrow q : F$

$p : T , q : F$

Q.6 **(3)**

$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$ is

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim p \wedge \sim q$	$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$\therefore$ neither tautology nor contradiction

Q.7 (4)

Fundamental concept of distribution law

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

Q.8 **(2)**

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$p \rightarrow q \quad \Rightarrow \sim q \rightarrow \sim p$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

hence

$p \rightarrow q \quad \Rightarrow \sim q \rightarrow \sim p$ is tautology

Q.9 (1)

Ram is smart and Ram is intelligent $\Rightarrow (p \wedge q)$

Q.10 (2)

It is a fundamental concept.

Q.11 (3)

Contrapositive of $p \Rightarrow \sim q$ is $q \Rightarrow \sim p$

Q.12 (4)

ΔABC is equilateral triangle if each angle is 60° $p \Leftrightarrow q$.

Q.13 (3)

$\sim (p \vee q) \Rightarrow \sim p \wedge \sim q$

Q.14 (3)

$s = p \Rightarrow q \wedge \sim q$ is contradiction

p	s	$p \rightarrow s$
T	F	F
F	F	T

neither tautology nor contradiction.

Q.15 (3)

$\sim (p \wedge q) \vee \sim (q \Leftrightarrow p)$

p	q	$\sim (p \wedge q)$	$\sim (q \Leftrightarrow p)$	s
T	T	F	F	F
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

Q.16 (2)

Equations are not a statement but 5 is natural no. is a statement.

Q.17 (1)

Q.18 (1)

p	q	$\sim p$	pvq	$\sim$ (pvq)	$\sim p \wedge q$	$\sim (pvq) v (\sim p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	F	T
F	F	T	F	T	T	T

So, $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to $\sim p$

Q.19 (2)

$p \rightarrow q$ is false only when p is true and q is false.

$p \rightarrow (\sim p \vee q)$ is false only when p is true and $(\sim p \vee q)$ is false.

$\sim p \vee q$ is false if q is false, because $\sim p$ is false.

Q.20 (1)

p	q	~ p	$p \Leftrightarrow q$	$\sim p \wedge (p \Leftrightarrow q) = s$	$\sim s = p \vee q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	T	T	F

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

Contrapositive of $A \rightarrow (B \rightarrow A)$ is

$\sim (B \rightarrow A) \rightarrow \sim A$

$(B \wedge \rightarrow A) \rightarrow \sim A$

Q.2 (2)

p : you work ward

q : you will earn

given $(p \rightarrow q)$

contrapositive of $(p \rightarrow q) = \sim q \rightarrow \sim p$

Q. 3. (2)

$\sim(\sim p \wedge (p \vee q))$

$= \sim (\sim p \wedge p) \vee (\sim p \wedge q))$

$= \sim (\sim p \wedge q) = p \vee \sim q$

Q.4 (1)

$A \wedge (\sim A \vee B) \rightarrow B$

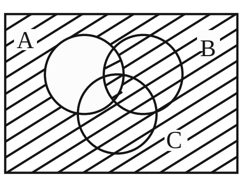
$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$

$= (A \wedge B) \rightarrow B$

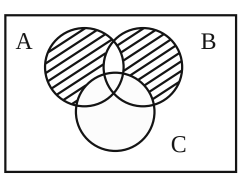
$= \sim A \vee \sim B \vee B \qquad = t$

Q.5 (3)

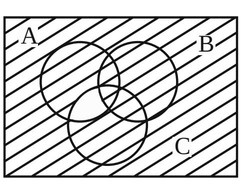
$(\sim A \vee B) \equiv$



$\sim C \wedge (A \vee B)$

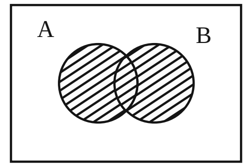


$\therefore F_1 :$

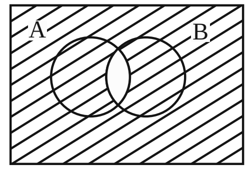


$\Rightarrow$ Not tautology

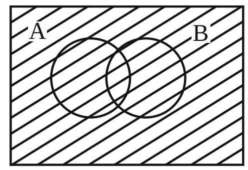
$A \vee B \equiv$



$A \rightarrow \sim B$



$\therefore F_2 \rightarrow$



Tautology

Truth tabel for F_1 (A,B,C)

A	B	C	~ A	~ C	$A \vee B$	$\sim A \vee B$	$\sim C \wedge (A \vee B)$	$(\sim A \vee B) \vee (\sim C \wedge (A \vee B)) \vee \sim A$
T	T	T	F	F	T	T	F	T
T	F	F	F	T	T	F	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F	F
F	T	T	T	F	T	T	F	T
F	F	F	T	T	F	T	F	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	T	F	T

Truth table for F_2

A	B	$A \vee B$	~ B	$A \rightarrow \sim B$	$(A \vee B) \vee (A \rightarrow \sim B)$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

F_1 not shows tautology and F_2 shows tautology.

Q.6 **(4)**

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$ is tautology

Q.7 **(1)**

Q$p \rightarrow q \equiv \sim p \vee q$

So, $* \equiv \vee$

Thus, $p * (\sim q) \equiv p \vee (\sim q)$

$\equiv q \rightarrow p$

Q.8 **(1)**

Option (1)

$(p \wedge q) \longrightarrow (p \rightarrow q)$

$= \sim (p \wedge q) \vee (\sim p \vee q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee (\sim q \vee q)$

$= \sim p \vee t$

$= t$

Option (2)

$(p \wedge q) \wedge (p \vee q) = (p \wedge q)$ (Not a tautology)

Option (3)

$(p \wedge q) \vee (p \rightarrow q)$

$= (p \wedge q) \vee (\sim p \vee q)$

$= \sim p \vee q$ (Not a tautology)

Option (4)

$= (p \wedge q) \wedge (\sim p \vee q)$

$= p \wedge q$ (Not a tautology)

Option (1)

Q.9 **(2)**

LHS of all the options are some i.e.

$((P \rightarrow Q) \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge \sim Q$

$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$

$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \rightarrow Q$

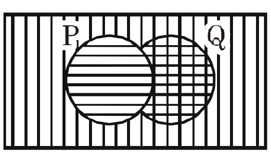
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$

$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \rightarrow \sim P$

$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$

$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \rightarrow P$

$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \rightarrow (P \wedge Q)$

$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	$P \vee Q$	$P \vee Q$	$\sim P$	$(P \vee Q) \vee \sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

Q.10 **(4)**

Q.11 (4)

Q.12 (3)

Q.13 (4)

Q.14 (1)

Q.15 (2)

Q.16 **(2)**

Q.17 (4)

p	q	~p	~q	p – q	$\sim(p \rightarrow q)$	$q \rightarrow p$	$\sim(q \rightarrow p)$
T	T	F	F	T	F	T	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	F	T	F

$p \wedge \sim q$	$\sim p \rightarrow \sim q$	$p \rightarrow \sim q$	$\sim (p \rightarrow \sim q)$
F	T	F	T
T	T	T	F
F	F	T	F
F	T	T	F

$p \wedge \sim q \equiv \sim (p \rightarrow q)$

Opation (4)

Q.18 **(1)**

Q.19 (3)

Q.20 (3)

Q.21 (3)

Q.22 (16)

Q.23 (1)

Q.24 **(3)**

EXERCISES

JEE-MAIN
OBJECTIVE PROBLEMS

Q.1 (1)

$P(n) : a^{2n-1} + b^{2n-1}$

$P(1) : a^1 + b^1 = a + b$, which is divisible by itself, i.e. by $(a + b)$.

$\therefore P(n) : a^{2n-1} + b^{2n-1}$ is divisible by $(a + b)$, and is true for $n = 1$

Let $P(k)$ be true, i.e. $P(k) : a^{2k-1} + b^{2k-1}$ is divisible by $(a + b)$

i.e. $a^{2k-1} + b^{2k-1} = m(a + b)$

Now,

$P(k + 1) = a^{2k+1} + b^{2k+1} = a^{2k-1} \cdot a^2 + b^{2k+1}$

$= a^2 [m (a + b) - b^{2k-1}] + b^{2k+1}$

$= m (a + b)a^2 - a^2b^{2k-1} + b^{2k+1}$

$= m(a + b)a^2 - b^{2k-1} (a^2 - b^2)$

$= m(a + b)a^2 - (a + b) (a - b) b^{2k-1}$

$= (a + b) [ma^2 - (a - b) b^{2k-1}]$

$\therefore P(k + 1)$ is divisible by $(a + b)$ whenever $P(k)$ is divisible by $(a + b)$.

Hence $P(n)$ is divisible by $(a + b)$ for all $n \in N$. **Ans.**

Q.2 (2)

$P(n) : (n + 1) (n + 2) .... (n + r)$

$P(1) : (2) (3) .... (r + 1) = r ! (r + 1)$, which is divisible by r !

Let $P(k) : (k + 1) (k + 2) .... (k + r) = r! (m)$

$\therefore P(k + 1) : (k + 2) (k + 3) .... (k + 1 + r) = r!(\lambda)$

L.H.S. of $P(k + 1)$

$= (k + 2) (k + 3) ... (k + r + 1)$

$$= \frac{(k+1)(k+2)(k+3)....(k+r+1)}{k+1}$$

$$= \frac{r!(m)(k+r+1)}{k+1} = r!(\lambda).$$

Thus, $P(k + 1)$ is divisible by r! whenever $P(k)$ is divisible by r!

Hence $P(n)$ is divisible by r! for all $n \in N$. **Ans.**

Q.3 (2)

$P(n) : 49^n + 16n - 1$

$P(1) : 49 + 16 - 1 = 64$, which is divsible by 64

Let $P(k) : 49^k + 16k - 1 = 64\,m$

$\therefore \quad P(k + 1) : 49^{k+1} + 16(k + 1) - 1 = 64\lambda$

L.H.S. of $P(k + 1) = 49^{k+1} + 16(k + 1) - 1$

$= 49 (64m - 16k + 1) + 16k + 16 - 1$

[Assuming P(k) to be true]

$= 64(49m) - 48(16k) + 64$

$= 64(49\,m - 12k + 1) = 64\lambda$

Thus, $P(k + 1)$ is divisible by 64 whenever $P(k)$ is divisible by 64.

Hence, $P(n)$ is divisible by 64. **Ans.**

Q.4 (3)

By Induction, $P(n)$ is true for all $n \in N$.

Q.5 (2)

$P(n) : \cos\alpha \cos 2\alpha \cos 4\alpha ..... \cos 2^{n-1}\alpha$

$$P(1) : \cos\alpha = \frac{\sin 2\alpha}{2\sin\alpha}$$

$$P(2) : \cos\alpha \cos 2\alpha = \frac{\sin 4\alpha}{4\sin\alpha}$$

Let $P(k) : \cos\alpha \cos 2\alpha \cos 4\alpha ..... \cos 2^{k-1}\alpha = \dfrac{\sin 2^k\alpha}{2^k \sin\alpha}$

$\therefore P(k + 1) : \cos\alpha \cos 2\alpha \cos 4\alpha .... \cos 2^k\alpha = \dfrac{\sin 2^{k+1}\alpha}{2^{k+1}\sin\alpha}$

L.H.S. of $P(k + 1)$

$= \cos\alpha \cos 2\alpha \cos 4\alpha ... \cos 2^k\alpha$

$$= \frac{\sin 2^k\alpha}{2^k \sin\alpha} \times \cos 2^k\alpha$$

[Assuming P(k) to be true]

$$= \frac{2\sin 2^k\alpha \cos 2^k\alpha}{2^{k+1}\sin\alpha} = \frac{\sin 2^{k+1}\alpha}{2^{k+1}\sin\alpha}$$

= R.H.S of $P(k + 1)$

Hence $P(n)$ holds true for all $n \in N$,. That is,

$$\cos\alpha \cos 2\alpha \cos 4\alpha .... \cos 2^{n-1}\alpha = \frac{\sin 2^n\alpha}{2^n \sin\alpha}.$$ **Ans.**

Q.6 (3)

For $n = 1$, $2^{3n} - 7n - 1 = 2^3 - 7 - 1 = 0$

For $n = 2$, $2^{3n} - 7n - 1 = 2^6 - 14 - 1 = 64 - 15 = 49$ which is divisible by 49. **Ans.**

Q.7 (1)

$f(n) = 10^n + 3 \cdot 4^{n+2} + k$

$f(1) = 10 + 3 \cdot 4^2 + k = 10 + 48 + k = 58 + k$

$= 9 \times 7 - 5 + k$

If f(1) is to be divisible by 9, then the least positive integral value of k has to be 5. **Ans.**

Q.8 (2)

$f(n) = 10^n + 3 \cdot 4^{n+2} + 5$

$f(1) = 10 + 48 + 5 = 63$, which is divisible by 7 and 3

$f(2) = 100 + 3(256) + 5 = 105 + 768 = 873$, which is divisible by 3.

So, $f(n) = 10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 3. **Ans.**

Q.9 (1)

Let $P(n) : x^n - 1 = \lambda(x - k)$

Now $P(1) : x - 1 = \lambda_1 (x - k)$

Also, $P(2) : x^2 - 1 = \lambda_2(x - k)$

$\Rightarrow P(2) : (x - 1)(x + 1) = \lambda_2(x - k)$

$\therefore$ The least value of k for which the proposition P(n) is true is k = 1. **Ans.**

Q.10 (2)

$$\text{Let } P(n) : \frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots \text{ (n terms)}$$

$$\Rightarrow P(n) : \sum \frac{1^3 + 2^3 + \ldots\ldots + n^3}{1+3+5+\ldots\ldots+(2n-1)}$$

$$\Rightarrow P(n) : \sum \left(\frac{\sum n^3}{n^2} \right)$$

$$\Rightarrow P(n) : \sum \left[\frac{1}{4} \frac{n^2(n+1)^2}{n^2} \right]$$

$$\Rightarrow P(n) : \frac{1}{4} \sum (n^2 + 2n + 1)$$

$$\Rightarrow P(n) : \frac{1}{4} \left[\sum n^2 + 2\sum n + \sum (1) \right]$$

$$\Rightarrow P(n) : \frac{1}{4} \left[\frac{n(n+1)}{2} + \frac{1}{3} n(n+1)(2n+1) + n \right]$$

$$\Rightarrow P(n) : \frac{1}{24} n \left[3(n+1) + 2(n+1)(2n+1) + 6 \right]$$

$$\therefore P(n) : \frac{1}{24} n \left(2n^2 + 9n + 13 \right). \textbf{ Ans.}$$

Q.11 (2)

$$\text{Let } P(n) = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$$

$$P(1) = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x} dx = \int_0^{\pi/2} \sin x \, dx = [-\cos x]_0^{\pi/2} = 1$$

$$P(2) = \int_0^{\pi/2} \frac{\sin^2 2x}{\sin x} dx = \int_0^{\pi/2} \frac{(2 \sin x \cos x)^2}{\sin x} dx$$

$$\Rightarrow P(2) = \int_0^{\pi/2} 4 \sin x \cos^2 x \, dx$$

$$\Rightarrow P(2) = 4 \left[\frac{-\cos^2 x}{3} \right]_0^{\pi/2} = \frac{4}{3} = 1 + \frac{1}{3}$$

$\therefore$ For any $n \in N$,

$$P(n) = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx = 1 + \frac{1}{3} + \frac{1}{5} + \ldots\ldots + \frac{1}{2n-1}.$$

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$P(n) = n^2 + 41$

$P(3) = 9 - 3 + 41 = 47$

$P(5) = 25 - 5 + 41 = 61$

Hence P (3) and P (5) are both prime

EXERCISES

JEE-MAIN
OBJECTIVE PROBLEMS

Q.1 (1)

Data x	Mean $\bar{x}$
$x = ap + bQ$	$\bar{x} = a\bar{p} \times b\bar{Q}$

Q.2 **(2)**

x_i	w_i	$x_i w_i$
1	1^2	1^3
2	2^2	2^3
3	3^2	3^3
⋮	⋮	⋮
n	n^2	n^3

$$\bar{x} = \frac{\sum xiwi}{\sum wi} = \frac{1^3 + 2^3 + 3^3 + ... + n^3}{1^2 + 2^2 + 3^2 + ... + n^2}$$

$$= \frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n(n+1)(2n+1)}{6}} = \frac{n^2(n+1)^2}{4} \times \frac{6}{n(n+1)}\ (2n+1)$$

$$= \frac{3n(n+1)}{2(2n+1)}$$

Q.3 (1)

$$\sum (x_i - \bar{x}) = \sum x_i - n\bar{x}$$

$$= n\bar{x} - \bar{x} \cdot n = 0$$

Q.4 **(3)**

x_i	f_i	$x_i f_i$
1	2	2
2	2	4
3	2	6
⋮	⋮	⋮
n	2	2n

$$\frac{\sum x_i f_i}{\sum f_i} = \frac{2 + 4 + 6 + ...2n}{2 + 2 + ...2}$$

$$= \frac{2(1 + 2 + 3 + ...n)}{2n} = \frac{2\frac{(n(n+1))}{2}}{2n} = \frac{n+1}{2}$$

Q.5 **(2)**

$P = P_1 . P_2 \ldots P_n$

Q.6 **(1)**

$$n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$

$$12 \times 6 = 6 \times 8 + 6 \times \bar{x}_2$$

$$\bar{x}_2 = \frac{72 - 48}{6} = \frac{24}{6} = 4$$

Q.7 **(4)**

According to question x_2 is replaced by t then

$$\bar{x} = \frac{n\bar{x} - x_2 + t}{n}$$

Q.8 **(4)**

Q.9 **(4)**

x_i	$(x_{(i+1)})x_i$
1	$(1 + 1)_1$
2	$(2 + 1)_2$
3	$(3 + 1)_3$
n	$(n + 1)_n$

$$\frac{\sum (x_i + 1)x_i}{n(n+1)} = \frac{2 + 6 + 12 + ...(n+1)^n}{n(n+1)}$$

Q.10 **(1)**

Arrange is accending order

$$\Rightarrow\ t - \frac{7}{2}, t - 3, t - \frac{5}{2}, t - 2, t - \frac{1}{2}, t + \frac{1}{2}, t + 4, t + 5$$

$$\Rightarrow\ \frac{1}{2}[4^{th} + 5^{th}\ \text{value}]$$

$$\Rightarrow\ \frac{1}{2}\left[2t - \frac{5}{2}\right]$$

$$\Rightarrow\ t - \frac{5}{4}$$

Q.11 **(4)**

Mode = 3 median – 2 Mean

121 = 3 median – 2 × 91

$$\frac{121+182}{3}=\frac{303}{3}=101$$

Q.12 **(1)**

x_i	S.D.(s)
$x_i \pm \lambda$	s
λx_i	$\lvert\lambda\rvert$ s
$\frac{x_i}{\lambda}$	$\frac{s}{\lvert\lambda\rvert}$

S.D of px + q is |p| s

Q.13 **(2)**

x_i	s
$x_i \pm \lambda$	s
$\lvert\lambda\rvert x_i$	$\lvert\lambda\rvert s$
$\frac{x_i}{\lvert\lambda\rvert}$	$\frac{s}{\lvert\lambda\rvert}$

S.D. of $\frac{a_x+b}{c}$ is $\left|\frac{a}{c}\right|s$

Q.14 **(3)**

$$\sigma=\frac{\sum f_i x_i^2}{\Sigma f_i}-\left(\frac{\sum f_i x_i}{\Sigma f_i}\right)^2$$

Q.15 **(2)**

$$n\overline{x}=n_1\overline{x}_1+n_2\overline{x}_2$$

$$=n_1\frac{k}{n_1}+n_2$$

$$\boxed{n_2=n\overline{x}-K}$$

Q.16 **(4)**

x_i	$\overline{x}$
$\frac{x_i}{\lambda}$	$\frac{\overline{x}}{\lambda}$

then new mean after each number is divided by 3 is $\frac{\overline{x}}{3}$

Q.17 **(3)**

x_i	w_i	$x_i w_i$
0	0	0
1	1	1^2
2	2	2^2
3	3	3^2
4	4	4^2
⋮	⋮	⋮
n	n	n^2

$$\frac{\sum x_i w_i}{\sum w_i}=\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}=\boxed{\frac{2n+1}{3}}$$

Q.18 **(2)**

A.M. = of 1 + 2 + 4 + 8 + 16 +...... 2^n

$$=\frac{2^{n+1}-1}{n+1}$$

Q.19 **(1)**

In central tendency we measure mean, mode, median.

Q.20 **(1)**

Most stable measure of central tendency is mean.

Q.21 **(3)**

x_i	f_i
1	1
2	1
3	1
:	:
n	1

$$\overline{x}=\frac{\sum f_i x_i}{\sum f_i}=\frac{1+2+3+....n}{n}=\frac{n(n+1)}{2n}=\left(\frac{n+1}{2}\right)$$

Q.22 **(3)**

$$n\overline{x}=n_1\overline{x}_1+n_2\overline{x}_2$$

$$10\overline{x}=7\times10+3\times5$$

$$\overline{x}=\frac{70+15}{10}=\frac{85}{10}=8.5$$

Q.23 **(3)**

A statistical measure which can not be determined graphically is harmonic mean it is a fandomental concept.

Q.24 **(1)**

The measure which takes into account all the data item is mean it is a fandamental concept of account

Q.25 **(3)**

$$\overline{x}=\frac{\Sigma x}{n}\Rightarrow \Sigma x=n\overline{x}$$

= 15 × 154 = 2310

$\Sigma x = 2310 - 145 + 175$
$= 2340$

correct mean = $\frac{2340}{15} = 156$ c.m.

Q.26 **(2)**
For median arrange
scored in order
0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

Median is $\left(\frac{n+1}{2}\right)^{th}$ term

$\frac{11+1}{2}$ = 6^{th} term = 27.

Q.27 **(1)**
Total $\Rightarrow \Sigma x = n\bar{x} = 10 \times 12.5 = 125$
First six $\Rightarrow \Sigma x = n\bar{x} = 6 \times 15 = 90$
Last five $\Rightarrow \Sigma x = n\bar{x} = 5 \times 10 = 50$
Last four $125 - 90 = 35$
6^{th} no is $50 - 35 = 15$

Q.28 **(1)**
$\Sigma x = n\bar{x} = 100 \times 50 = 5000$

S.D. = $\sqrt{\sigma^2}$

$4 = \sqrt{\sigma^2}$

$= \sqrt{\frac{1}{n}\Sigma x_i^2 - \bar{x}^2}$

$= \sqrt{\frac{\Sigma x^2}{100} - (50)^2}$

$16 = \frac{\Sigma x_i^2}{100} - 2500$

$(16 + 2500) \cdot 100 = \Sigma x_i^2$

$\boxed{251600 = \Sigma x_i^2}$

Q.29 **(1)**

S.D. = $\sqrt{\frac{1}{N}\Sigma x_i^2 - \bar{x}^2}$

x_i	f_i
1	${}^nC_0$
a	${}^nC_1$
a^2	${}^nC_2$
⋮	⋮
a^n	${}^nC_n$

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{{}^nC_0 + {}^{an}C_1 + {}^{a^2\,n}C_2 + {}^{a^n\,n}C_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}$$

$$\frac{\Sigma f_i x_i^2}{N} = \frac{{}^nC_0 + {}^{a^2\,n}C_1 + {}^{a^4\,n}C_2 + {}^{a^6\,n}C_3 + ... a^{2n\,n}C_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}$$

Q.30 **(1)**

AM = $\frac{a+b}{2} = 10$

G.M. = $\sqrt{ab} = 8$

H.M = $\frac{2ab}{a+b}$ = ?

H.M. = $\frac{(G.M.)^2}{A.M.}$

$= \frac{64}{10} = 6.4$

And number are 16, 4

Q.31 **(3)**
$n_1 = 100$
$n_2 = 150$
$\bar{x}_1 = 50$
$\bar{x}_2 = 110$
$\sigma_1^2 = 5$
$\sigma_2^2 = 6$
$n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$
$= 100 \times 50 + 150 \times 40$
$= 5000 + 6000$

$\bar{x} = \frac{11000}{250} = 44$

$\sigma^2 = n_1 \frac{(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$

$d_1 = 50 - 44 = 6$
$d_2 = 40 - 44 = -4$

$\sigma^2 = 100\frac{(25+36)+150(36+16)}{250}$

$= \frac{6100+7800}{250} = 55.6$

$\sigma = \sqrt{55.6} = 7.46$

Q.32 **(1)**
$CV_1 = 58\%$
$CV_2 = 69\%$
$\sigma_1 = 21.2$

$\sigma_1 = 15.6$

$CV = \frac{\sigma}{x} \times 100$

$CV_1 = \frac{\sigma_1}{x_1} \times 100 \Rightarrow \bar{x}_1 = \frac{\sigma_1 \times 100}{CV_1} = \frac{21.2 \times 100}{58} =$

$\frac{2120}{58} = 36.55$

$CV_2 = \frac{\sigma_2}{x_2} \times 100 \Rightarrow \bar{x}_2 = \frac{\sigma_2 \times 100}{CV_2} = \frac{15.6 \times 100}{69} =$

22.60

Q.33 **(3)**

n = 10

$\bar{x} = 12$

$\Sigma x^2 = 1530$

$\sigma^2 = \frac{1}{n}\Sigma\left(x_1^2 - \bar{x}^2\right)$

$\sigma^2 = \frac{1}{10}$ [1530 – 10(144)] = $\frac{90}{10} = 9$

$\sigma = 3$

$\bar{x} = 12$

C.O.V. = $\frac{\sigma}{x} \times 100 = \frac{3}{12} \times 100 = 25\%$

Q.34 **(1)**

$AM = \frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}{n+1}$

$= \frac{2^n}{n+1}$

Q.35 **(3)**

$\bar{x}_1 = 50$	$\sigma_1^2 = 15$
$\bar{x}_2 = 48$	$\sigma_2^2 = 12$
$\bar{x}_3 = 12$	$\sigma_3^2 = 2$

Most consistant is kapil

Q.36 **(2)**

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{\Sigma(x_i + 2i)}{n} = \frac{\Sigma x_i}{n} + \frac{2\Sigma i}{n} = \bar{x} + \frac{2n(n+1)}{2n}$

$= \bar{x} + (n+1)$

Q.37 **(2)**

$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$= \frac{1^2 + 2^2 + 3^2 + ... + n^2}{n} - \left(\frac{1+2+3+....+n}{n}\right)^2$

$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$

$= \frac{(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4n^2}$

$= \frac{n^2 - 1}{12}$

Q.38 **(1)**

$\bar{x} = \frac{\Sigma x}{n} \Rightarrow M = \frac{\Sigma x}{n} \Rightarrow \Sigma x = nM$

sum of n – 4 observations is a

mean of remaing 4 observation is $\frac{nM - a}{4}$

Q.39 **(3)**

Mean of series is

$\bar{x} = \frac{a + (a+d) + (a+2d) + ... + (a+2nd)}{(2n+1)}$

$\bar{x} = a + nd$

$\therefore \sum_{i=0}^{2n} |x_i - \bar{x}| \Rightarrow \frac{2d(n)(n+1)}{2}$

$\Rightarrow$ n(n + 1)d

$\therefore$ Mean deviation = $\frac{n(n+1)d}{(2n+1)}$

Q.40 **(3)**

$\bar{x} = \frac{\Sigma x_i}{n}$

$= \frac{x_1 + 1 + x_2 + 2 + ...}{n}$

$= \frac{x_1 + x_1 + ... + x_n}{n} + \frac{1 + 2 + ...n}{n} = \bar{x} + \frac{n(n+1)}{2n}$

$= \bar{x} + \left(\frac{n+1}{2}\right)$

Q.41 **(4)**

Quartile deviation = $\frac{\theta_3 - \theta_1}{2} = \frac{40 - 20}{2} = 10$

Q.42 **(1)**

x_i	S.D.
$x_i \pm \lambda$	s
λx_i	$\lvert\lambda\rvert s$
$\frac{x_i}{\lambda}$	$\frac{s}{\lvert\lambda\rvert}$

then S.D. of ax + b is |a| s

where s is staindered deviation.

Q.43 **(1)**

r = range

S.D. = $S^2 = \frac{1}{n-1}\sum_{i=0}^{n}(x_i-\overline{x})^2$ then $S \le r\sqrt{\frac{n}{n-1}}$

Q.44 **(3)**

If x_1, x_2 .... x_n are n observations with frequencies f_1, f_2 .... f_n, then mean deviation from mean (m) is given by

Mean deviation = $\frac{1}{N}\Sigma f_i\,|x_i - M|$

Q.45 **(4)**

x_i	σ
x	4
$\frac{x}{4}$	$\frac{4}{\lvert 4 \rvert}=1$

KVPY
PREVIOUS YEAR'S

Q.1 (A)
Q.2 (B)
Q.3 (B)
Q.4 **(C)**

Let $x_1 < x_2 < x_3 \ldots\ldots x_{11}$

median of $x_1, x_2 \ldots\ldots x_{10}$ is $\frac{x_5+x_6}{2}$

Now the new set of number are $x_1, x_2, \ldots\ldots x_5$

$\frac{x_5+x_6}{2}, x_6, \ldots\ldots x_{10}$

Hence median is $\frac{x_5+x_6}{2} < x_6 \Rightarrow$ median decreases

JEE MAIN
PREVIOUS YEAR

Q.1 (11)

$$\sigma^2 = \frac{\Sigma x^2}{n} = \left(\frac{\Sigma x}{n}\right)^2$$

$$\sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k^2}{10}\right)^2 < 10$$

$(90 + k^2)10 - (81 + k^2 + 8k) < 1000$
$90 + 10k^2 - k^2 - 18k - 81 < 1000$
$9k^2 - 18k + 9 < 1000$

$(k-1)^2 < \frac{100}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

Q.2 (4)

$\Sigma x_i - 18\alpha = 36$

$\Sigma x_i = 18(\alpha+2)$ ...(i)

$\Sigma x_i^2 + 18\beta^2 - 2\beta\Sigma x_i = 90$

$\Sigma x_i^2 + 18\beta^2 - 2\beta\times 18(\alpha+2) = 90$

$\Sigma x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$ ....(ii)

$\sigma^2 = 1 \Rightarrow \frac{1}{18}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{18}\right)^2 = 1$

$\Rightarrow \frac{1}{18}\left(90 - 18\beta^2 + 36\alpha\beta + 72\beta\right) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\alpha\beta + 72\beta - 18(\alpha+2)^2 = 18$
$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (\alpha+2)^2 = 1$
$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4 - 4\alpha = 1$
$-\alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$
$-(\alpha-\beta)^2 - 4(\alpha-\beta) = 0$
$-(\alpha-\beta)(\alpha-\beta+4) = 0$
$\Rightarrow \alpha - \beta = -4 \quad (\alpha \ne \beta)$
$|\beta - \alpha| = 4$

Q.3 **(4)**

For a, b, c

mean = $\frac{a+b+c}{3}$ $(=\overline{x})$

b = a + c

$\Rightarrow \overline{x} = \frac{2b}{3}$ ......(1)

S.D. (a + 2, b + 2, c + 2) = S.D. (a, b, c) = d

$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - (\overline{x})^2$

$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - \frac{4b^2}{9}$

$\Rightarrow 9d^2 = 3(a^2+b^2+c^2)$? $4b^2$

$\Rightarrow b^2 = 3(a^2+c^2)$? $9d^2$

Q.4 **(5)**

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{(n_1+n_2)}(\overline{x}_1 - \overline{x}_2)^2$$

$n_1 = 10,\ n_2 = n,\ \sigma_1^2 = 2,\ \sigma_2^2 = 1$

$\overline{x}_1 = 2,\ \overline{x}_2 = 3,\ \sigma^2 = \frac{17}{9}$

$$\frac{17}{9} = \frac{10\times 2 + n}{n+10} + \frac{10n}{(n+10)^2}(3-2)^2$$

$\Rightarrow \frac{17}{9} = \frac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340\ n = 90n + 9(n^2+30n+200)$

$\Rightarrow \quad 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow \quad (2n + 5)(n - 5) = 0 \Rightarrow n = \underset{\downarrow}{\frac{-5}{2}}, 5$

(Rejected)

Hence n = 5

Q.5 **(35)**

$\frac{\sum x_i}{25} = 40\ \&\ \frac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

Q.6 **(1)**

Let observations are denoted by x_i for $1 \le i < 2n$

$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a+a+...+a)-(a+a+...+a)}{2n}$

and $\sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+...+a^2}{2n} - 0 = a^2$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y} = \bar{x} + b = 5$

$\Rightarrow b = 5$

and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$

Q.7 **(4)**

Q.8 (164)

Q.9 (3)

Q.10 (4)

Q.11 (3)

Q.12 (1)

Q.13 **(4)**

Q.14 **(3)**

Q.15 [398]

Q.16 **(12)**

Q.17 (4)

Given :

Mean $= (\bar{x}) = \frac{\sum x_i}{20} = 10$

or $\sum x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \sum x_i$ (Correct)

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given S.D. = 2.5 (σ)

$\sigma^2 = \frac{\sum x_i^2}{20} - (10)^2 = (2.5)^2$

or $\sum x_i^2 = 2125$ (incorrect)

or $\sum x_i^2 = 2125 - 25^2 + 35^2$

$= 2725$ (correct)

$\therefore$ correct $\sigma^2 = \frac{2725}{20} - (10.5)^2$

$\underline{\sigma}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

Q.18 (40)

Q.19 **[100]**